# Housing Services and Volatility Bounds with Real Estate Returns

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**Abstract**

We investigate the role of a real estate market in consumption based asset pricing models. A long-standing issue in this context is the inability of these models to match the equity premium observed in the data. Piazzesi, Schneider, and Tuzel (2007) use a model with housing services that comes closer to matching the premium. We look at the performance of this model from a different perspective. Rather than focusing on the model implied returns, we use historical returns to construct the Hansen-Jagannathan volatility bounds for the inter-temporal rate of substitution (a stochastic discount factor). Our returns include real estate returns in addition to the typically included returns on stocks and a risk-free asset. Then we calculate moments of a stochastic discount factor implied by the model from Piazzesi, Schneider, and Tuzel (2007). These moments are shown to be above the Hansen-Jagannathan volatility bounds.

KEY WORDS: Equity premium puzzle; Hansen-Jagannathan volatility bounds; housing services; stochastic discount factor; real estate returns.

JEL CLASSIFICATION: G12

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# 1 Introduction

A standard Consumption based Capital Asset Pricing Model (CCAPM) with Constant Relative Risk Aversion (CRRA) preferences cannot produce equity premium which can match the data for plausible (i.e. low) values of risk aversion. This problem is referred to as the equity premium puzzle. Mehra and Prescott (1985) illustrate the problem by using a partial equilibrium approach. They calibrate a consumption process to the US aggregate consumption data and compare the model-implied moments for asset returns with historical returns. Hansen and Jagannathan (1991) use a reverse sequence. They start with data on returns and derive restrictions for a stochastic discount factor (SDF). SDF for the CCAPM with CRRA utility function is the intertemporal rate of substitution, which depends on consumption. The restrictions are known as the Hansen-Jagannathan volatility bounds. The bounds usually are constructed using returns on stocks and risk-free government bonds. We add real estate to the set of considered assets and use its return to derive the bounds, which are now slightly more restrictive. The newly calculated bounds can be used to test a specific CCAPM. A natural choice for a model to be tested is one that explicitly treats housing both as a consumption good and as an asset. We use the model from Piazzesi, Schneider, and Tuzel (2007). We extend and re-construct the dataset employed in Piazzesi, et. al. (2007) from the US National Income Product Accounts (NIPA) data. We show that the volatility of the model-generated SDF is above the Hansen-Jagannathan bounds.

The rest of the report is organized as follows. Section (2) characterizes in some detail the equity premium puzzle and volatility bounds and provides a brief update on the current status quo with respect to established research results. Section (3) uses the model and extended data from Piazzesi, et al. (2007) to show that SDF from this model an be sufficiently volatile to satisfy Hansen-Jagannathan bounds. Section (4) discusses robustness of the results and issues for future research.

# 2 Equity Premium Puzzle and Volatility Bounds for SDF

The equity premium puzzle arises in the context of the CCAPM. This model imposes restrictions on the covariance between the stochastic discount factor and asset returns. The stochastic discount factor is given by the marginal utility growth of consumers. The restrictions are captured by the Euler equation (see Guvenen and Lustig 2007a for a

summary of theoretical arguments):

$$E_t[m_{t,t+1}R^j_{t,t+1}] = 1, \tag{1}$$

where $m_{t,t+1} = \beta\frac{U_c(C^i_{t+1},W_{t+1})}{U_c(C^i_t,W_t)}$. $C^i_t$ is the optimal consumption of consumer $i$ and $R^j_{t,t+1}$ is a return on asset $j$. $W$ denotes items other than consumption, which enter the utility function. The equation (1) can be understood using simple intuition. By investing a small amount $\xi$, the investor reduces consumption today in return for extra consumption $\xi R_{t+j}$ tomorrow. For time-separable preferences, this translates into an optimality condition $-U_c(C^i_t, W_t\xi + E_t[\beta U_c(C^i_{t+1}, W_{t+1})\xi R^j_{t,t+1}]$ which can be rearranged, yielding equation (1). Rubenstein(1976) and Lucas (1978) derived the Euler equation in discrete time and Breeden (1979) in continuous time.

The equity premium puzzle was identified by Mehra and Prescott (1985) who demonstrated it using CRRA preferences. The puzzle was also inherently behind the rejection of the same model by Hansen and Singleton(1983) using the Maximum Likelihood Estimation. Let us define returns $R^e_{t+1}$ on risky assets such as equity and $R^f_{t+1}$ returns on a risk free asset. The equity premium is then $EP_{t+1} = R^e_{t+1} - R^f_{t+1}$. Assuming that returns and consumption growth are jointly log-normal, it can be shown that:

$$E[EP_{t+1}] = \varphi Cov(R^e_{t+1}, \Delta c_{t+1}), \tag{2}$$

where $\varphi$ is the coefficient of relative risk aversion and $\Delta c_{t+1}$ is the log consumption growth. The covariance between equity premium and consumption growth rate was 0.0024 using the US data from Mehra and Prescott(1985). Since the equity premium was about 6% at the time, this implies a risk aversion coefficient $\varphi$ equal to 25. This is an unrealistically large number: consumers with a risk aversion of this size would prefer a certain 18% reduction in consumption to a bet with 50% chance of winning and 50% chance of loosing 20% of consumption. The puzzle appears across the world and has stood against time. Dimson, Marsh, and Staunton (2006) show using data from 17 countries and the sample from 1900 to 2005 that the worldwide premium is 4.5-5% even after the collapse of the internet bubble in the early 2000s.

Hansen and Jagannathan (1991) view the Euler equation from a different perspective. SDF can be projected on a space spanned by asset returns:

$$m = X'\pi_0 + \epsilon, \tag{3}$$

where $X = (1\ R')$ is a vector of gross asset returns with $R = (1 + r_1, ..., 1 + r_N)'$. Since $E[X\epsilon] = 0$, it follows that $\pi_0 = (E[XX'])^{-1}E[Xm]$. Imposing (1) results in

$$\pi_0 = P^{-1}\begin{pmatrix} Em \\ e \end{pmatrix}, \tag{4}$$

where $P = E[XX']$ and $e$ is an $(N \times 1)$ vector of ones. Orthogonality of $\epsilon$ to $R$ together with non-negativity of its variance implies

$$\begin{aligned} V(m) &\geq \pi_0' E[(X - EX)(X - EX)']\pi_0 \\ &= (e - EmER)'\Omega^{-1}(e - EmER), \end{aligned} \tag{5}$$

where $\Omega$ denotes the covariance matrix of $R$. The expression (5) can be written in the terms of the standard deviation of the stochastic discount factor as

$$\sigma_m \geq [(e - EmER)'\Omega^{-1}(e - EmER)]^{1/2}. \tag{6}$$

The restriction (6) does not require the stochastic discount factor to be specified and parametrized. The bound depends entirely on observable moments of returns

$$\bar{R} = \frac{1}{T}\sum_{t=1}^{T} R_t, \quad \hat{\Omega} = \frac{1}{T}\sum_{t=1}^{T}(R_t - \bar{R})(R_t - \bar{R})'. \tag{7}$$

The bound then can be calculated for a given $Em$. However, the choice of returns included in the volatility bound is up to a researcher. Most researchers include some measure of the US stock market performance such as the return on the S&P 500 Index. However, a large portion of household wealth is invested in real estate and this report argues that the return on housing should be included as well to study implications of the housing market fluctuations for consumption. In the context of asset pricing theory summarized here by the volatility bounds, the impact of housing prices on consumption are both direct and indirect. The direct impact is via equation (1), which captures the interaction between the housing return and the discount factor. The indirect impact is via its effect on the stock market return. It manifests itself in the off-diagonal terms of the variance-covariance matrix of returns $\Omega$. There is also the question of whether a risk-free return should be included. If the risk-free rate $r_f$ exists, the equation (1) implies that $Em = 1/(1 + r^f)$. Since $r_t^f$ varies over time with its return to be riskless only between periods $t$ and $t + 1$, it can be approximated by its sample mean.This determines the mean of the stochastic discount factor as well. A typical proxy for the risk free rate is the rate of return on the short-term Treasury Bills. Alternatively, one can treat this return as risky due to inflation and due to a tiny but non-zero probability of default. In this case, the Treasury Bill would be treated as another asset in addition to stocks and to real estate. Both possibilities are explored here.

Guvenen and Lustig (2007b) summarize numerous attempts to resolve the equity premium puzzle. They classify the attempts based on what is modified: (i) preferences, (ii) the markets and asset structure, and (iii) the endowment process. Here we focus on the various preference specifications designed to generalize the CRRA utility function in a

way which allows to generate a larger equity premium without using an unreasonably high relative risk aversion parameter.

A popular approach to resolve the puzzle is based on the following specification of the utility function:

$$U_t = \frac{(C_t - W_t)^{(1-\varphi)}}{(1-\varphi)}, \tag{8}$$

where $W_t$ is a function of the investor's past consumption or of consumption of some reference group. We speak of internal habit in the former case (see Sundaresan 1989 and Constantinides 1990) and catching up with the Joneses or external habit in the latter case (see Abel 1990 and Campbell and Cochrane 1999). Recent research considers preferences generating dynamics of the SDF similar to those produced by the external habit formulation. We concentrate on the inclusion of housing services into the utility function in Piazzesi, Schneider, and Tuzel (2007). Later we would like to focus on the impact of durable consumption in Yogo (2006). Both papers claim significant improvement of asset pricing performance of their model.

# 3 Housing Services and Volatility Bounds with Real Estate Returns

This section first derives the restrictions on the moments of a stochastic discount factor. The bounds are calculated using not only the stock market returns but also the returns on housing. The restrictions are then applied in a special case where the stochastic discount factor is specified using a model with housing consumption. The restrictions are used to find parameters of this model satisfying the volatility bounds.

## 3.1 Parametrization of the Pricing Kernel

While the stochastic discount factor (the pricing kernel) does not have to specified for the volatility bound to be computed, including real estate returns raises the issue of a proper treatment of housing. A house is not only an asset but it also affects the utility of a consumer by providing housing services. Therefore, rather then focusing on implications of the Hansen-Jagannathan bounds for a general pricing kernel, the discount factor is specified here using an asset pricing model with housing. Piazzesi et al. (2007) supplies a convenient framework for this type of analysis.

The utility function in an economy with many identical agents is given by

$$E\left[\sum_{t=0}^{\infty}\beta^t u(C_t)\right] \tag{9}$$

where $C_t$ is aggregate consumption and

$$u(C_t) = \frac{C_t^{1-1/\gamma}}{1-1/\gamma}. \tag{10}$$

$\gamma$ represents the intertemporal elasticity of substitution. $1/\gamma = \varphi$ is the relative risk aversion coefficient. $\beta$ characterizes the time preference. The aggregate consumption is a function of nonhousing consumption of nondurables and services $c_t$ and shelter $s_t$:

$$C_t = w(c_t, s_t) = (c_t^{(\lambda-1)/\lambda} + \phi s_t^{(\lambda-1)/\lambda})^{\lambda/(\lambda-1)}. \tag{11}$$

$\lambda$ denotes the intratemporal elasticity of substitution. Shelter captures consumption of housing services. The life-time utility is maximized subject to the budget constraint

$$p_t^c c_t + p_t^s s_t + q_t^e \xi_t^e + q_t^h \xi_t^h + q_t^f \xi_t^f = (q_t^e + p_t^e \bar{c}_t)\xi_{t-1}^e + (q_t^h + p_t^s \bar{s}_t)\xi_{t-1}^h + \xi_{t-1}^f, \tag{12}$$

where $\xi_t^e$, $\xi_t^h$, and $\xi_t^f$ are asset holdings for equity, housing stock, and a risk-free asset, respectively. The corresponding asset prices are $q_t^e$,$q_t^h$ and $q_t^f$, respectively. $p_t^c$ and $p_t^s$ are prices for nonhousing consumption and for shelter, respectively. In equilibrium, $c_t = \bar{c}_t$, $s_t = \bar{s}_t$, $\xi_t^e = \xi_t^h = 1$ (positive net supply), and $\xi_t^f = 0$ (zero net supply).

The Euler equation for the agents' optimization problem is again

$$1 = E[m_{t+1}(1 + r_{i,t+1})], \quad i = 1, 2, 3, \tag{13}$$

where $r_{1,t+1} = r_{t+1}^e = (q_{t+1}^e + k_{t+1})/q_t^e - 1$, $r_{2,t+1} = r_{t+1}^h = (q_{t+1}^h + s_{t+1})/q_t^s - 1$, and $r_{3,t+1} = r_{t+1}^f$. $k_t$ and $s_t$ are dividends and rents, respectively. The pricing kernel is given by:

$$m_{t+1} = \beta\,\frac{u'(C_{t+1})\,w_1(c_{t+1}, s_{t+1})}{u'(C_t)\,w_1(c_t, s_t)} = \beta\left(\frac{c_{t+1}}{c_t}\right)^{-1/\gamma}\left(\frac{1+\phi(\frac{s_{t+1}}{c_{t+1}})^{(\lambda-1)/\lambda}}{1+\phi(\frac{s_t}{c_t})^{(\lambda-1)/\lambda}}\right)^{(\gamma-\lambda)/(\gamma(\lambda-1))}. \tag{14}$$

The stochastic discount factor (14) is expressed in real terms for $c_t$ and $s_t$, which are difficult to measure precisely. The static first-order conditions imply that the marginal rate of substitution between housing and nonhousing consumption is equal to their price ratio, i.e.

$$\frac{p_t^c}{p_t^s} = \frac{w_1(c_t, s_t)}{w_2(c_t, s_t)} = \phi^{-1}\left(\frac{c_t}{s_t}\right)^{-1/\lambda}. \tag{15}$$

Using (15), the expenditure ratio is obtained as follows:

$$z_t = \frac{p_t^c c_t}{p_t^s s_t} = \phi^{-1} \left(\frac{c_t}{s_t}\right)^{1-1/\lambda} = \phi^{-\lambda} \left(\frac{p_t^c}{p_t^s}\right)^{1-\lambda}. \tag{16}$$

The expenditure share on non-housing consumption is defined as

$$\alpha_t = \frac{z_t}{1+z_t}. \tag{17}$$

The intertemporal marginal rate of substitution (14) then can be expressed in terms of nonhousing consumption and the expenditure share on nonhousing consumption:

$$m_{t+1} = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-1/\lambda} \left(\frac{\alpha_{t+1}}{\alpha_t}\right)^{(\lambda-\gamma)/(\gamma(\lambda-1))}. \tag{18}$$

Given historical asset returns and observed consumption and nonhousing consumption, the parameters needed to calculate the expected value and variance of the pricing kernel (18) are $\beta, \gamma$, and $\lambda$. The objective is to find values for which the Hansen-Jagannathan bound (6) is satisfied, assuming such values exist.

## 3.2 Results

In this section, the empirical Hansen-Jagannathan bound is calculated using various combinations of the US stock, housing, and risk-free returns. Then we generate the time series for the stochastic discount factor implied by the housing model from above and the US housing and nonhousing consumption data. The Intertemporal Marginal rate of Substitution (IMRS) is parametrized to satisfy the bound with all three returns. Finally, the interaction of asset returns and the pricing kernel is discussed. The data used in this report are annual from 1929 to 2008. This time period includes the most recent cycle of the boom and a collapse on the housing market, as well as the decline of the stock market in the early 2000s.

The housing return is constructed using data from National Income Product Accounts (NIPA). The value of the housing stock is given by the depreciation-adjusted current-cost of residential structures. The real estate-related cash-flow is represented by the aggregate rental income. The return is adjusted to property tax. It is the only available return on housing in the US, which takes into account the rental cash-flow. The exact source series for this return and all the other data are described in detail in Data Appendix. The peaks and troughs of the nominal housing return are displayed in Figure 1, which shows high property returns after the end of the World War II, in the late 1970s and in

the early 2000s. There is also a peak in the mid 1980s, which is relatively small. Note that high-returns do not necessarily coincide with high real estate prices. This is due to the rental income as well as due to the fact that prices have to grow fast to generate high returns but they do not have to be high in absolute terms.

The NIPA real estate return is compared to all major US housing returns used elsewhere in literature (see the Data Appendix for details). Flavin and Yamashita (2002) use the average risk-free rate to substitute for the lack of rental income in their definition. We calculate the property return according to their definition but employing the NIPA data. In addition, other nominal returns are used, which simply define the housing return based on a real estate price appreciation. Sources for these are the website of Robert Shiller, the Federal Housing Finance Agency (FHFA), and the National Association of Realtors (NAR), respectively. Summary statistics comparing these series using a common sample from 1976 to 2008 are in Table 1 and the series are plotted in Figure 1. The series have all very similar properties with returns based on the Shiller data being a potential outlier. The graph however illustrates that it moves with the other series most of the time even though the time of peaks and troughs is somewhat different.

Performance of the stock market is measured by returns on the S&P 500 index. The returns on three-months T-Bills approximate the risk-free rate. Finally, all nominal returns are adjusted to inflation, which is measured by the difference in growth rates between nominal and real non-housing consumption from NIPA. The summary statistics for real returns are shown in Table 2. The mean values for returns are as expected, with the equity risk premium at about 7%. The housing return is smaller than the stock market return but it seems also less risky based on its standard deviation. Interestingly, the mutual correlations are fairly low.

The real returns are used to construct the volatility bounds for the stochastic discount factor. The bounds are calculated for all seven combinations of the three considered returns for Em=0.80 to 1.20 in intervals of 0.01. Only three combinations are displayed in Figure 2 - these sufficiently illustrate the relevant pattern. Adding one asset at a time gradually restricts the subset of plausible combinations of $\sigma_m$ and $Em$. The minimum for the Hansen-Jagannathan bound for the three returns is close to the point of $\frac{1}{1+r_t^f} = 0.9979$ which equals to $Em$ assuming there is a risk-free asset. This turns out to be relevant for the parametrization of the pricing kernel.

The objective is now to find parameter values for the stochastic discount factor (18), for which the volatility bound (6) is not violated. Rather than attempting to estimate preference parameters subject to the volatility restriction, a simple grid search is conducted over the set of 'reasonable' parameter values. This search sets the rate of time

preference $\beta$ to 0.99. Plausible levels of the risk aversion $\gamma$ are typically considered to be between 0 and 5. For the risk-aversion of 5, a consumer with the standard power utility function is willing to pay 9% to avoid facing a bet, where she has 50-50 chance of winning or loosing 20% of her wealth. Piazzesi et al. (1997) set the elasticity of substitution between housing services and nonhousing consumption $\lambda$ to 1.05 and 1.25, respectively. The data for consumption and housing expenditures (shelter) are from NIPA tables. Details are in Data Appendix. Summary statistics as well as correlations with the three returns are given in Table 2. The expected value of the stochastic discount with $\lambda = 1.06$ and $\gamma = 4.66$ is 0.9981 and its standard deviation is 0.7603, which is greater than 0.6998, the Hansen-Jagannathan bound for our three assets. This stochastic factor has the expected value closest to $\frac{1}{1+r_t^f} = 0.9979$ for the defined subset of parameters. Figure 3 depicts the relationship between $Em_{t+1}$ and the standard deviation of $m_{t+1}$ for $\lambda = 1.06$ and $\gamma = 0.01, 0.02, ..., 5.00$. For illustration, we also show a similar curve for $\lambda = 1.12$ and $\gamma = 7.10, 7.11, ..., 12.10$. This indicates that higher values of the elasticity of substitution require higher levels of risk aversion to generate a pricing kernel satisfying the volatility condition.

# 4 Robustness of the Results and Future Research

In this paper, we show that the CCAPM with housing services can generate sufficient volatility of the stochastic discount factor even with stricted volatility bounds including real estate returns. However, results from Davis and Martin (2005) suggest that the empirical success of the model from Piazzesi, et al. (2007) may be sensitive to the used data frequency. To investigate the issue, we construct quarterly US data from NIPA. The data include per capita consumption of nondurables and services, as well as consumption of housing services. We also construct quarterly series of durables' consumption, following Yogo (2006) and Pakoš (2011a). The series needs to be constructed since the stock of durables is only available annually. We calculate the quarterly stock by assuming constant depreciation in a given year. The average quarterly depreciation is 5.2%. We then proceed to use the same algorithm to construct residential stock, which we combine with imputed rents and price indices for residential investment to calculate real estate returns. The average quarterly residential depreciation is 0.33%. We believe that we are the first one to calculate quarterly housing returns with rents. We add returns on stocks (including dividends) on S&P 500 Index and US government bonds. This means we have a complete set of data for testing various versions of the CCAPM.

We have conducted some preliminary tests with our quarterly data and CCAPM with consumption of nondurables and services, durables, and housing services. We have not been able to find plausible parameter values with sufficiently high volatility of SDF to be above the Hansen-Jaganntha volatility bounds. This seems to be the case for both power utility and Constant Elasticity of Substitution (CES) preferences. For power utility, we use the fact that Hansen-Jagannthan bounds imply that the ratio of standard deviation to the expected value of SDF is greater than the maximum Sharpe ratio, i.e. the ratio of an asset risk premium to its standard deviation. This ratio is highest for housing returns. The power utility model can produce SDF with sufficient volatility only for risk aversion greater then 136. Therefore, simply adding housing and durables does not resolve the equity premium puzzle. We have estimated the CES model by GMM and rejected it using the Hansen J test. We have also randomly searched for parameter values, which would generate sufficient volatility of SDF but so far have not found them. This is interesting since Piazzesi, et al. (2007) uses CES preferences with housing services (with no durables) and the model generate sufficient variance of SDF, as we have shown. The difference is either due to slightly different definitions of data or different data frequency. For example, Piazzesi, et al. (2007) do not use per capita consumption while Yogo (2006) does. The definitions of various NIPA series can be slightly different at different frequencies and some parameters of CCAPM may have different meaning for different frequencies.

We intend to investigate this issue thoroughly. We will look at three models with consumption of nondurables and services, durables, and housing services with power utility, CES, and Epstein & Zin (1989) preferences. There will be annual and quarterly data frequencies, and total consumption vs per capita consumption. Overall, this is 12 combinations. We would like to test the models using Hansen J test plus using a classical test based on Hansen-Jagannathan bounds (e.g. see Burnside 1994). If we do find a plausible SDF we can model it individually or jointly with stock and real estate premia using a Markov chain.[1]

[1]We have plenty of experience with Markov switching models, e.g. see Pakoš (2011b), Zemčík (2001,2006).

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## Data Appendix: Annual Data

The housing return is constructed using NIPA tables. Note that the numbering of NIPA tables has changed recently and therefore there is not exact one-to-one matching of the data series presented here to the data in Piazzesi et al. (2007). The nominal housing return is calculated as:

$$r_t^h = (q_t^h \; \xi_t^h + p_t^s \; s_t)/q_{t-1}^h \; \xi_{t-1}^h - (1 - \tau_i) \; \tau_p - 1. \tag{19}$$

where $q_t^h \; \xi_t^h$ is the current-cost of *residential structures* taken from the Fixed Assets Table 2.1. The table provides the year-end estimates (in billions of dollars) of Current-Cost Net Stock of Private Fixed Assets, Equipment and Software, and Structures by Type. The costs of residential structures are in line 59 of the table. Same information is given in line 1 of Fixed Assets Table 5.1, which is Current-Cost Net Stock of Residential Fixed Assets by Type of Owner, Legal Form of Organization, Industry, and Tenure Group. The costs of residential structures account for both depreciation and the size of the housing stock and therefore there is no need to further adjust the formula for $r_t^h$. $q_t \; s_t$ is the rental income of persons with capital consumption adjustment from line 21 in Table 7.4.5. entitled Housing Sector Output, Gross Value Added, and Net Value Added. $\tau_i = 0.33$ and $\tau_i = 0.025$ are the marginal income tax rate, and the property tax rate, respectively.

The nominal housing return constructed according to equation (19) has been compared to several widely used measures of appreciation of the real estate value. The first such measure is computed following Flavin and Yamashita (2002) according to

$$\frac{q_t^h}{q_{t-1}^h} + \bar{r}^f + \tau_i \; \tau_p - 1. \tag{20}$$

The housing price $q_t^h$ is calculate using the current cost of residential cost structures $q_t^h \; \xi_t^h$ discussed above, which is divided by the the chain-quantity index for *residential fixed assets* from line 1 in Fixed Assets Table 5.2 named Chain-Type Quantity Indexes for Net Stock of Residential Fixed Assets by Type of Owner, Legal Form of Organization,

Industry, and Tenure Group. $\bar{r^f} = 0.075$ is from the risk-free rate used in this paper. $\tau_i = 0.33$ and $\tau_i = 0.025$ as for the NIPA housing return. Several other nominal housing returns are calculates as a change in real estate prices, i.e. $q_t^h/q_{t-1}^h - 1$. This data are from the website of Robert Shiller, from the FHFA, and from the NAR (Median Sales Price of Existing Single-Family Homes), respectively.

The stock returns are from the web-site of Robert Shiller and are computed as

$$r_t^e = (q_{t+1}^e + k_{t+1})/q_t^e - 1 \tag{21}$$

where $q_t^e$ is the S&P 500 Index and $k_t$ is the corresponding dividend series. The risk-free rate is from Mehra and Prescott (1985) from 1930 to 1933. From 1934 to 2008, it is the 3-Month Treasury Bill Secondary Market Rate from the Board of Governors of the Federal Reserve System. All nominal returns are adjusted to inflation by the difference between the real and nominal non-housing consumption growth rates.

NIPA is the data source for consumption growth. The real aggregate consumption $c_t$ is calculated using lines from the nominal Personal Consumption Expenditures in Table 2.3.5. adjusted to inflation by the Personal Consumption Index from Table 2.3.4. Specifically, the non-housing consumption series is given by the consumption of *nondurable goods* (line 6) and *services* (line 13) minus *clothing and shoes* (line 8) and minus *housing services* $s_t$ (line 14). The expenditure ratio $z_t$ is the ratio of the nominal non-housing consumption and expenditures on housing services from Table 2.3.5 (line 14). Given $z_t$, the expenditure share on non-housing consumption is calculated using (17). The total consumption $C_t$ is the sum of nondurable goods and services (i.e. lines 6 and 13).

Table 1: Housing Returns Statistics

Notes:
(1) Annual data, sample 1976-2008.
(2) NIPA denotes returns calculated using National Income Product Accounts; FY - stands for returns calculated using NIPA data and the Flavin Yamashita definition of housing returns; FHFA are real estate returns computed using data from Federal Housing Finance Agency; and NAR - returns using the data from the National Association of Realtors.

| | NIPA | FY | SHILLER | FHFA | NAR |
|---|---|---|---|---|---|
| Mean | 6.20 | 6.01 | 0.87 | 5.65 | 5.62 |
| Std. Dev. | 4.24 | 3.71 | 6.77 | 3.51 | 4.48 |
| Correlations | | | | | |
| NIPA | 1.00 | | | | |
| FY | 0.99 | 1.00 | | | |
| SHILLER | 0.43 | 0.36 | 1.00 | | |
| FHFA | 0.90 | 0.89 | 0.38 | 1.00 | |
| NAR | 0.89 | 0.89 | 0.39 | 0.85 | 1.00 |

Table 2: Data Summary Statistics

Notes:
(1) Annual data, sample 1929-2008.
(2) $c_t$ and $C_t$ are the real non-housing and total consumption expenditures for nondurables and services.
(4) $r_t^h, r_t^s$, and $r_t^f$ are real housing, stock, and risk-free returns.

| | $\Delta \ln c_t$ | $\alpha_t$ | $\Delta \ln \alpha_t$ | $\ln z_t$ | $\Delta \ln s_t$ | $\Delta \ln C_t$ | $r_t^h$ | $r_t^e$ | $r_t^f$ |
|---|---|---|---|---|---|---|---|---|---|
| Mean (%) | 3.01 | 82.14 | 0.04 | 153.09 | 3.43 | 3.08 | 2.83 | 7.39 | 0.35 |
| St. dev. | 2.30 | 1.74 | 0.71 | 12.36 | 2.04 | 2.16 | 4.08 | 19.38 | 3.90 |
| Correlations | | | | | | | | | |
| $\Delta \ln c_t$ | 1.00 | | | | | | | | |
| $\alpha_t$ | 0.41 | 1.00 | | | | | | | |
| $\Delta \ln \alpha_t$ | 0.46 | 0.14 | 1.00 | | | | | | |
| $\ln z_t$ | 0.37 | 1.00 | 0.13 | 1.00 | | | | | |
| $\Delta \ln s_t$ | 0.48 | 0.57 | -0.01 | 0.56 | 1.00 | | | | |
| $\Delta \ln C_t$ | 0.99 | 0.44 | 0.41 | 0.41 | 0.58 | 1.00 | | | |
| $r_t^h$ | 0.24 | 0.19 | -0.06 | 0.20 | 0.27 | 0.25 | 1.00 | | |
| $r_t^e$ | 0.13 | 0.02 | 0.07 | 0.01 | -0.08 | 0.10 | 0.17 | 1.00 | |
| $r_t^f$ | -0.39 | -0.74 | -0.57 | -0.73 | -0.48 | -0.41 | -0.09 | 0.07 | 1.00 |

Figure 1: Nominal Housing Returns

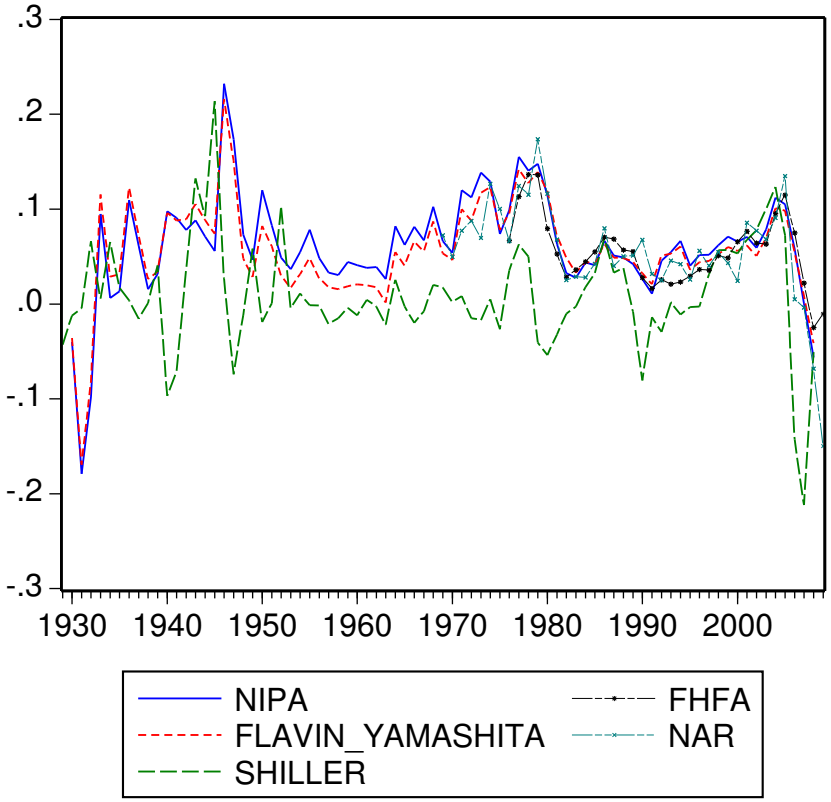

Figure 2: Volatility Bounds

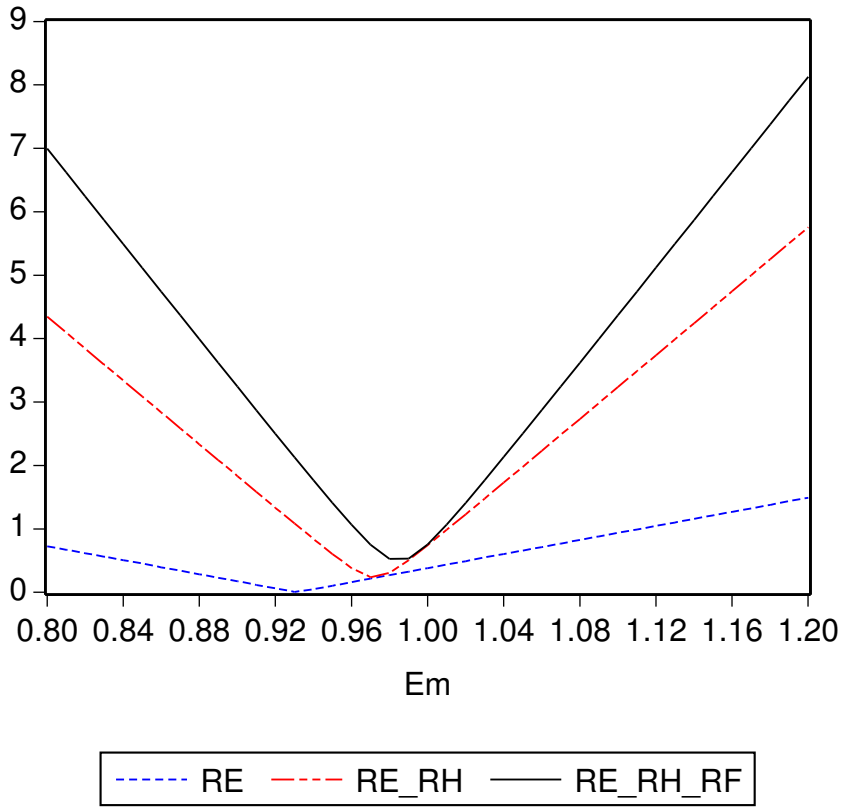

Figure 3: Pricing Kernel Volatility

Notes:
(1) ra is risk aversion $\gamma$ and elasticity is the intratemporal elasticity of substitution between housing and nonhousing consumption $\lambda$.
(2) re_rh_rf is the volatility bound implied by the data on the equity, housing, and risk-free returns, respectively.