Explaining changes in transaction volumes in Hungarian settlements during the crisis

Áron Horváth†, Aliz McLean‡

May 15, 2013

Abstract

In this paper, we analyse one slice of the 2008 economic crisis in the Hungarian residential real estate market. We show that less frequent offers can qualitatively explain some of the observed phenomena. The decreasing volume of transactions and the fall in transaction prices are explained in an optimal stopping framework, where less frequent offers force sellers to decrease their reservation price. Our empirical data reveals the connection between settlement size and transaction volumes. Post-crisis adjustment in transaction volumes is different in smaller settlements than in larger ones: there appears to be a smaller relative decrease in transaction volumes in smaller settlements. The unexpected negative relationship between size and the change in volume is robust to the inclusion of controls (e.g. for NUTS 2 region, or the distance from the nearest larger town). A possible explanation is the following. The fact that transaction volumes fall after a crisis means that sellers will receive fewer offers over a given space of time. In smaller settlements, offers are generally less frequent than in larger settlements. Therefore, in a post-crisis smaller settlement, if an offer does come along, sellers will be likely to accept it even if it is low - since they anticipate that this offer will be the only one they receive for some time. Sellers in larger settlements, however, may still find it worth their while to hold out and wait for a better offer. This means that while post-crisis adjustment in smaller settlements happens less in volume than in price, the converse is true of larger settlements. The above argument fits into an optimal stopping framework, in which sellers receive differing offers at various intervals (each offer only being available for a limited amount of time), and must decide which offer to accept. We therefore address our puzzle using an optimal stopping model.

1 Introduction

The 2008 economic crisis had a profound impact on the real estate market in Hungary, including (though not limited to) the residential market segment. In this paper, we turn our attention to one puzzling aspect of the impact, and investigate it in an optimal stopping framework.

We begin by underpinning the observation that post-crisis adjustment transaction volumes was different in smaller settlements than in larger ones. Our data

*Preliminary version, please do not quote without authors’ consent.
†ELTIGA Centre for Real Estate Research
‡ELTIGA Centre for Real Estate Research, corresponding author: mcleana@eltinga.hu
shows, however, that contrary to popular belief, transaction volumes decreased relatively less in smaller settlements than in larger ones. This result is robust to the inclusion of various controls, various econometric specifications, and even differing approaches to measuring the dependent variable.

A possible explanation is the following. The fact that transaction volumes fall after a crisis means that sellers will receive fewer offers over a given space of time. In smaller settlements, offers are generally less frequent than in larger settlements. Therefore, in a post-crisis smaller settlement, if an offer does come along, sellers will be likely to accept it even if it is low - since they anticipate that this offer will be the only one they receive for some time. Sellers in larger settlements, however, may still find it worth their while to hold out and wait for a better offer. This means that while post-crisis adjustment in smaller settlements happens less in volume than in price, the converse is true of larger settlements. The above argument fits into an optimal stopping framework, in which sellers receive differing offers at various intervals (each offer only being available for a limited amount of time), and must decide which offer to accept. We therefore address our puzzle using an optimal stopping model, based on the well-known "house-selling problem".

"Optimal stopping" entails choosing a point in time to take a given action to maximize expected payoffs. The decision is based on random variables observed sequentially. In our case, a person attempting to sell their house is the decision-maker, who receives various offers for her house. According to Ferguson (2013), the house-selling problem was first introduced by MacQueen and Miller (1960), Derman and Sacks (1960), Chow and Robbins (1961) and Karlin (1962) in the case when the seller can go back and accept offers from previous time periods. The case where there is no such possibility (an offer once rejected is lost forever) was described by the aforementioned Chow and Robbins (1961). As noted in Ferguson (2013), the house-selling problem shares much with the job search problem in economics, in which a worker is searching for a job, and must decide whether to accept a current offer or wait for another. This problem is attributed to George Stigler (1961) and (1962).

The structure of the paper is the following. Section 2 introduces the empirical results we have reached, first introducing Hungary’s settlement structure and then focusing on transaction volumes. Section 3 shows the essentials of our theoretical model. Section 4 concludes.

2 Some empirical observations on the Hungarian residential housing market

In this section, we present and describe our data and calculate some descriptive statistics, focusing on the relationship between settlement size and transaction volumes. We first introduce the very basic facts regarding settlements in Hungary. We then move on to transaction volumes and how they changed in response to the 2008 economic crisis.

2.1 Hungary’s settlement structure

There are 3152 settlements in Hungary altogether for a population of around 10 million. The country is divided into 7 regions (NUTS 2 according to the European

---

1While the number of settlements changes very slightly over time due to settlements joining and breaking up, these changes are not relevant to our investigation, and we can therefore ignore them.
Union’s statistical classification), 19 counties (in addition to the capital city, NUTS 3), and 174 micro-regions (NUTS 4). Budapest is by far the largest settlement, with a population of over 1.7 million people, and a housing stock close to 900,000. The runner-up is county centre Debrecen with its population of just over 200 thousand people in around 90 thousand dwellings - a mere tenth of Budapest’s. In the remainder of this paper, we will focus on housing stock as the descriptor of size.

Table 1 shows the number of settlements in various size categories. 61.6% of settlement fall into the smallest category, and have less than 501 dwellings, and 96.5% have at most 5000. We draw the line between "small" and "large" settlements at 5000 dwellings.

<table>
<thead>
<tr>
<th>Housing stock (number of dwellings)</th>
<th>Number of settlements</th>
<th>Proportion of settlements</th>
</tr>
</thead>
<tbody>
<tr>
<td>500 or less</td>
<td>1943</td>
<td>61.6%</td>
</tr>
<tr>
<td>501-1000</td>
<td>609</td>
<td>19.3%</td>
</tr>
<tr>
<td>1001-5000</td>
<td>491</td>
<td>15.6%</td>
</tr>
<tr>
<td>5001-20000</td>
<td>88</td>
<td>2.8%</td>
</tr>
<tr>
<td>20001 or more</td>
<td>21</td>
<td>0.7%</td>
</tr>
<tr>
<td>Sum</td>
<td>3152</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 2 shows the distribution of the housing stock by region. Regions are relatively well-balanced in terms of housing stock with the exception of Central Hungary, which contains the capital city Budapest and thus contains a disproportionate number of dwellings.

<table>
<thead>
<tr>
<th>Region</th>
<th>Housing stock</th>
<th>Proportion of housing stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Southern Great Plain</td>
<td>595 138</td>
<td>14%</td>
</tr>
<tr>
<td>Southern Transdanubia</td>
<td>402 414</td>
<td>9%</td>
</tr>
<tr>
<td>Northern Hungary</td>
<td>514 452</td>
<td>12%</td>
</tr>
<tr>
<td>Central Hungary</td>
<td>1 332 512</td>
<td>31%</td>
</tr>
<tr>
<td>Central Transdanubia</td>
<td>440 872</td>
<td>10%</td>
</tr>
<tr>
<td>Western Transdanubia</td>
<td>425 368</td>
<td>10%</td>
</tr>
<tr>
<td>Northern Great Plain</td>
<td>619 925</td>
<td>14%</td>
</tr>
<tr>
<td>Sum</td>
<td>4 330 681</td>
<td>100%</td>
</tr>
</tbody>
</table>

We now turn our attention to our primary focus, transaction volumes in the settlements.
2.2 Transaction volumes

The housing market in Hungary was booming before the crisis. By 2008, prices had doubled compared to 2000, the equivalent to a 30% increase in real terms. By the third quarter of 2010, however, prices had plummeted to a level lower than in 2000 in real terms, and have continued to fall since.

Transaction volumes also reflect the changes wrought by the crisis. To track these changes, we will analyse data on the number of transactions completed in each settlement in Hungary, for each year from 2007 to 2011. We define 2007 and 2008 as "pre-crisis" and 2009 to 2011 as "crisis" years. Data on settlement size is also available, both for the housing stock (number of dwellings) and the population. Our data on housing stock is from 2009, and we will assume that it remains constant over the period under investigation.

Based on our sources, we first calculate the mean and median number of transactions per year per settlement pre- and post-crisis. The results can be seen in Table 3.

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Mean</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-crisis (2007-2008)</td>
<td>32</td>
<td>8%</td>
</tr>
<tr>
<td>Crisis (2009-2011)</td>
<td>18</td>
<td>6%</td>
</tr>
</tbody>
</table>

Two things are apparent from Table 3. Firstly, that the number of transactions per year decreased dramatically following the outbreak of the crisis. We will investigate this relationship further in this section. Secondly, that the median is very much smaller than the mean. This is a natural result, since the distribution of transactions, similarly to the distribution of settlement size, is strongly skewed to the left.

Table 4 gives an overview of how transaction volumes have changed year by year. The dramatic drop post-2008 is immediately clear: volumes dropped over 40%.

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of transactions</th>
<th>Number of settlements where transactions took place</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>191 170</td>
<td>2633</td>
</tr>
<tr>
<td>2008</td>
<td>154 097</td>
<td>3010</td>
</tr>
<tr>
<td>2009</td>
<td>91 137</td>
<td>2964</td>
</tr>
<tr>
<td>2010</td>
<td>90 271</td>
<td>2890</td>
</tr>
<tr>
<td>2011</td>
<td>87 730</td>
<td>2905</td>
</tr>
</tbody>
</table>

It is worth looking at the fraction of dwellings traded compared to the total number of dwellings in each settlement: this results in comparable numbers across settlements. Figure 1 shows these numbers for five settlement size categories, for

---

2Data for 2012 was still incomplete at the time of writing, and we therefore omit it.

3Housing stock does, of course, change from one year to the next. However, the change is minor (especially post-crisis), and would have no important effect on our results.
2008 and 2011, and demonstrates that larger settlements have more active housing markets: a larger proportion of dwellings is traded each year. This is true both before and during the crisis, the difference is only one of magnitude, and again reflects the dramatic drop in transaction volumes due to the crisis.

We shall now focus on calculating the change in transaction volumes before and during the crisis. Since our central theme is the relationship of transaction volumes and settlement size, we are interested in relative changes in transaction volumes. Consider the following two ways for calculating a relative change for each settlement:

1. The difference in the average number of transactions per year before and during the crisis, divided by the number of transactions pre-crisis.

$$\Delta T_{Relative} = \frac{T_{during} - T_{before}}{T_{before}}$$

2. The difference in the average number of transactions per year before and during the crisis, divided by housing stock.

$$\Delta T_{Housing} = \frac{T_{during} - T_{before}}{Housing}$$

The first case is a classical relative change, which eliminates units of measurement. The second controls for size more explicitly by measuring the change in the proportions of dwellings traded before and during the crisis. Arguments can be made in favour of using either, and we will make use of both. However, we generally prefer the second case.

To investigate how the drop in transactions was distributed among smaller and larger settlements, we compare the average yearly transaction volume between 2009-2011 with that between 2007-2008, using the \((T_{during} - T_{before})/Housing\) method of calculating changes, for each settlement. Figure 2 shows the results for various settlement sizes.

It is apparent that a larger housing stock implied a larger relative change in transaction volumes, that is: transactions volumes decreased less, relatively, in smaller settlements than in larger ones. The cut-off point between "small" and "large" settlements can be drawn at around 5000 dwellings. The next step is to verify that this relationship is robust to the inclusion of controls. Several possible controls may come to mind. As location is always key in real estate, firstly it makes sense to
introduce controls with regard to it. We control for "absolute" location by introducing dummy variables to represent the seven NUTS 2 regions of Hungary. "Relative" location may also be important, and we control for it by including the distance (in kilometres) from the nearest centre of a micro-region (NUTS 4), which essentially corresponds to the nearest town where most services are available.

The exact results of regressions are not especially important and we will not interpret them numerically. Prices and transaction volumes influence one another, and their complex relationship is one of the main themes of this paper. We make no claim that price were exogenous in such regressions. However, is it noteworthy that in every specification of the regressions, our main explanatory variable, housing stock, is significant and has a negative sign. Table 5 shows some of the specifications and results.

### Table 5: Regression results

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>$\Delta T_{\text{Housing}}$</th>
<th>$\Delta T_{\text{Housing}}$</th>
<th>$\Delta T_{\text{Relative}}$</th>
<th>$\Delta T_{\text{Relative}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing stock</td>
<td>$-5.12 \times 10^{-7}***$</td>
<td>$-4.32 \times 10^{-7}***$</td>
<td>$-1.19 \times 10^{-5}***$</td>
<td>$-8.97 \times 10^{-6}***$</td>
</tr>
<tr>
<td>Region dummies</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Distance to nearest centre of a micro-region</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Difference in average price</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Number of observations</td>
<td>3140</td>
<td>2285</td>
<td>2987</td>
<td>2283</td>
</tr>
<tr>
<td>R-squared</td>
<td>10.97%</td>
<td>15.51%</td>
<td>11.01%</td>
<td>14.71%</td>
</tr>
</tbody>
</table>

Significance levels: *: 10%, **: 5%, ***: 1%

The crisis, essentially a demand-side shock, implies that some adjustments in price and quantity must take place. If the crisis affected each settlement in the same way (empirically, of course, this is not so), it would make sense that if adjustment happens to a lesser extent in quantity (that is, in smaller settlements), it could happen more in price.

According to our data, however, there is no robust relationship between settlement size and price changes. Price adjustments, it appears, differ strongly by
location, but in contrast to transaction volumes, not by settlement size. Table 6 shows the correlation coefficient between the number of dwellings and the relative change in average price in the settlement between 2009-2011 and 2007-2008.

<table>
<thead>
<tr>
<th>Table 6: Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation coefficient Settlemen t size (number of dwellings)</td>
</tr>
<tr>
<td>Difference in average price</td>
</tr>
</tbody>
</table>

Measuring change in price and creating price indices is of course a complex issue in itself, especially when there are is not a sufficient number of transactions in order to reach reliable estimates. Several methods exits, but this is not the topic of this paper. The numbers we use below are "raw" price differences, which implies they are susceptible to composition bias, that is, that the composition (in terms of size, location and type, for example) of dwellings traded has changed in the wake of the outbreak of the crisis.

The basic statistics shown above served as inspiration for the theoretical part of this paper.

3 Optimal stopping: a possible modelling approach

3.1 The house-selling problem

The basic problem is the following. The owner of a house wishes to sell his property. Offers are made to him, and following each offer, the owner decides whether to accept (1) or reject (0) the offer. This decision is final, that is, if an offer has been rejected, then it can never be recalled. The distribution of the offers is known to the seller. In our case, this distribution will be a uniform distribution with a given minimum \(a\) and maximum \(b\). There is, however, a cost associated with waiting for a further offer. This cost will be expressed through a discount factor. Theoretically, the seller may wait forever - there are an infinite number of periods. However, unless he sells, the house is worth nothing.

Solving the house-selling problem entails finding the owner’s optimal stopping rule. Specifically, we will calculate the owner’s reservation price (the smallest offer that he will accept) and the expected time of his decision to accept (how many periods he waits, in expected value, before receiving an offer which reaches his reservation price).

The size of settlements enters into picture in the form of the frequency of offers: in smaller settlements, offer arrive less frequently than in larger settlements. The fact that offers arrive at different frequencies for different settlements means that we must be very explicit about what we mean by a "time period". Consider the following definition: a period is defined from the point of view of the seller. For each seller, an offer arrives each period. If offers arrive once a year, then the time period for the given seller is one year, if they arrive monthly, then a period is one month long.
3.2 Deriving the reservation price

First, we will solve for the reservation price. Price will be denoted by $P$, the reservation price by $P^*$. There is also a discount factor. Traditionally, a yearly discount factor is used in the form $1/(1 + r)$, and this is what we will use here. This entails making the implicit assumption that the period for this particular seller is one year. For the sake of simplicity, for now we can make this assumption, but it is by no means necessary. In theory, $r$ could be defined to mean a discount rate for any length of time.

Analytically, the solution looks like this. The Bellman equation of the problem is the following:

$$V(P) = \max_{(b,1)} \left[ 0 + \frac{1}{1 + r} EV(P'|P), P \right] \tag{3}$$

This equation defines $P^*$, the reservation price. We can find $P^*$ from the following equation:

$$\frac{1}{1 + r} EV(P'|P^*) = P^* \tag{4}$$

Since we have assumed that the offer are drawn from a uniform distribution with parameters $(a,b)$, $EV(P'|P^*)$ can be expressed in the following way:

$$(1 + r)P^* = EV(P'|P^*) = P^* \frac{P^* - a}{b - a} + \frac{b - P^*}{b - a} P^* + \frac{b}{2} \tag{5}$$

This leads to the following quadratic equation for $P^*$:

$$P^{*2} - 2((1 + r)b - ra)P^* + b^2 = 0 \tag{6}$$

The solutions are:

$$P^{*1,2} = ((1 + r)b - ra) \pm ((1 + r)b - ra)^2 - b^2)^{\frac{1}{2}} \tag{7}$$

One of these solutions will fall outside the $[a,b]$ interval, but the other, the reservation price we seek, will fall within it. To show how this works, let us look at a few numerical examples. The simplest case is, when in a uniform distribution, $a = 0$ and $b = 1$. Further, we will make the assumption that $r = 0.1$, that is, 10%. Then the solutions are the following:

$$P^*_1 = 1.56 \tag{8}$$
$$P^*_2 = 0.64 \tag{9}$$

Of these, $P^*_2$ is relevant. If we shift $b$, the maximum upward to 2, leaving all else unchanged, the solutions are:

$$P^*_1 = 3.12 \tag{10}$$
$$P^*_2 = 1.28 \tag{11}$$

Of these, $P^*_2$ is possible. It is, in fact, exactly twice the result we got previously, where $b = 1$. Equation 7 explains this: the results react linearly to changes in $b$, as long as $a = 0$. Now let us shift the distribution to offers between $a = 1$ and $b = 2$, that is, the spread is once again one, and $r$ is unchanged at 0.1. The solutions are
the following:

\[ P^*_1 = 2.74 \quad (12) \]
\[ P^*_2 = 1.46 \quad (13) \]

Of these, \( P^*_1 \) is possible. Once both \( a \) and \( b \) are larger than 0, results do not react in a linear way to changes in the parameters. Even without looking at Equation 7 the reason is intuitively clear, and lies in the existence of a discount factor. If \( r \) is the same, but the minimum and maximum price shift upward, there is more to lose by waiting another period. It is interesting to note that the result is smaller than the simple expected value of the offer.

The analytical calculations presented here can be confirmed by simple simulations.

As discussed above, these results do not rely on periods being one year long. The important element is that the discount factor relate to the period at hand. If the yearly discount rate is \( r \) and the period is \( k \) years long, then the appropriate discount rate is \( (1 + r)^k - 1 \). This means, for example, that the monthly discount rate (where \( k = 1/12 \)) is \( (1 + r)^{1/12} - 1 \). The following equation shows the solutions in this form:

\[
P^*_{1,2} = ((1 + r)^k b - ((1 + r)^k - 1)a) \pm ((1 + r)^k b - ((1 + r)^k - 1)a - b^2)^{1/2} \quad (14)\]

### 3.3 Deriving the expected time of stopping

The seller accepts the offer, i.e. stops, when the offer reaches his reservation price. Therefore, the probability that he stops in the first period is simply

\[
\frac{b - P^*}{b - a} \quad (15)
\]

The probability that he stops in the second period is the product of two probabilities: the probability that he did not stop in the first period, and the probability that he will stop in the second:

\[
\frac{P^* - a}{b - a} \frac{b - P^*}{b - a} \quad (16)
\]

For the next periods, the situation is similar. The owner stops in the \( n \)th period, for example, with the following probability.

\[
\left( \frac{P^* - a}{b - a} \right)^{n-1} \frac{b - P^*}{b - a} \quad (17)
\]

It is simple to verify that the sum of the probabilities equals 1:
\[
\frac{b - P^*}{b - a} \sum_{i=1}^{\infty} 1 + \frac{P^* - a}{b - a} + \left( \frac{P^* - a}{b - a} \right)^2 + ... =
\]
\[= \frac{b - P^*}{b - a} \frac{1}{1 - \frac{P^* - a}{b - a}} =
\]
\[= \frac{b - P^*}{b - a} \frac{b - a}{b - P^*} =
\]
\[= 1
\]

The expected period of stopping (denoted by \(T\)) can then be calculated from the formula for expected value.

\[
E(T) = \frac{b - P^*}{b - a} + 2 \left( \frac{P^* - a}{b - a} \right) \frac{b - P^*}{b - a} + ... + n \left( \frac{P^* - a}{b - a} \right)^{n-1} \frac{b - P^*}{b - a} + ... =
\]
\[= \frac{b - P^*}{b - a} * S,
\]

where \(S\) is defined as

\[S = 1 + 2 \left( \frac{P^* - a}{b - a} \right) + ... + n \left( \frac{P^* - a}{b - a} \right)^{n-1} + ...
\]

To gain the simple solution to \(E(T)\), we multiply \(S\) by \((P^* - a)/(b - a)\):

\[S \left( \frac{P^* - a}{b - a} \right) = \left( \frac{P^* - a}{b - a} \right) + 2 \left( \frac{P^* - a}{b - a} \right)^2 + ... + n \left( \frac{P^* - a}{b - a} \right)^n + ...
\]

and then subtracting Equation 25 from 24 results in:

\[S - S \left( \frac{P^* - a}{b - a} \right) = 1 + \left( \frac{P^* - a}{b - a} \right) + ... + \left( \frac{P^* - a}{b - a} \right)^{n-1} + ...
\]

which simplifies to

\[S \left( 1 - \frac{P^* - a}{b - a} \right) = \frac{1}{1 - \frac{P^* - a}{b - a}}.
\]

This can also be written as

\[S \left( \frac{b - P^*}{b - a} \right) = \frac{b - a}{b - P^*}.
\]

Therefore, \(S\) can be expressed as

\[S = \left( \frac{b - a}{b - P^*} \right)^2.
\]

This means that the expected period of stopping is

\[E(T) = \frac{b - P^*}{b - a} * S = \frac{b - a}{b - P^*}.
\]

As expected, if the distribution is fixed, \(E(T)\) depends only on the reservation price \(P^*\), as defined in Equations 7 and 14. \(P^*\) in turn depends on the discount rate.
Thus, \( a, b, r \) and \( k \) (all exogenous variables) define \( E(T) \) and \( P^* \) (our endogenous variables).

### 3.4 Transaction volumes

Up until now, we have focused on the case of one seller. We now consider the case of settlements, where multiple sellers reside. As observed above, we identify settlement size as the frequency with which offers are made on a specific house. The larger the settlement, the more frequent the offers are. For the sake of simplicity, we assume that within a settlement, the frequency of offers received by sellers is identical.

A further assumption we make is that the number of houses for sale at any given time in a settlement is fixed. This implies that the number of transactions in any period is also fixed. Effectively, this means that if a house is sold, another one appears immediately on the market - altogether \( N \) houses per period. This assumption, while stringent, is not without merit. We are basically assuming that adjustment due to, for example, the crisis, does not affect the supply of houses for sale, but instead affects the frequency of offers (meaning that a period would become longer or shorter).

Under these assumptions, the long run equilibrium in the market can be expressed in the following way:

\[
N \ast \pi_{\text{out1}} + N \ast \pi_{\text{out2}} + N \ast \pi_{\text{out3}} + ... = N, \tag{31}
\]

where \( N \) is the number of houses sold in equilibrium in a given period, and \( \pi_{\text{outx}} \) is the probability that a house is sold from the "generation" that entered the market \( x \) periods ago. We can also calculate the number of houses for sale in a given period, which equals:

\[
N + N \ast \pi_{\text{in1}} + N \ast \pi_{\text{in2}} + N \ast \pi_{\text{in3}} + ..., \tag{32}
\]

where \( \pi_{\text{inx}} \) is the probability that a seller has refused \( x \) offers on the market. This can be further expressed as

\[
N \left( \pi_{\text{in1}} + \pi_{\text{in2}} + \pi_{\text{in3}} + ... \right) =
N \left( \frac{P^* - a}{b - a} + \left( \frac{P^* - a}{b - a} \right)^2 + \left( \frac{P^* - a}{b - a} \right)^3 + ... \right) = \tag{33}
N \left( \frac{1}{1 - \frac{P^* - a}{b - a}} \right) = \tag{34}
N \frac{b - a}{b - P^*} \tag{35}
\]

To sum up, this means that \( N \) transactions takes place out of a for sale housing stock of \( \frac{b-a}{b-P^*} \) houses each period, and settlements differ in how long that period actually is. The fraction of houses traded in a period (which we will denote by \( A \)) is therefore

\[
A = \frac{N}{N \frac{b-a}{b-P^*}} = \frac{b-P^*}{b-a}. \tag{36}
\]

This is actually an acceptance rate: in a period, everyone receives an offer, and
the fraction of them shown in Equation 37 accept. As a numerical example, the convex curve in Figure 3 shows this acceptance rate for various offer frequencies, for the case where the yearly discount rate $r$ is 3%, and $b = 2a$.

To understand the concave curve in Figure 3, we return to the notion of several settlements. We have introduced the idea of several settlements, but until now have always focused on one settlement only, with its own individual period length. However, if we wish to compare settlements with different period lengths, that is, different frequencies of offers arriving, we need to standardise the above result for the acceptance rate. A logical approach is to standardize to a year. In Section 3.2 we introduced the possibility of using $k$ to signify the length of a period as compared to one year (at one year, $k = 1$). If a period is one month (ie. an offer arrives every month), then $k = \frac{1}{12}$, and if it is 2 years long, then $k = 2$. A standardised expression for the transaction volume over stock for sale in a year (which we will denote by $F$), is then:

$$F = \frac{b - P^*}{b - a} \times \frac{1}{k}. \quad (38)$$

Figure 3: Transactions and offer frequency: a numerical example

The fact that $A$ is increasing in $k$ and $F$ is decreasing in it as seen in Figure 3 is not dependent on the specific values chosen for $a$, $b$ and $r$. $P^*$ increases with the frequency of offers, that is, the earlier the next batch of offers is expected in a settlement, the higher the reservation price. It follows that the probability of acceptance will be lower. $F$ is decreasing in $k$: if offers arrive less frequently the fraction of houses traded from among those available, standardised to a year, will grow smaller.

The formulas above make it possible to simulate our pre-crisis and crisis situation described in Section 2. We introduce this with a simple, numerical example and will expand on the topic at a later date. Imagine that pre-crisis in a "large" settlement, offers are made every six months ($k = \frac{1}{2}$), whereas in a "small" settlement, offers arrive only once every year ($k = 1$). However, due to the crisis, the time period lengthens by one month in each case. Assuming, as in Figure 3 that $r = 3\%$ and $b = 2a$, this means that $F$ decreases by 7.82% for the large settlement but only by 4.21% for the small settlement.
4 Conclusions and further work

In this paper, we analysed one slice of the 2008 economic crisis in the Hungarian residential real estate market. We showed that less frequent offers can qualitatively explain part of the observed phenomena. The decreasing volume of transactions and the fall in transaction prices were explained in an optimal stopping framework, where less frequent offers force sellers to decrease their reservation price. Our empirical data revealed the connection between settlement size and transaction volumes. Post-crisis adjustment in transaction volumes is different in smaller settlements than in larger ones: there appears to be a smaller relative decrease in transaction volumes in smaller settlements. The optimal stopping framework could also explain this surprising phenomenon.

This work is preliminary and incomplete. Further work will give a detailed description of housing price behaviour according to settlement size, and quantitative analysis is also needed for the identification of the size of the shock, specifically, income change in settlements. In relation to the model, further work will analyse the results in a more detailed fashion, and attempt to calibrate the model using plausible empirical data.

This is a work in progress, please do not quote without authors’ consent.
References


