Game Theory and Real Options: An alternative to the replicating portfolio

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Eidhovenn, 17th June 2011
1. Motivation

2. Case Study
   - NPV valuation
   - Real Option valuation and related problems

3. Competition and Game Theory

4. Some results
This paper contributes to a novel literature which joins Real Option Theory and Game Theory.

Literature on Real Estate:
- Only RO: Titman (1985), Williams (1993), Grenadier (1995) and many others
- RO-GT: Smit and Ankum (1993), Grenadier (1996) and few others
- Constant BIG PROBLEM: short sales/replicable portfolio

We focus on Multiple optimal investment decisions
Offer a first solution to the big problem
The site was acquired at the price of £12.78m.

The difference between the annual cost of £150k to keep the strategic option open, and the annual income generated by a car park managed on the site is marginal.

- We assume that there is no either cost or income in deferment other than financial costs related to discounting (i.e. the dividend is equal to zero).

The local authority wishes to see the site completely developed and has already granted planning permissions for the actual development to be started within the next 5 years. Whenever the investor wishes to abandon the scheme within the next 5 years, she has to sell it back to the local authority at a fixed price of £8m.
### NPV valuation

**Month**

<table>
<thead>
<tr>
<th>Time Index (t)</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
<th>18</th>
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<tr>
<td>Property Sale</td>
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<tr>
<td>Land Acquisition</td>
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<tr>
<td>Construction Costs</td>
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<td>Prof Fees</td>
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<td>-0.22</td>
<td>-0.24</td>
<td>-0.25</td>
<td>-0.35</td>
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**FCF_t**

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**Annual WACC (k)**: 9.00%

**Quarterly WACC (kQ)**: 2.18%

|  |  |  |  |  |  |  |  |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| NPV<sub>r</sub> | 10.26          |  |  |  |  |  |  |
| NPV<sub>c</sub> | 22.98          |  |  |  |  |  |  |
| PV (selling price) | 79.93          |  |  |  |  |  |  |
| PV (construction phase) | 56.95          |  |  |  |  |  |  |
NPV reconstruction

Simple model:
The market can either go up or down with probabilities $q$ and $(1-q)$

- Upward jump $\equiv u = \exp^{\sigma \sqrt{\Delta t}}$
- Downward jump $\equiv d = \exp^{-\sigma \sqrt{\Delta t}}$
Deferral Option Value (incl. NPV)

Option value
\[ C_t = \exp^{-r\Delta t} \left( q \max[V_u, C_{t+\Delta t, u}] + (1 - q) \max[V_d, C_{t+\Delta t, d}] \right) \]

EMM
\[ q = \exp^{rF \Delta t - d} \]
Decision Tree Analysis

- Proposed solution by Boris, *JACF 2005*

- EMM $q = \frac{\exp^{rW*\Delta t} - d}{u-d}$

\[\text{t: 0.00 1.00 2.00 3.00 4.00 5.00}\]

\[23.46 \quad 28.82 \quad 35.25 \quad 42.92 \quad 52.00 \quad 62.63\]

\[9.92 \quad 12.89 \quad 16.73 \quad 21.72 \quad 28.20 \quad 1.57\]

\[0.69 \quad 0.91 \quad 1.19 \quad \text{abandon} \quad \text{abandon} \quad \text{abandon}\]
A problem of arbitrage (DTA)

- Problem: Risk-adjusted discount rates are constant
- Arbitrage: Is not risk changing along the tree??
A problem of arbitrage (*back to ROA*)

- Risk-adjusted discount rates are **not** constant
- No Arbitrage given **replicable portfolio**
Real Option and Game Theory Analysis

Cournot Model: \( P = a - b\overline{Q} \), \( Q_L > Q_S > Q_F \)

Two options: (i) Defer and (ii) Decide the Size

No replicable portfolio assumed, EMM \( q = \frac{\exp^{rW\Delta t} - d}{u-d} \)
Obtained Decision Tree and Valuation

- Which *number* when multiple equilibria? Gabrieli and Marcato, 2010

- No replicable portfolio assumed, what about arbitrage?
Risk-adjusted discount factor varies

Arbitrage opportunities **not** based on **replicable portfolio** have been excluded
The impact of competition

<table>
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<th>DEFER/NPV</th>
<th>b=0.1</th>
<th>b=0.3</th>
<th>b=0.5</th>
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<tr>
<td></td>
<td>20 %</td>
<td>24 %</td>
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The impact of equilibrium selection rules

<table>
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<th>DEFER/NPV</th>
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<th>Average</th>
<th>Pessimistic</th>
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<tr>
<td></td>
<td>11.5 %</td>
<td>9.2 %</td>
<td>8.56 %</td>
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Game Theory and Real Options
Conclusion

- Contribution
  - Comparison of various approaches
  - Risk-varying discount rates
  - No evident arbitrage opportunities
- Questions ? Suggestions ?