Rental Dynamics in Madrid Office Market: An Approach from the Cointegration and Long-Term Equilibrium

Ramiro J. Rodríguez

Summary

Recent data availability has allowed real estate economists to dig deeper into the property market dynamics. This is the case of Madrid where, until recently, lessons on office markets have been extracted from the studies of other countries as United Kingdom, Germany or US. This paper shows that effectively office space demand is rigid with regard to short term rent variations and that its main driver is the physical space necessity, on which businesses have to accomplish their activities. Equally important is the dynamics of letting rents; they move in response to demand pressures, with the background of a rigid or semi-rigid supply. It means that it is a market where demand determines price adjustments and these, in turn, determine developer’s decisions in how much office stock is added.

Further, using cointegration and vector error correction models it is possible to estimate the grade of rents overvaluation or undervaluation existing in the market, with respect to the long term trend. Accordingly, it has been possible to compute a 15% of overvaluation in the Madrid’s office market in the expansive phase of the years 2000-2001 and a price adjustment process from the bust of the current crisis. Moreover, it is suggested that the rents remain overvalued at the end of 2011, despite falling by 35% since the start of the recent economic downturn.

Key words: Rents, office rents, Madrid office market, real estate markets, commercial property market, vector error correction model, cointegration.

Introduction

The study of non-residential real estate markets (retail premises, warehouses and offices) has gained momentum in the economic literature since the 80's, with the base of previous works analyzing the economics of residential real estate which began in the 60's in the U.S. Research on the effects of economic cycles in the residential construction of Alberts (1962) and the price estimation of housing by Blanck & Winnick (1953), Pritchett (1977) and Ferri (1977) were the seeds of economic analysis of non residential markets. In the last 20 years certain conditions have been met prompting the investigation into the non-residential property markets (Ball, Lizieri, & MacGregor, 1998):

- The global economic boom of the late 80's and early 90's and its impact on development of offices, high street and shopping centers and industrial and logistics warehouses

- The development and diffusion of new statistical analysis tools, including cointegration and Vector Error Correction models

1 Universidad Complutense de Madrid, Student of the Doctorate in Economics – Research Analyst at BNP Paribas Real Estate
- Greater availability of longer time series of supply, prices and demand of property markets

In this context, sprang the seminal works on cycles in office markets in the United Kingdom and United States of Rosen (1984) and Wheaton (1987) seeking better adjustments to the forecast of fundamentals (rent, availability, absorption of space and construction) and a deeper understanding of the relationships with macroeconomic variables. In light of these works, substantial amount of literature has been developed, extending the analysis onto other European markets since the late 90's.

Published research for the Spanish commercial property market is not abundant. It can be mentioned the work of Brounen and Jennen (2009) and Fuerst (2010), that seek to explain the rents dynamics in different European cities (10 and 19 cities, including Madrid, respectively). Brounen and Jennen use an error correction model on maximum rents and Fuerst uses linear regression models to analyze the elasticity of supply.

The objective proposed in this paper is, by using time series analysis (cointegration and error correction models), to describe the dynamics of demand for space offices, supply of office buildings and the average rents and to pinpoint long-term relationships that exist in the Madrid office market. This is achieved developing models capable of:

- Predicting future market developments in both the short and long term and identifying phases in which rents have been appreciated or depreciated against the long-run equilibrium.

- Quantifying the possible overvaluation or undervaluation of rents.

This work has adapted the theoretical model developed by Wheaton, Torto and Evans (1997) to the Madrid office market to support the econometric specification of two models employed: one using the methodology of cointegrating regression through Fully Modified OLS proposed by Phillips and Hansen (1990); and another that uses the Johansen’s methodology to estimate cointegration relationships and error correction mechanisms. After this introduction, the second section is a description of how tertiary property markets work and the third section details the econometric models employed. The fourth section describes the data used and the fifth make estimates of model parameters. Finally, the sixth section discusses the results of the two estimation methodologies employed and, finally, shows the main conclusions.

**Economics of the commercial property markets**

Nonresidential real estate markets are composed of the interaction of four sub-markets (Ball, Lizieri, & MacGregor, 1998):

- **Users**, through which employers choose the locations where they develop their productive activity. They buy or lease space to owners of available office stock. In turn, these owners have acquired these properties recurring to the:

- **Investment market**, whereby institutional or private investors (or the users) acquire real estate assets based on their performance relative to other assets (Opportunity cost). They have bought their properties using the second hand market or to the:

- **Promotion market**, through which new buildings are added to the existing stock. This promotion of new offices is activated when employers require additional space, in a market of completely inelastic
short term supply. In effect, these new buildings are constructed to be traded among investors some time after the start of the promotion, between 18 and 24 months later, explaining the inelasticity of the supply. The spaces on which to build new buildings are acquired in the:

- **Land market**, corresponding to the (limited) locations on which the new supply will be developed. The type of building to be built depends on the opportunity costs of alternative uses that may be given to the final construction. Consequently, every possible use (residential, commercial, industrial, offices, etc.) is *competing* with the others, thereby determining cost of the soil.

This work aims to analyze the final office-user market, where rents are formed each period. The following describes the operation of the office market, which will support us in specifying our empirical models.²

Demand for offices is mainly derived from the need to use space as input of production, mainly of non-industrial activities, needing a specific location to harbor the labor force. Among the main activities demanding office space we can mention:

- Business services sector
- Financial, insurance and real estate
- Support for industrial production (management, human resources, etc.)
- Public Administration

The labor absorbed by these activities can be classified as *office employment* due to about 75% of total employment in these sectors may be housed in office buildings (Wheaton W., 1987). Accordingly, the occupied stock depends deeply on the labor cycle.

Businesses demand office space from landlords aiming to obtain the maximum return on their investment that participate in the market offering space to let to businesses.

According to BNP Paribas Real Estate (2011) the general practice in the user market in Madrid is 80% of transactions as offices leases, 5% as pre-lets and the rest as sales. It is therefore a reasonable assumption in most empirical studies (including this) that the owners are limited to rent space (never sell) and end users to take leased space (never purchase it). This will facilitate the analysis and focus of the dynamics of rent, letting alone selling prices, which are balanced in the investment market.

**Office stock** is the market supply. It has the characteristics of a fixed asset subject to depreciation (destruction and change of use) as well as accumulation through new construction. There will be new stock added when property prices charged by developers exceed construction costs (interest rate, land, construction, materials, etc.). In other words, once the shortage in the available stock is transferred to rental increases in the user market, and finally to the selling market, developers will begin the

² This model attempts to give theoretical explanation to the formation of office rents, but may be easily extrapolated to other non-residential property markets, as retail/malls and logistics/industrial warehouses.
construction of new buildings to benefit of the higher property prices. Developments cease at the moment in which the stock available caters all demand, causing prices of the property falling back to the level of replacement costs. In this sense, the office promotion market can be considered as an "imbalance phenomenon" (Ball, Lizieri, & MacGregor, 1998). Once the imbalance is observed in the user market, new stock is added in the next period, thus forming a real estate cycle.

(Source: BNP Paribas Real Estate) Figure 1 shows the (delayed) response in the supply of new development to the dynamics of real rents in Madrid. The response is clearly observed in 2001 and 2002 when deliveries responded to the peak rents observed in 2000 (second half). The same reaction was seen after the rents peak in 2007, with an impact in the first half of 2009, although with less intensity than in 2001/2002.

Developers will construct new buildings according the balance of the asset price and their replacement costs. That is, office supply responds positively to higher property prices and negatively to the production costs and financing, which are assumed exogenous.

Meanwhile, property prices are higher the more scarce the available stock is (once exhausted the resource of reduction of space per employee), that is, the lower the vacancy rate, which is the ratio between the total floor area and stock, the higher the rental values. In turn, this shortage is greater in periods of increased economic activity. In summary, the office market holds a close correlation with the real business cycle and employment market.

(Source: BNP Paribas Real Estate)
Chart 2 shows two cycles of the office market in Madrid in its variables of vacancy rate (ratio of available surface area on the total stock) and level of office jobs. A countercyclical movement of the two variables is clearly observed. The analysis indicates that in times of high demand for labor the floor area of offices is reduced; having the respective impacts on land, investment and promotion markets.

The high correlation between activity variables (production, economic sentiment, etc.) and employment as well as the correlation between national and local employment allow for obtaining similar adjustments in the commercial real estate models. According to Brounen and Jennen (2009) no significant differences are obtained. Therefore, following these authors we have chosen the office employment in Madrid as the main indicator determining the need for space and therefore office demand.

**Theoretical model**

Scheme 1 shows a conceptual framework that helps to explain the key relationships of an office property market, which can be applied to any empirical modeling of office market or any other non-residential market (Brooks & Tsolacos, 2010). The direction of the arrows indicates whether a variable affects or is affected by other(s). For example, the level of employment directly affects the occupied office stock, but is not affected by other variables. The employment variable will therefore be considered as exogenous to the office market. The sign accompanying the arrow corresponds to the effect of a change in the origin variable on the target variable. Continuing the example, an increase (decrease) in level of employment will increase (decrease) the occupied stock. The endogenous variables, therefore, are occupied stock (demand), new development (supply) and rental levels. In the following sections we specify the equations derived of this scheme.
Occupied space, employment and rents

As mentioned above, this paper will use the theoretical model by Wheaton, Torto and Evans (1997). The first relationship to comment is that between occupied space (demand), office employment and rental levels. When office employment increases or rents decrease, two immediate effects are triggered: An upward pressure on the space occupied per employee and a sub-optimal use of the current occupied space. Consequently, companies will want to adjust the space used. However, long term leases (5 to 10 years in Madrid’s market) generate frictions in the adjustment of the space occupied (DiPascuale & Wheaton, 1995). In other words, there is no instant adjustment to changes in employment and rents. Recon this relationship it is proposed the following space demand function:

\[
OS_t^* = \alpha_0 EMP_t^\alpha_1 R_t^{\alpha_2} \\
\log(OS_t^*) = \alpha_0 EMP_t + \alpha_1 EMP_t + \alpha_2 R_{t-1}
\]  

(1)

Where \(OS_t^*\) is the level of required space, that businesses would take-up if they had no space adjustment restrictions. Such restrictions do not exist in long-term, in the end of the lease contracts, therefore (1) can be associated to the long-term demand.

From the discussion on the functioning of the market we acknowledge that this demand depends on the (office) employment, \(EMP_t\) and the rent level observed in the last period, \(R_{t-1}\).

Short term changes of employment and rents will make companies want to change their space used. However, given the rigidities imposed by the contracts, some cannot adjust, whereby the current market demand differs from the long term. Accordingly, there is an adjustment process in each period in the occupied area, represented by:

\[
\frac{OS_t}{OS_{t-1}} = \left(\frac{OS_t^*}{OS_{t-1}}\right)^{\tau_1} \\
\log\left(\frac{OS_t}{OS_{t-1}}\right) = \log(\alpha_0 EMP_t + \alpha_1 EMP_t + \alpha_2 R_{t-1}) = AN_t
\]  

(2)

Where \(OS_t\) is the space currently occupied and its log difference equivalent to the short-term dynamics of the demand or net absorption, \(AN_t\). \(\tau_1\) represents the proportion of firms that, by contracting or regrouping space, have reached their long-term demand in the period \(t\). At steady state there is no net absorption, because the space occupied is set equal with the required space. Replacing (1) in (2) we obtain a new expression for the dynamics of short-term net absorption:

\[AN = \gamma_0 + \gamma_1 EMP_t + \gamma_2 R_{t-1} - \tau_1 OS_{t-1}\]  

(3)

In equation (3), the mechanism by which the short-term demand is adjusted to achieve the long-term demand depends on rents and employment\(^3\). Estimation of the coefficients of (1) and (3) (\(A\), \(\gamma\), \(\tau\)) will be discussed in the section on econometric specification.

\(^3\) As mentioned above, also depends on the intensity of land use. This is why you can specify (1) in depending on the surface of the previous period
Rents and vacancy rate

There is a strong correlation between rents levels and availability of space. The more (less) the availability (or vacancy rate) the less (greater) tend to be the rents (Wheaton, Torto and Evans, 1997). The equilibrium rents (long term) can be represented by:

\[ R_t^* = P_0 VAC_t^F \]
\[ \log(R_t^*) = r_t = \rho_0 + \rho_1 VAC_t \]

(4)

Where \( R_t^* \) is the real rent in the steady state (long term) defined from the VAC or vacancy rate:

\[ \text{STOCK}_t = OS_t - VS_t \]
\[ VAC_t = \frac{VS_t}{\text{STOCK}_t} \]  

(4a)

Where STOCK\(_t\) is the total office stock and and VS is the vacant surface in period t.

Short term rental levels may differ from the steady state. In this case, there will be an adjustment process that will lead to rentals to their steady state or long-term equilibrium. Therefore, it is convenient to use some error correction mechanism in the specification of rents’ short-term dynamics, to account for the cycle adjustments.

The dynamics of the short-term rents shall be specified by an Error Correction Model, in which the percentage change of each period will depend, among other factors, of the discrepancy between the rent and the equilibrium rent determined in (4):

\[ \Delta r_t = \mu_0 \Delta r_{t-1} + \sum_{l=1}^{k_1} \mu_1 \Delta r_{t-l} + \sum_{l=2}^{k_2} \mu_2 \Delta VAC_{t-l} + \mu_3 + \epsilon_t \]  

(5)

\[ \Delta r_t = \mu_0 (r_{t-1} - \rho_0 - \rho_1 VAC_{t-1}) + \sum_{l=1}^{k_1} \mu_1 \Delta r_{t-l} + \sum_{l=2}^{k_2} \mu_2 \Delta VAC_{t-l} + \mu_3 + \epsilon_t \]  

(5a)

The expressions (5) and (5a) show how rent variation in each period depends on the variations in employment and the availability rate in previous periods, and how far from the equilibrium is the observed rent. Estimating the coefficients (4) and (5) (\( \mu, \rho \)) and the selection of the delays (k) will be analyzed in the section on econometric specification.

New building starts and deliveries

As discussed in the chapter of economics of the real estate markets, developers start new projects based on the balance of the asset market price (offices) and the replacement costs. The office supply responds positively to higher property prices and negatively on production and financing costs, which in this paper are assumed exogenous. The sale price is in turn determined by the lease rents and a discount rate. In addition, developers will take into account the intensity of use of the current stock as an indicator of demand they will face once the project is developed. In short, the supply function should take into account rents, vacancy rate, interest rate and replacement costs:
Where BS<sub>t</sub> is the building starts, cc<sub>t-n3</sub> and i<sub>t-n4</sub> are the long-term construction costs and interest rate observed certain periods before.

At the same time office stock varies as projects started certain periods before are deliveries and destruction (depreciation) has taken place. Thus, the level of offices stock will be given by the following identity:

$$STOCK_t = (1 + \delta)STOCK_{t-1} + \sum_{k=1}^{t} BS_{t-k}$$  (7)

It is worth noting that the start of new buildings modeled here are a different series than those of deliveries of new buildings. Data from BNP Paribas Real Estate correspond to deliveries and not starts. For this reason the econometric treatment can vary in terms of delays in the variable and even in the statistical representation.

To close the modeling section, we specify the identities of the variables endogenous to the model:

$$VACR_t = \frac{(STOCK_t - OS_t)}{STOCK_t}$$  (8)

$$VAC_t = (STOCK_t - OS_t)$$  (9)

**Madrid office market**

The database used in this report was provided by BNP Paribas Real Estate and contains biannual data from 1995 through the second half of 2011. It conveniently collects two complete cycles of the Spanish economy, as the boom and the subsequent "dot-com crisis" and the boom of the years 2004-2007 and the subsequent crisis since then until today. Although data are usually presented in quarterly series, the first statistical collections were made semiannually, so the series are presented in this way in this work. A positive effect of this constraint is avoiding some inherent volatility of higher frequency series.

The geographical scope corresponds to the offices within metropolitan area of Madrid, plus municipalities of Las Rozas, Pozuelo, Alcobendas and San Sebastian de los Reyes. The detail of the variables included in the study is provided in Annex II and the following table presents a summary of key statistics:
Table 1 lists the variables of the model to be estimated in the next section. The main property-related variables are measured in meters squared. Except the vacancy rate that expresses the available surface as a percentage of the stock. The average rent is measured in €/m²/year and is deflated using GDP deflator (2005=100) published by the IMF. Values in parentheses show extreme points.

Figure 3 shows a simple but significant story of the recent crisis in Spain and Madrid. The maximum level of office employment is registered in the second half of 2007, causing the maximum occupied space level reached in the first half of 2008. This generated a decrease in vacancy rate from 10% to 7% between first 2005 and 2007 (Figure 2). Once the crisis hit, the beginnings of new construction was halted for reaching a low of 10,000 square meters in the second half of 2010. Standard deviation
figures indicate high volatility in the supply of new development, in line with the variables of investment in physical capital.

It also shows that the maximum and minimum size of such series as occupied stock, stock and employment. They have their maximum values at the beginning and end of the series, indicating a tendency to the long-term growth, which is the same as non-stationarity in the series average, such as many economic variables.

Rent and vacancy rate series show two cycles: the first seems to start in the mid-nineties, reaching its peak in 2000, coinciding with the "dot com crisis". The second began around 2004, to reach its peak in the year 2008. It is also possible to observe the continuing growth trend in variables as occupation, stock and employment. Deliveries of new development are the most volatile component.

The possible co-movements of the series have been traced through their correlations and collected in Table 2. On the demand side, a close-to-one (and very significant) correlation between employment and the occupied stock makes clear the role of economic activity but also the fact that the series are not stationary. Meanwhile, the average rent does not display its negative effect in demand. This can be explained by the co-movements in times of boom and slowdown which linear correlation captures as co-movements of the variables. However, changes in rents do affect negatively the demand for occupied space, an effect that can be recovered in the econometric specification.

The correlation of -0.88 between average real rent and vacancy rate (p-value of zero) confirms other sound interplay of the real estate variables. The effect of both the stock and the available stock (components of the vacancy rate) is clear and intuitive. That is why the coefficient is less than zero.

To conclude this section one can say that new deliveries have no strong correlation with the selected variables. As already noticed in Table 1 the high volatility of the series can reduce their correlation with the other fundamentals.

<table>
<thead>
<tr>
<th>Correlation p-value</th>
<th>Occupied space</th>
<th>Vacant space</th>
<th>Office stock</th>
<th>Vacancy rate</th>
<th>Deliveries</th>
<th>Real Rent</th>
<th>Office employment</th>
<th>Euribor (12 months)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Occupied space</td>
<td>1.0000</td>
<td>-----</td>
<td>0.3332</td>
<td>0.0581</td>
<td>-----</td>
<td>0.5112</td>
<td>0.0024</td>
<td>0.9807</td>
</tr>
<tr>
<td>Vacant space</td>
<td>0.0581</td>
<td>1.0000</td>
<td>0.0024</td>
<td>-----</td>
<td>0.9807</td>
<td>1.0000</td>
<td>0.9250</td>
<td>0.2300</td>
</tr>
<tr>
<td>Office stock</td>
<td>0.9807</td>
<td>0.5112</td>
<td>1.0000</td>
<td>0.5112</td>
<td>0.9807</td>
<td>1.0000</td>
<td>0.9250</td>
<td>0.2300</td>
</tr>
<tr>
<td>Vacancy rate</td>
<td>0.9250</td>
<td>0.0024</td>
<td>0.9126</td>
<td>-----</td>
<td>0.9807</td>
<td>1.0000</td>
<td>0.9250</td>
<td>0.2300</td>
</tr>
<tr>
<td>Deliveries</td>
<td>0.2125</td>
<td>0.8381</td>
<td>-0.0199</td>
<td>1.0000</td>
<td>0.1978</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Real Rent</td>
<td>0.2698</td>
<td>0.0402</td>
<td>0.5677</td>
<td>0.0000</td>
<td>0.2591</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Office employment</td>
<td>0.1978</td>
<td>-0.3599</td>
<td>0.1059</td>
<td>-0.4674</td>
<td>0.2698</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Euribor (12 months)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
Table 2 contains the correlations between different variables involved in the model, with p-values for each correlation. Interest rate data were added for analyzing whether or not they are related to the new construction deliveries. The analysis suggests that the linear relationship between two variables is negative, although not very strong. However, the stock reflects higher correlation and in the same direction.

Econometric Specification

In this section we present the results of two methods selected for estimating the equations of Demand, Rents and Deliveries. These methods are:

- Cointegrating regression, using the method of Fully Modified OLS estimation proposed by Phillips and Hansen (1990). Under this method, the standard assumptions of the asymptotic analysis are valid in the presence of 1st-order non-stationary and cointegrated series. The inference on the estimated coefficients is possible because the t-statistic and f- distributions behave optimally. In this way a structural modelling in a multivariate system is performed, explaining the changes in endogenous variables by the values (delayed or not) of other explanatory variables. In this case, the functions used replicate equations (1) to (7).

- Maximum likelihood proposed by Hansen BE (1992a) and (1992b) which is the formulation of a restricted VAR model. This approach results in Error Correction Models, which restrict the cointegrating vectors assumed for all variables involved in the model as relationships determined by the theory. In this case, the restrictions fit theory explained above.

In both cases the variables involved in the model must be first-order integrated (I(1)). In other words, the variables must be stationary in first difference. This is a necessary condition for the existence of cointegration⁴, concept on which to VEC models and the cointegration regression are based (Brooks & Tsolacos, 2010).

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⁴ In this paper we use the concept of weak stationarity or mean stationarity, without requiring stationarity in variance nor autocovariance.
The concept of cointegration and its role as evidence of long-term equilibrium

According to Engle and Granger (1987) a vector of variables is cointegrated when (a) all its components have the same order of integration and (b) there is (at least) a vector of coefficients constituting a stationary linear combination (of the variables vector).

If a set of (non-stationary) series moves in a coordinated manner over time (obey the same laws of nature, for example) can be said that these series have along-term relationship. The existence of cointegration may be a reflection of this relationship and identify that the co-movements are sustained in the long term, although fluctuations differ short term. This is why we will proceed to verify the existence of unit root and cointegration between the variables involved in the model.

The following is the assessment of the existence of unit roots in the model variables, which is made by the Augmented Dickey-Fuller Test.

Table 3: Augmented Dickey-Fuller Test

Null Hypothesis: Variable has unit-root
Lag Length: 0 (Automatic - based on SIC, maxlag=8)

<table>
<thead>
<tr>
<th>Integration Order</th>
<th>Occupied space</th>
<th>Vacant space</th>
<th>Office stock</th>
<th>Vacancy rate</th>
<th>Deliveries</th>
<th>Real Rent</th>
<th>Office employment</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-value</td>
<td>0.2825</td>
<td>0.8193</td>
<td>0.8165</td>
<td>0.3828</td>
<td>0.0325</td>
<td>0.0898</td>
<td>0.9999</td>
</tr>
<tr>
<td>P-value for the first difference</td>
<td>0.0002</td>
<td>0.0200</td>
<td>0.0000</td>
<td>0.0284</td>
<td>0.0000</td>
<td>0.3169</td>
<td>0.0497</td>
</tr>
</tbody>
</table>

* The same order of integration results are obtained with the variables in logarithms

** The Real Rent variable reached its stationary level with the first difference with a P-value of 0.0000

Cointegrating Regression results

This method assumes for the estimation the existence of cointegration between the variables implied in the model. This is the statistical representation of the assumption of a long-term equilibrium relationship in the studied variables. In this sense the coefficients estimated have the suitable properties for inference, with the conventional asymptotic properties. The prove of existence of cointegrating relationships is found in annex II.
Estimation of the long-run demand (equation 1):

\[
\text{OS} = \beta_1 \text{EMP} + \beta_2 \text{RRENT} + \epsilon
\]

Where OS is the occupied space, EMP is the office employment and RRENT is real rent.

The estimated coefficients are significant and show the effect of employment and rents as predicted by theory.

It is interesting analyzing the effects of the right side variables throughout their coefficients. The main driver of the long-run demand is the office employment, as a 1% change in its level yields a change of 0.6% change in “desired” occupied space. On the contrary a similar change in real average rents will drive a contraction in occupied space by less than 0.01% signalling low price elasticity of demand.

As the properties of the estimation, the Durbin-Watson is over 2 indicating low serial correlation which is translated into good performance of residuals. In fact, the sum of squared residuals is nearly zero.

The estimate of difference equations – short-term demand:

\[
\Delta \text{OS}_t = \gamma_0 + \gamma_1 \Delta \text{EMP}_{t-1} + \gamma_2 \Delta \text{RRENT}_{t-1} + \delta_0 \Delta \text{OS}_{t-1} + \epsilon_t
\]
Since the model series are I(1) the estimation method for the short term demand has been ordinary least squares. C(1) represents the coefficient of the cointegrating vector and gives information of the speed-of-adjustment of the short-term demand to its long-term trend. Here, the negative sign tells us that the adjustment will offset the imbalance and the coefficient magnitude says that each period 17% of the demand reaches the steady-state occupied space. That is, if demand is below its long-term trend, we expect positive net absorption in the six following periods (3 years) for all firms reach their optimal space usage. Comment at this point that although the sign of C(1) is as expected, speed of adjustment can be described as high, away from the intuition of rigidities imposed by enforceable contracts.

C(2) is the influence the first difference of employment in the first difference of the occupied space. A constant and the effect of rents were not significant in this model specification, confirming the dominant effect of employment in the demand cycle.

The coefficient of determination does not have a very high level (nor very low), remembering that it is a model in differences. The Durbin-Watson statistic indicates that it has obtained low serial correlation in the residuals. In this way we have an expression for the fluctuations in the short term depending of an exogenous variable (employment) and other endogenous (real rent)

**Estimation of the Rents equations (equation 4):**

By the method of Fully Modified Least Squares the effect of vacancy rate (VACR) in equilibrium real rent (RRENT) has been estimated.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOG(VACR)</td>
<td>-0.458036</td>
<td>0.032040</td>
<td>-14.29590</td>
<td>0.0000</td>
</tr>
<tr>
<td>C</td>
<td>4.146098</td>
<td>0.082729</td>
<td>50.11678</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

In this estimation the coefficients are significant and have the expected signs. In particular, a change of 1% in the vacancy rate (VACR) will lead to a change of half a point in the equilibrium rental level. This denotes a high sensitivity of rents to changes in vacancy rates, which is quite normal due to the total inelasticity of the real estate supply (i.e. new buildings take around two year to be delivered).
Using this output for the estimation of the equation (5) of short term rental dynamics:

<table>
<thead>
<tr>
<th>Dependent Variable: DLOG(RRENT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method: Least Squares</td>
</tr>
<tr>
<td>Date: 05/31/12</td>
</tr>
<tr>
<td>Time: 23:23</td>
</tr>
<tr>
<td>Sample (adjusted): 1996S2 2011S2</td>
</tr>
<tr>
<td>Included observations: 31 after adjustments</td>
</tr>
</tbody>
</table>

\[ DLOG(RRENT) = C(1)*LOG(RRENT(-1)) + 0.458035883805*LOG(VACR(-1)) - 4.14609839363 + C(2)*DLOG(RRENT(-2)) \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
<td>-0.733010</td>
<td>-4.790524</td>
<td>0.0000</td>
</tr>
<tr>
<td>C(2)</td>
<td>0.269199</td>
<td>2.094043</td>
<td>0.0451</td>
</tr>
</tbody>
</table>

Here C(1) represents the effect of the speed of adjustment of the short-term rent levels to the real underlying rent. The coefficient has the expected sign and denotes that each period the short term rents will offset long-term deviations in a prompt fashion. This is normal because it is quite immediate a ‘price tag’ change in presence of a vacancy rate variation.

C(2) is the impact of lagged rents variations on the current rent variation and tells us that past rent dynamics have repercussions in current rent dynamics.

We get very low correlation in the residuals (Durbin Watson near 2) and an acceptable coefficient of determination (55%).

Estimation of building starts (equation 6):

The estimated supply function has not been carried out, because the model uses building starts which currently are not available for Madrid. We have obtained construction deliveries and for this reason we have used them as an exogenous variable. The advantage of this new specification is that data are very precise because the developers put into the public domain their future deliveries, seeking investors/occupants of the spaces they build. For this case, it has been registered the future office supply for the 2011-2016 period in each semester (COMP).

Results of the Johansen approach

In this case we have specified a VEC model with the following variables (in logs): Occupied space, Vacancy Rate, real Rents and Employment.
As we have found three cointegrating relations (available in the appendix), we have incorporated them into the estimation and we have restricted their cointegrating vectors and coefficients according to economic theory explained at first.

Under the new model the long term occupied stock depends on the employment and rents (cointegration equation 1) and rents depend on the vacancy rate (cointegration equation 2). We have used one delay, following the Akaike Information Criterion, rejecting the use of 2 and 3 delays.

Vector Error Correction Estimates

<table>
<thead>
<tr>
<th>Cointegrating Eq</th>
<th>CointEq1</th>
<th>CointEq2</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOG(OS(-1))</td>
<td>1.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>LOG(RRENT(-1))</td>
<td>-0.016177</td>
<td>1.000000</td>
</tr>
<tr>
<td></td>
<td>(0.03732)</td>
<td>[-0.43343]</td>
</tr>
<tr>
<td>LOG(VACR(-1))</td>
<td>0.000000</td>
<td>0.506265</td>
</tr>
<tr>
<td></td>
<td>(0.02411)</td>
<td>[20.9968]</td>
</tr>
<tr>
<td>LOG(EMP(-1))</td>
<td>-1.066003</td>
<td>0.000000</td>
</tr>
<tr>
<td></td>
<td>(0.02991)</td>
<td>[-35.6394]</td>
</tr>
<tr>
<td>C</td>
<td>-8.634708</td>
<td>-4.021758</td>
</tr>
</tbody>
</table>

The cointegrating equation 1 (CointEq1) was standardized in the occupied space and depends inversely on rents and positively on employment. The strong significance of the coefficient of employment tells us that the needs of physical space are paramount for the demand, giving rent levels a secondary roll.

In the cointegrating equation 2 (CointEq2) it has been demonstrated that equilibrium rents depend on vacancy rate, and that for instance, a 1% increase in the vacancy rate will lower long-term rents by half a point.
As for the short-run dynamics, we have restricted the difference equation of demand (Occupied Space) to not be affected by CointEq2, while the equation of short-term price (Real Rent) is only restricted to not be affected by CointEq1. The results of short-term equations are showed below:

In this model the coefficient of CointEq1 is significant in the cyclic demand equation. In this equation, lags of occupied space are highly significant, showing autoregressive behaviour, natural in real estate figures. Variations in employment also impact variations of occupied space. Rent variations are not significant in the short run, maybe due to rigidities imposed by lease contracts.

The coefficient of CointEq2 is significant for the cyclic equation of Rents and gives information of how short term rents fit to the long-term trend. In this case, short-term rents ‘seek’ the equilibrium correcting 80% of the deviation each period. The speed of adjustment can be justified through the easiness of price-tag change. CointEq2 is the main driver of short-term rents as the other coefficients tend to be non significant.

Rent oscillation around the equilibrium path

One of the utilities of the error correction models is the analysis of the oscillation of the observed series (cycle) around the estimated long-run equilibrium. Next we make this analysis for rental series in Madrid, but can be done to other endogenous model variables.
Figure 4 shows the two long-term estimated paths for each methodology. The blue line corresponds to the equilibrium path of the structural model and the Johansen green. The green line describes the observed rent path. The oscillations of both methods are quite similar, except for the “dot-com” bubble period where Johansen approach predicts a high value of equilibrium rents. Thus, the forecast adjustments are high (see forecasts section) to these models with the deviations between long and short term are low. Both models agree in warning that between 1996 and 1999 market rents were below its equilibrium value. Also that after the the "dot com" crisis property (2001-2002) was overrated. Thereafter, the equilibrium path is almost identical in both models, pointing alternating phases of over and undervaluation of the offices. Two years before the current crisis market fundamentals pointed undervaluation, to reach the corresponding overreaction of 2008 (it is normal to see these 'over-reaction' in prices during periods of transition in many economic series). Since then we have seen a period of overvaluation through the end of 2010. At this point the structural model indicates the end of the overvaluation, while Johansen tells us that there is still room for more negative rent adjustments.

Next an analysis of rental deviations is displayed where observed rents is weighted against the equilibrium calculated by both methodologies.
Figure 5 shows the rate of deviation of each observation of the series of real rent against its long-term path. Negative values indicate underestimation of the property and positive overvaluation. Both models predict the same periods of under and overvaluation. It is clear that in 1995 the rents were entered the ‘undervaluation zone’ and remained there until the burst of the dot-com hype. The burst of the 'dot com' bubble led to a sharp imbalance and the property came to be valued over 15% its fundamental value. This imbalance was reduced almost as quickly as it came. However, between 2002 and 2008 short-term rents bounced between under and overrating. However, the rebounds are skewed to the positive quadrant. After the current crisis, rents have not been able to reach their long-term path and keep above it. Nevertheless rents ended 2011 below 10% of overvaluation.

**Johansen Vs. Structural Methodology: Out-of-sample forecasts**

Below are the results of the dynamic forecasts, using the previous predicted value for the next forecasted period. Forecasts have been made for the span H2-2008 to H2-2011 yielding the following remarks:

- Demand is predicted with some accuracy in both cases. In the Johansen's methodology, occupied space prevision remains below the actual value for the last 3 periods

- Rents tend to present the same trend as the observed value, however, with the Johansen approach they remain over the observed rent.

- Vacancy rates fit better with the structural approach having all the forecasted values of the Johansen approach well over the actual value.

In summary, both models have some level of gap, but the structural methodology seems to be more accurate than that of Johansen’s.
In general, the fluctuations with Johansen methodology are lower, but fail in predicting rents and vacancy rate. Both models predict well the demand, although the Johansen model the gap is greater.

**Conclusion and discussion**

Empirical literature about the long-term relationships in real estate markets is still scarce. There is still room for best performances in modeling commercial property markets. This deficiency is even higher in Spain, where little research has been done in this matter. It is in this sense that this work has been proposed, using the theoretical framework of Wheaton, Torto and Evans (1997) and taking into account the characteristics of the Madrid office market, modeling econometrically its short and long term dynamics. In addition to the three classical endogenous variables (Occupied space, vacancy rates and rents) an exogenous variable - employment in the service sector of the Community of Madrid –
has been used as an indicator of the economic activity and needs of office space. We estimated two models that include dynamic cointegration relationships in the long run. One utilizes the cointegration regression, using the estimation method of Fully Modified Least Squares OLS proposed by Phillips and Hansen (1990), and another using the methodology of Johansen cointegration and error correction model. Both models are capable of capturing much of the dynamics of short and long term of the series studied. In the dynamic projections outside the sample, the cointegrating regression method has better performance. Additionally, the structural model equations are well supported by the economic theory as the functional form and regresors (in the equations of long and short term) are taken from this base. It is worth noting that both models do well for the analysis of deviations of the trend versus cycle. In this way an application has been studied on rents dynamics concluding that after the rise of the dot-com hype rents became overvalued, with an overrating reaching 15 in early 2001. From the beginning of the current crisis rents have stayed oversized compared to the equilibrium rent. One would expect to see further adjustments in the coming periods towards their equilibrium levels, therefore presenting added discounts.

The extension to a larger number of observations over time will shed more light on the assertiveness of these models. It is desirable that further work integrates more economic variables in the estimates, such as interest rates and construction costs the better model the supply side of the Madrid’s office market.
Bibliography


Annex 1 – Details of the model variables

Frequency: semi-annual


<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Definition</th>
<th>Source</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Office employment in Madrid</td>
<td>Persons</td>
<td>People employed in activities demanding office space</td>
<td>Ministerio de trabajo e inmigración, Oxford Economics</td>
<td>Employment in financial sector, insurance, business services and real estate</td>
</tr>
<tr>
<td>Office stock</td>
<td>m²</td>
<td>Surface area of offices inside Madrid’s metropolitan area</td>
<td>BNP Paribas Real Estate</td>
<td></td>
</tr>
<tr>
<td>Deliveries</td>
<td>m²</td>
<td>Added new buildings each period</td>
<td>BNP Paribas Real Estate</td>
<td></td>
</tr>
<tr>
<td>Occupied surface</td>
<td>m²</td>
<td>Part of the stock actually taken-up</td>
<td>BNP Paribas Real Estate</td>
<td></td>
</tr>
<tr>
<td>Vacant surface</td>
<td>m²</td>
<td>Surface area immediate available for lease</td>
<td>BNP Paribas Real Estate</td>
<td></td>
</tr>
<tr>
<td>Vacancy rate</td>
<td>%</td>
<td>Ratio of vacant space over total stock</td>
<td>BNP Paribas Real Estate</td>
<td></td>
</tr>
<tr>
<td>Average real rents</td>
<td>€/m²/year</td>
<td>Headlight rents</td>
<td>BNP Paribas Real Estate</td>
<td>Deflated using IMF GDP deflator</td>
</tr>
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</table>
## ANEX 2

**Johnson Cointegration Test Summary**

<table>
<thead>
<tr>
<th>Data Trend</th>
<th>Test Type</th>
<th>None</th>
<th>None</th>
<th>Linear</th>
<th>Linear</th>
<th>Quadratic</th>
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<tbody>
<tr>
<td>No Trend</td>
<td>No Intercept</td>
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<td>2</td>
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<td>Linear</td>
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<tr>
<td>Trend</td>
<td>Intercept</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

*Critical values based on MacKinnon-Haug-Michelis (1999)*

**Information Criteria by Rank and Model**

<table>
<thead>
<tr>
<th>Data Trend</th>
<th>Rank or No of GEs</th>
<th>None</th>
<th>None</th>
<th>Linear</th>
<th>Linear</th>
<th>Quadratic</th>
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</thead>
<tbody>
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<td>120.0551</td>
<td>120.0551</td>
<td>120.0551</td>
<td>120.0551</td>
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<tr>
<td>Intercept</td>
<td>No Trend</td>
<td>158.5388</td>
<td>160.2559</td>
<td>144.5034</td>
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<td>162.3526</td>
</tr>
<tr>
<td>Intercept</td>
<td>Intercept</td>
<td>160.7109</td>
<td>162.5847</td>
<td>155.6284</td>
<td>153.8528</td>
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<tr>
<td>Intercept</td>
<td>Intercept</td>
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<td>150.2067</td>
<td>150.2067</td>
<td>150.2067</td>
<td>150.2067</td>
</tr>
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</table>

**Akaike Information Criteria by Rank (rows) and Model (columns)**

<table>
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<tr>
<th>Schwarze Criteria by Rank (rows) and Model (columns)</th>
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<th>1</th>
<th>2</th>
<th>3</th>
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<tbody>
<tr>
<td>-6.204392</td>
<td>-6.204392</td>
<td>-6.204392</td>
<td>-6.204392</td>
<td>-6.204392</td>
</tr>
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