Liquidity Black Hole and Optimal Behavioral Model

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Abstract

In turmoil periods, market liquidity can experience sudden dry ups connected with a significant price movements. This unexpected changes in liquidity patterns, often driven by irrational investors' behaviour,and it is normally defined as Liquidity Black Hole (LBH). So far relevant research in this area explored macro-market level rather than explaining micro-agent decisions.

In this study we show - both theoretically and empirically - that the LBH effect at market micro-level is originated by agents' decisions made at a mutual fund level. We present a model on the behaviour of investors as a function of expected market risks and returns. The causes of a LBH are analyzed and the model is also tested under real case scenario, i.e. UK Real Estate Mutual Fund industry. Price creation is modeled both endogenously and exogenously, and it shows that the relationship between fund flows and expected liquidity risk follows an exponential function. Finally, we demonstrate that areas of absolute LBH exist and cannot be hedged. In those areas neither the available "cash-like cushion" nor the managerial skills of the market maker can avoid the "economic failure" of a fund.

1 Introduction

Liquidity is becoming a key phenomenon to explain the development of asset pricing and market movements. Over the last decade several studies have addressed relationship between asset pricing and market movements. Some empirical studies such as one by Goetzman and Ivkovic (2001) demonstrate the performance-driving effect of flows on mutual fund pricing, focusing on the difficulties in price determination due to international investment and differences in liquidity levels of underlying assets. Mutual funds, in particular, represent an interesting case study for such issues because they are characterized by inflows and outflows that are not necessarily and instantaneously matched, showing temporary imbalances in liquidity. Among others, two main determinants of liquidity have been identified in the literature: price signalling when investors reveal future price movements in their investment/disinvestment activities; return chasing behavior when market players read into past fund performances and take subsequent investment decisions on the basis of momentum or contrarian strategies.

In explaining the reasons behind liquidity movement and pricing, the behavioral aspects are by far the most interesting ones. Investors fear illiquidity for various reasons such as the possibility of facing personal liquidity shocks and consequently being forced to liquidate in a bearish market environment; the tendency to move the asset allocation of a portfolio from illiquid to more liquid investments when a liquidity crisis is approaching (i.e. flight to quality); the possible redemption of stocks at a price below expected values; and denial of redemptions due to wing up solutions or the presence of insufficient resources. This latter phenomenon is known as Liquidity Black Hole (i.e. LBH) and was first illustrated by Morris and Shin (2004), the same phenomenon has also been called liquidity run, liquidity spiral and flight to quality. Substantially, a LBH represent a run to liquidity due to a shock in the market. This shock is originated by the fear of an event or by a violation of specific conditions. Moreover, if the performance and price of an asset are determined by the behavior of other agents, investors can be influenced - contagion - by the general behavior, even if their individual conditions are not violated. This run to liquidity causes a sudden and deep change in the market driven by a strongly one-sided flow of money (i.e. outflow). The immediate effect is a sudden dry-up of liquidity, with consequences on the pricing of such assets. Metaphorically, it is as if the liquidity is completely sucked up by an invisible hole, exactly as it happens in astrophysics for black holes and matter.

In this paper we apply the LBH concept to mutual funds and study the process in which a LBH is formed, identifying the main driving factors, its possible causes, determining the extent to which each single factor contributes to the formation of a LBH. Starting from the investment pattern of mutual funds, the origination of inflows and outflows is studied. The optimal behavior of an investor is examined as a function of both exogenous and endogenous market events. Through this procedure we are accounting for evidences of sources for the origination of a LBH. This study will also then be able to raise some concerns about policy issues and implications and needs of self-regulation in mutual fund markets

The price is first obtained starting from the normal interaction between buyers and sellers (i.e. market clearing conditions). Subsequently, the price is imposed as exogenous information by the fund manager. The main difference of the two approaches are considered and analyzed. The fund manager and market maker can react to the creation of a LBH, and therefore modify the likelihood of a liquidity run. More specifically, the manager can interact with the idiosyncratic risk and can use some instruments or policies to minimize the impact. However for higher values of idiosyncratic risk there are no possibilities of hedging against losses and therefore a liquidity black hole can be originated. Anyway, as Hawkings' radiation theory suggests, the black hole reduces its energy over time and it disappears in the long-run, bringing values back to pre-hole conditions. The time for a complete recovery depends on the magnitude of the hole, and on the wealth of the market. Hameed, Kang, and Viswanathan (2010) demonstrate that a LBH lasts 1-2 weeks on average in the US equity market. The period is identified by external intervention (e.g. government) deemed to pre-empt a deterioration of the situation through injection of liquidity in the system. We argue that this timing depends on the underlying assets' liquidity, with less liquid markets (e.g. real estate) showing longer periods than more liquid ones (e.g. equity).

For this reason, after presenting a theoretical model, we use the real estate mutual fund (i.e. REMF) industry to run an empirical analysis on an existing fund in order to test the ability of our theoretical model to predict market phenomena. During the most recent period, in fact, we have noticed a steep rise of REMFs market performance, followed by a sudden decline in prices due to the presence of a very high liquidity in the early-mid 2000s and a sudden liquidity dry-up from the second half of 2007.

Several motivations are behind this choice. Firstly, the liquidity issue is more relevant

than for other funds due to the very illiquid nature of underlying assets. When there is a sudden increase of unit redemptions (i.e. outflows), fund managers cannot easily sell assets in a short space of time (normally it takes 6 to 9 months to complete a transaction in real estate markets) and hence they try to maximize inflows to match potential outflows. On one hand, this feature could cause acceleration in the LBH phenomenon - see Huang and Wang (2010) - due to the lack of actions able to generate immediate cash. However, on the other hand, this type of assets could attract more long-term conservative investors, who may be more able to absorb liquidity shocks in the short run and then cause a deceleration of a LBH. Secondly, a recent paper by Marcato and Tira (2010) using a panel VAR model demonstrates that inflows and outflows reflect different behavioral attitudes of investors (respectively return chasing and pricing signal). Hence there is asymmetric information content between flows in and out of the fund. Even if in our model we do not model this asymmetry and treat buyers and sellers similarly, a natural extension of our model could incorporate this new assumption and theoretically find solutions explaining different behavioral attitudes. A third reason for using REMFs lies on the fact that they represent one of the very few industries where the fund manager and market maker coincide. The manager of an open ended REMF determines the bid-ask spread of the unit and is obliged to provide and redeem units to investors asking for it. As already mentioned above, since the secondary market is limited and it is mainly managed by in- and outflows matching, market makers take decisions in relation to the fund performance and they are not only interested in widening the bid-ask spread. This co-participation provides a considerable change in the utility function of the market maker. Vayanos (2004), for example, demonstrates that the market maker is not simply risk averse and subjected to a problem of maximization of the bid-ask spread. In this context, the utility problem for the market maker becomes the maximization of the bid-ask spread under the optimization of the manager's utility function (i.e. maximization of net flows). Furthermore the market maker is averse to the LBH because it reduces the performance of the fund, while in the existing liquidity literature, the market maker is only considered in a LBH problem for its inventory skills. In the classical approach, he is not damaged by the LBH because he simply adjusts the inventory and changes prices in response to investors' choices having a long-term investment horizon.[1] Fourth, the execution order in REMFs follows a FIFO (first in first out) rule. Therefore the possibility of redemption is dependent upon the decision

[1]However, Huang and J.Wang (2010) demonstrate that, under precise assumptions, the market making sector can hedge against the problem of liquidity shocks.

of all other investors in the fund. This feature can create panic and ultimately originate a LBH as Bernardo and Welch (2004) demonstrate. If an investor is willing to redeem but is aware that the likelihood of redemption depends on the decision of other investors, her fear of future liquidity shocks may lead her to redeem before the possibility of a liquidity run, causing the liquidity run itself. It is also true however that even equity mutual funds sometimes are not perfectly sequential in their execution orders. This would be an explanation of why a LBH could also be originated in stock markets. Following this assumption we decide to develop our model for the overall mutual fund industry and to use the real estate case only for empirical purposes. Finally, since we consider the existence of a cost of participation in the market, a phenomenon allude to by Huang and Wang (2009, 2010), the illiquidity is originated by a non-full agents' participation - these costs (e.g. brokerage, fund raising, entry, management and redemption fees) tend to be higher for REMFs than for other industries.

Our paper brings a new contribution to the existing literature as it represents the first attempt to model a LBH in a mutual fund context. Previous literature analyzed the liquidity run in a general financial market contest focusing on agents inside that market. The driving factors are normally derived from the general market condition (often simplified in a single-asset market) and this approach does not consider that, although a LBH may exist, there may also be some very liquid instruments. Consequently, we argue that a LBH first originates at a fund level and then can spread to other funds or markets through a contagion effect. At the same time, we may find evidence of a global negative economic situation increasing the likelihood of a LBH in many funds, without however providing the concrete evidence of a contagion effect. Klaus and Rzepkowski (2009) expore the spillover effect on hedge funds and find that a liquidity run can occur for both internal redemption issues and through contagion effect from other funds. They show that bad market conditions are more likely to drive a consistent redemption flow in the fund rather than a contagion effect in the market.

The balance of the paper is as follows: section 2 presents the relevant literature; section 3,4 and 5 respectively present the theoretical model, the relative market equilibrium and the optimal investment decisions. Section 6 discusses the possibility of a liquidity black hole and an application of the model to a real case scenario.

2 Literature review

Liquidity risk is becoming more relevant in asset pricing and particularly in real estate markets as Brounen, Eichholtz, and Ling (2009) demonstrate. They define liquidity as the speed of sale and price impact. They describe three measures of liquidity as trading (connected with selling speed), turnover and illiquidity. They show that the market capitalization is an important variable to define liquidity because of the higher volume connected with specific assets. Finally they discover a relationship between liquidity and asset pricing: the smaller the bid-ask spread is and the faster the movement of the security is (i.e. higher liquidity).

On one hand several papers focus on the effect of liquidity on asset pricing within stock and real estate markets, as well as for specific instrument (e.g. Brounen, Eichholtz, and Ling (2009), Marcato and Ward (2007), Subrahmanyam (2007), Bollen, Smith, and Whaley (2004), Hameed, Kang, and Viswanathan (2010), Aragon (2007), Sadka (2010)). Particularly Allen and Carletti (2008) demonstrate that during downturns, such as the current credit crunch, prices exchanged in the market are more representative of asset liquidity than future payoffs. Furthermore they show that prices are a function of liquidity, which can modify the asset performance. Finally, some articles specific to the real estate sector demonstrate the significance of liquidity in the pricing of assets such as REMFs - e.g. Marcato and Tira (2010), Gullet and Redman (2005), Tomperi (2009), O'Neal and Page (2000).

On the other hand there is an abundant literature on the effect of liquidity on investors' decisions. The behavioral finance/economics literature concentrates on both the origination of such phenomenon and its solutions. The origination of the decision process differs between studies: some present the fear for the worst scenario as the dominant effect. This fear causes liquidity run or a LBH, which is represented by a significant amount of outflows driven by sentiment and capable to create a consistent loss (or economic failure) in the market. In periods of liquidity dry-ups or low performance, investors fear funds seizing unit redemptions due to the pressure of fund managers to sell assets in a distressed market at a price smaller than their fair value. In particular Morris and Shin (2004) present a model with short- and long-term investors. They state that irrational behavior and the subsequent creation of a LBH are caused by the fear that asset prices may fall below the limit loss of the short-term investor. When the price decreases significantly, a short-term investor - not knowing the limit loss of other investors - fears the worst loss scenario because other agents may decide to redeem. Therefore investors sell even if their limit is

not broken and they cause a liquidity run themselves (even if there would not be rational reasons to explain it). Furthermore Bernardo and Welch (2004) build a model in which the LBH is caused by the fear of investors to have a liquidity shock in a period of a run. In this situation they will not been able to liquidate their assets at a fair value. Therefore, they prefer to sell their assets now instead of facing the possibility of incurring in a sale during the run period. Hence LBHs are not caused by a realized liquidity shock, but by the fear of a possible future liquidity shock. Finally, Huang (2003) presents a model of optimal asset allocation, which takes the possibility of a liquidity shock randomly affecting investors into account. She defines the boundaries for the liquidity premium and the risk-free premium in order to establish a correct asset allocation.

Several studies, instead, identify the uncertainty in the market as the main cause for liquidity runs. Uncertainty creates an imbalance in flows and investors' behavior, leading to a reduction in prices driven by illiquidity (i.e. investors would be willing to invest only if there is a reward for their liquidity risk). In accordance with this theory, Easley and O'Hara (2010) affirm that liquidity issues arise because of a non full agents participation in the market. Net flows are then originated from this participation asymmetry and ultimately illiquidity is brought about by uncertainty due to the lack of trading. Moreover, they observe that illiquidity is caused by uncertainty about future performances and the value of underlying assets. Moreover, Routledge and Zin (2009) show that the uncertainty related to the performance of the underlying financial asset sensibly reduces the liquidity of that asset and consequently of the market. Therefore, crisis are originated for both a liquidity effect (i.e. through the bid-ask spread) and uncertainty. As a consequence, people stop trading and this increases the bid-ask spread, and ultimately leads to a reduction in liquidity.

Other researchers argue that illiquidity and order imbalances are caused by the cost to participate in the market. Potential investors have to face both personal and market constraints. These costs generate imbalances in players' flows and the market moves away from strong and semi-strong efficiency.Therefore, participation costs impede a full participation in the market. Among these researches we can find J. Huang and J.Yan (2007),A. W. Lo and Wang (2001) and Vayanos and Wang (2009). Particularly, Huang and Wang (2009) demonstrate that costly market presence generates a trading imbalance. This imbalance is overwhelmed by the sell side (outflow) and then causes a need for liquidity.

This endogenous need can therefore cause a market crash without the necessary condition of a specific aggregate to happened. Moreover, Huang and J.Wang (2010) demonstrate that the cost to participate in the market creates a liquidity issue due to the flow imbalance and, ultimately, it represents a serious problem for the welfare. However they also show that when the cost to participate for a market maker is below a specific threshold, the liquidity problem does not exist because the market making sector is able to absorb order imbalances. Along with this, Brunnermeier and Pedersen (2009) demonstrate that the costs of funding speculators' investments can cause liquidity spirals for two reasons: a funding shock moves the margin and reduces market liquidity, with subsequent further increase in the margin; if a speculator has a significant market share, an induced sale of her assets corresponds to a further price change.

Finally some studies focus on the solution to the illiquidity phenomenon by looking at flows and price movements after and/or during a LBH. This is the case for the phenomena such as flight to quality: in periods when liquidity really matters, the manager and investor tend to move their asset allocation to more liquid assets. This effect could be observable with the increase in price recorded for such instruments. Caballero and Krishnamurthy (2008) show that during a crisis investors prefer to hold liquid assets, mostly because of a Knightian uncertainty about the future. This uncertainty then causes illiquidity, which originates a flight to quality behavior. In addition Caballero and Krishnamurthy (2008) describe the effect of agents' decisions (for both investors and managers) on the welfare and the optimum central bank reaction, under the assumption of an economy with up to two waves of liquidity shocks. Moreover, Vayanos (2004) develops a model with a fund manager being subjected to outflows generated by random and unexpected personal reasons (e.g. investor liquidity need) and fund performances. As a consequence, the manager prefers liquid assets in periods of market illiquidity because of the higher likelihood of seeing a high request of unit redemptions by investors - i.e. investors are willing to pay a premium to invest in a liquid asset during periods of illiquid markets, as in Acharya and Pedersen (2005). In fact, during periods of high volatility the liquidity premium increases, the beta of asset increases and investors become more risk adverse, hence tending to redeem more.

3 The model

We model the decision making process of investors in a mutual fund, given their preferences and the state of the economy. We show that a LBH can be an outcome in specific states of the economy.

3.1 The market

The economy has three dates, $t = 0, 1, 2$. At time 0 the various agents are originated and their portfolios are defined. At time 1 agents choose whether to buy, sell or hold. At time 2 the economy terminates, final wealth is realized and portfolios are liquidated. In this economy there are only two possible investments choices: a single monopolistic mutual fund and cash which is used as a numeraire. Cash gives a fixed return equal to 1 at date 2 and hence has no volatility. The mutual fund gives a return equal to N at time 2, where N is normally distributed with mean $\tilde{N}$ and standard deviation σ_N.

3.2 Agents

The economy is populated by a large number I of investors and each investor i, $i = 1...I$, is a price taker. For each agent i, preferences are described by the following constant absolute risk aversion(CARA) expected utility function over her final wealth:

$$u^i = -e^{-\alpha W_2^i}, \tag{1}$$

where W_2^i represents the wealth of agent i at time 2 and α is the coefficient of absolute risk aversion.

The return for the investors is realized at time 2 and it is function of their allocations of cash and shares of mutual fund at time 1. The return N_t of the fund is defined as

$$N_t = y_t + z_t, \tag{2}$$

where y and z represent, respectively, the systematic and the specific component of risk. In particular y is considered the un-diversifiable market risk which defines a specific state of the economy.[2] The value of y varies between negative and positive values and the absence of systematic risk in the market is represented by $y = 0$. The value of y is common

[2]Changes in inflation or in the general level of rents are examples of this variable.

knowledge for the investors at time 1. The variable z represents the specific risk of the fund. We assume that each investor does not know the true value of z at time 1 but forms an expectation. We model investors with heterogenous expectations and this feature will imply the possibility of LBH. Both y and z are normally distributed, with expected value Y and $\tilde{z}$, standard deviation σ_y and σ_z respectively and zero correlation.

More specifically, we assume that the specific risk z of the fund is composed by two components: an endogenous and an exogenous one. The exogenous component includes modifications of the fund which are not the direct result of managerial decisions, such as the change in value of a specific assets in the portfolio. These shocks are realized at time 1 and are public knowledge. The endogenous component includes instead the actions on the structure of the fund taken by the manager as a reaction of the shocks at time 1.[3] The manager decides these variables at time 1 but they become public knowledge only at time 2. Therefore at time 1 an investor does not know the true value of N but forms an individual expectation based on a public signal (exogenous component) and an expectation on the manager's decision (endogenous component).

In order to model trading we distinguish between two types of investors: buyers, sellers, with each type accounting for half of the investors. Buyers and sellers have identical preferences but different initial holdings: buyers are endowed with cash and sellers with shares and the value of their initial endowment is identical. In the model we do not allow for leverage and short sales: buyers can only decide how many shares to buy and sellers decide how many shares to sell given their initial endowments.[4]

3.3 Participation costs

Following the literature[5], we model a participation cost c that represents the cost for an investor to buy or sell in the market. In reality, an investor supports her purchase of a financial asset with a specific capital structure. Retail investors invest using an equity-prevailing capital structure while the institutional ones leverage their positions in line with the target capital structure of the company. Both equity and debt have a cost that

[3]Within this component, the capital structure and other specific variables are considered. Marcato and Tira (2010) have shown that some variables, such as leverage, cash and asset concentration, drive the return in REMF and are consequences of specific decisions of the manager.

[4]This simplifying assumption keeps the model simple but does not restrict the analysis of LBH. The study of the implications of leverage and short sales surely constitutes an interesting avenue for further research.

[5]See for example Huang and Wang (2009, 2010).

varies according to the wealth of the investor and the status of the economy. Moreover investments in mutual fund are subjected to fees, payable to the manager for managing flows and fund structure. Furthermore also the secondary market requires a fee to support the brokerage.

For these reasons it is not possible to affirm that investments are free of the participation cost. In this paper a cost c is applied on a transaction. The cost is considered, for simplicity, as a lump sum and summarize all the above mentioned costs. The cost is applied for participation in the market only. An investor deciding to participate in the market must undergo a cost c of participation. We define $\eta_i = 1, 0$ respectively as the choice to whether participate or not in the market for the investor i. If an investor does not participate no additional costs are incurred. in accordance with Easley and O'Hara (2010), we will show that the cost does not influence the equilibrium price and allocation of the economy but it influences the buy-sell-hold decision.[6]

3.4 Timeline

At time 0 buyers and seller are defined and no proper action is taken by the agents. Buyers and sellers differ by whether their initial endowment is expressed in either cash or shares in the fund, but the value of the initial endowment is the same for both classes of agents. We define $\gamma_{i,t}$ and $\theta_{i,t}$ respectively the amount of cash and the number of shares, expressed as a percentage of the total outstanding shares, that agent i holds at time t. At time 0 buyers are endowed with a quantity γ_0 of units of cash and sellers with a fraction θ_0 of shares. In order to assure market clearing in the market we impose that each seller holds twice the amount of the per-capita quantity of outstanding shares in the economy. Labeling the per-capita fraction of outstanding shares in the economy by $\bar{\theta}$, we therefore set

$$\theta_0 = 2\bar{\theta}. \tag{3}$$

In order to assure that all the outstanding shares are possibly traded, even without allowing for short sales and leverage, we impose that the unitary price of shares at time 0, P_0, is such that buyers can in principle afford to buy all the shares owned by the sellers, hence

[6]The cost c can be influenced by the manager, i.e. entry/exit fee. In this paper this case is not considered because the focus of the research is on the investor behavior rather than on the optimal fee policy.

we set

$$P_0 = \frac{\gamma_0}{\theta_0}. \tag{4}$$

This implies that the value of the initial endowment of buyers and sellers is identical, i.e. $W_0 = \gamma_0 = P_0\theta_0$. Furthermore we treat cash as the numeraire and hence we impose the equilibrium price $P_0 = 1$. Given that at time 0, before expectations about the future returns are formed, cash and shares have the same expected return, in equilibrium they must have the same unitary price.

[INSERT FIGURE 1 HERE]

At time 1 the idiosyncratic risk y is revealed and investors observe a public signal v over the specific risk of the fund z. Given this signal, each investor i creates her own expectation $\tilde{z}_i$ on the specific risk of the fund at time 2. Investors are risk averse and decide to trade their position in order to achieve their optimal allocation, given their expectations of the future value of the fund. At this date, buyers (sellers) decide to buy (sell) some units of the funds or to hold their position. Agents are not obliged to invest the full amount of their endowment and an optimal mix of the two assets can be achieved. As we do not allow for leverage or short sales the buyer (seller) can not increase her endowment of cash (shares) beyond γ_0 (θ_0).

After trading in the market is completed, every investor owns her optimal allocation of the two assets defined as $A^i(\theta^i, \gamma^i)$. At date 2 the value of z is revealed, the final wealth is achieved and all portfolios are liquidated. This is the end of the economy. The final wealth of the investor is determined by the asset allocation A traded at time 1:

$$W_2^i = \theta_1^i P_1(1+N) + \gamma_1^i - \eta_i c. \tag{5}$$

where

$$\begin{aligned} \gamma_{B,1}^i = &\quad \gamma_0^i - \theta_1^i P_1, \\ \gamma_{S,1}^i = &\quad (\theta_0 - \theta_1^i) P_1 \end{aligned} \tag{6}$$

respectively represent the cash for a buyer and a seller at $t = 1$. Plugging (6) into (5) we obtain the new expressions for the final wealth of a buyer and a seller respectively:

$$\begin{aligned} W_{B,2}^i = &\quad \theta_1^i P_1 N + \gamma_0 - \eta_i c, \\ W_{S,2}^i = &\quad \theta_1^i P_1 N + \theta_0 P_1 - \eta_i c. \end{aligned} \tag{7}$$

3.5 Restrictions

In this economy investors have a fixed wealth m that represents the value of their initial endowment. Neither buyers nor sellers can increase the value m of their initial endowment, however at time 2 an investor may achieve a positive or negative payoff depending on the return on the risky investment. At time 0 the economy is defined as the initial endowment of investors (i.e. cash for buyers and risky units for seller). Therefore we can represent the per-capita endowment of the economy as follows:

$$P_0\theta_0 + \gamma_0 = m, \tag{8}$$

where m is a fixed given quantity. At time 1 investors decide concerning the participation in the market, given the following budget constraint :

$$P_1\theta_1 + \triangle\gamma - P_0\theta_0 + c = 0. \tag{9}$$

This condition implies that there is no leverage in the aggregate economy and that the economy is closed.[7]

Shortage and leverage are not allowed at the individual level as well. In order to impose such restrictions we impose some specific conditions. Particularly the ownership of unit of mutual fund can not be less than 0 at any time. While at time 0 the endowment of risky asset is given by definition, at time 1 for both the actors must be respected:

$$\theta_1 \geq 0 \tag{10}$$

This inequality guarantees no shortage in the economy. For what concerns the no leverage policy, in addition to (9), for the buyer the initial endowment of cash must be greater or equal to the value of risky stock that she can buy at time 1 adjusted for the cost to participate in the market:

$$\gamma_0 \geq \theta_1 P_1 + c. \tag{11}$$

In order to avoid leverage on the seller side, it must be that $\theta_0 \geq \theta_1$. In this case ownership of unit at time 1 can not exceed the initial given endowment of risky asset. However the investor can still leverage her position and use the excess cash to increase the return of the portfolio given the sure payoff of 1. This condition can not be violated by the Buyer because the (9) guarantees this eventuality. For the seller instead, the cash at time 1 must

[7]For a not leverage policy in the economy it is necessary that $P_1\theta_1 + \triangle\gamma - P_0\theta_0 + c \geq 0$. Therefore the condition (9) is sufficient to guarantee the condition of no leverage in the aggregate economy.

not exceed the maximum number of units that the seller can sell with the cost to participate in the market:

$$\gamma_1 \leq P_1 max(\theta_i \in \Theta) + c,$$

with i=1..I and Θ represent the set of all feasible allocation in the economy. Given that $\theta_0 \geq \theta_1$ implies that $max(\theta_i \in \Theta) = \theta_0$, we have that

$$\gamma_1 \leq P_1 \theta_0 + c. \tag{12}$$

These restrictions are applied in order to distinguish the agents between buyers and sellers through out the tree periods. In fact, shortage may allow buyers to became sellers and leverage may allow sellers to become buyers. These restrictions do not restrict our analysis of LBH and are applied in order to keep the tractability of the model.[8]

4 Equilibrium

In equilibrium the market clearing condition has to be verified. At any given time, in equilibrium the per capita demand has to equal the per capita supply. This condition must be verified for both the assets (cash and risky asset), hence:

$$\frac{1}{I} \sum_{i=1}^{\mathrm{I}} \theta_i^* = \bar{\theta}, \tag{13}$$

$$\frac{1}{I} \sum_{i=1}^{\mathrm{I}} \gamma_i^* = \bar{\gamma}. \tag{14}$$

At time 1 each investor observes the value of the systematic risk y and a signal v over the specific risk of the asset (exogenous component). We model agents with heterogenous expectations: each investor i has a different expectation over the final value of the mutual fund due to the signal v and due to their expectation of the managerial action. More precisely, we define the expectation of agent i as :

$$E_i[z|v] = \tilde{z}_i. \tag{15}$$

[8]Moreover, in the specific case of the Real Estate market short selling is hardly practised because of the limited secondary market. Gearing, instead, is possible in the RE unlisted market but it is not very popular among investors (while it is for the funds): REMF investors are mainly institutional investors and also among retailers investor, REMF is still considered a low grade risk investment. Gullet and Redman (2005) demonstrate that RE it is used to reduce the volatility of mixed portfolios and is considered as an inflation-hedge investment.

The specific risk z is distributed as a normal with expected value $\tilde{z}$ and standard deviation σ_z. We assume that also the individual expectation $\tilde{z}_i$ is distributed as a normal across agents.

Plugging (7) into (1) we find the maximization problem of the agents at time 1:

$$\max_{\theta_{1,i}} -e^{-\alpha * E[\theta_{1,i} P_1 (y+z)+k_0-\eta c]}, \tag{16}$$

where $k_0 = \gamma_0, \theta_0 P_1$, for the buyer and the seller respectively. Given that both z and y follow a normal distribution and plugging (15) into (16) we obtain that the maximization problem can be re-expressed as:

$$\max_{\theta_{1,i}} -e^{-\alpha * (\theta_{1,i} P_1 (y+\tilde{z}_i)-\frac{1}{2}\alpha\theta_{1,i}^2 P_1^2 \sigma_z^2+k_0-\eta c)}. \tag{17}$$

Using the first order condition we obtain the optimal allocation for each investor i:

$$\theta_{1,i} = \frac{y+\tilde{z}_i}{P_1 \alpha \sigma_z^2}. \tag{18}$$

In equilibrium, condition (13) must be verified. In order to compute the aggregate demand we compute the average expectation of the specific risk z:

$$\hat{z} = \frac{1}{I}\sum_{i=1}^{\mathrm{I}} \tilde{z}_i. \tag{19}$$

Aggregating over the individual demand (18) and using (19) we obtain the expression for the per-capita demand:

$$\frac{1}{I}\sum_{i=1}^{\mathrm{I}} \theta_{1,i} = \frac{y+\hat{z}}{P_1 \alpha \sigma_z^2}. \tag{20}$$

Imposing condition the market clearing condition (13) we find the equilibrium price:

$$P_1 = \frac{y+\hat{z}}{\bar{\theta}\alpha\sigma_z^2}. \tag{21}$$

Plugging the equilibrium price (21) in the optimal individual allocation (18) we find the equilibrium individual allocation:

$$\theta_{1,i}^* = \frac{y+\tilde{z}_i}{y+\hat{z}}\bar{\theta}. \tag{22}$$

Few things are worth noticing. First, the participation cost c does not influence the individual desired allocation (18). The reason is that participation costs represent a lump

sum fee and therefore influence only the decision of whether to operate on the market but not the individual optimal trade. Second, the first order condition and therefore the individual desired allocation (18) is identical for both a buyer and a seller. The reason is that the optimal mix between shares and cash does not depend on the initial endowment but only on the current equilibrium price of shares P_1 and on the expected return N. Third, P_1 is a relative price given $P_0 = 1$. A current equilibrium price P_1 which is lower (higher) than P_0 expresses the relative expected loss (gain) of the return on the fund's equity relatively to the zero risk-free return on cash.[9] According to (21) the equilibrium price can be expressed as the market expected return on the fund (idiosyncratic and specific risk) adjusted for the coefficient of risk aversion α and the per-capita endowment $\bar{\theta}$ of the agents. The reason why the per-capita endowment shows up in the denominator of (21) is precisely because P_1 is a relative price given P_0. Using (4) and (3) we see that $P_0 = \frac{\gamma_0}{2\bar{\theta}}$, hence changing the value of $\bar{\theta}$ does not change the ration between P_0 and P_1. Fourth, we notice that the equilibrium allocation (22) is given by the average endowment adjusted for a difference between the believe of investors and the average belief of the market . In fact an investor with higher expectation than the average belief of the market will own an amount of shares greater than the average holding of the market.

5 Optimal participation decision

The optimal participation decision for a buyer and a seller is now explored. Each investor at time 1 realizes the dimension of the idiosyncratic risk y and observes a signal v over the specific risk of the fund: this combination originates her expectation on the final value of the fund at time 2. Therefore, at time 1, a buyer (seller) can decide to either buy (sell) shares in the fund and pay a participation cost or to hold the current portfolio and realize a risky (zero certain) return. The optimal decision can be obtained analyzing the difference of the utility in the two possible scenarios. We define the indirect utility function of non-participating and participating as J_{np} and J_p respectively. It is possible to calculate the difference of payoffs from the two strategies. Given that the investors are risk averse the natural notion for such comparison is the difference in the certain equivalents from the two strategies. We define such difference as the net gain function from participation and we label it $g(.)$.

[9]The zero risk free return on cash can be naturally interpreted as the return on an inflation adjusted zero-coupon bond.

Proposition 1. *The net gain function from participation for each investor i is:*

$$g_i = -\frac{1}{\alpha} ln \frac{J_p^i}{J_{np}^i}. \tag{23}$$

Proof in appendix 1.

Notice that the gain from participation $g(.)$ is positive if and only if $\frac{J_p}{J_{np}} < 1$. Given the specified utility function, J_p, J_{np} take negative values and therefore $\frac{J_p}{J_{np}} < 1$ when $J_p > J_{np}$. Therefore the gain from participation is positive when the expected utility from participation is greater than the expected utility form non-participation. Given that the participation cost c decreases J_p, the higher the participation and the less individuals are willing to enter the market. As J_p, J_{np} are in principle different from buyer and seller, the impact of the costs may be in principle different on he participation of the two classes of agents.

5.1 Optimal decision for the buyer

If a buyer decides not to participate her final allocation will be given only by her initial amount of cash, hence :

$$\begin{aligned} \theta_1 = \theta_0 = 0, \\ \gamma_1 = \gamma_0 = P_0 \bar{\theta} \end{aligned} \tag{24}$$

If we substitute (24) in (17) we obtain the indirect utility from non-participating for a buyer:

$$J_{B,np} = -e^{-\alpha \gamma_0}. \tag{25}$$

If the buyer decides instead to buy some shares in the market we plug the equilibrium price (21) and the equilibrium allocation (22) in the expected value of (17) and we obtain we obtain the indirect utility from participating for a buyer:

$$J_{B,p} = -e^{-\alpha(\frac{\frac{1}{2}(y+\tilde{z})^2}{\alpha \sigma_{\tilde{z}}^2} + P_0 \bar{\theta} - c)}. \tag{26}$$

In figure 7 we give an example of how the net gain function changes as a function of $\tilde{z}$, given different values of the cost c.

[INSERT FIGURE 2 HERE]

From the figure it is possible to see that a buyer finds profitable to buy and incur costs when the expected return is higher than a certain threshold. Moreover, we notice that for higher values of c, the participation threshold to buy is moved to higher values of $\tilde{z}$.

5.2 Optimal decision for the seller

If a seller decides not to participate her final allocation will be given by the initial amount of shares only, hence :

$$\begin{aligned} \theta_1 &= \theta_0 = 2\bar{\theta}, \\ \gamma_1 &= \gamma_0 = 0. \end{aligned} \tag{27}$$

If we substitute (27) in (17) we obtain the indirect utility from non-participating for a seller:

$$J_{S,np} = -e^{-\alpha(\frac{2(y+\hat{z})(y+z)-2(y+\hat{z})^2}{\alpha\sigma_{\tilde{z}}^2}+2P_1\bar{\theta})}. \tag{28}$$

If the seller decides instead to sell some shares in the market we plug the equilibrium price (21) and the equilibrium allocation (22) in the expected value of (17) and we obtain the indirect utility from participating for a seller:

$$J_{S,p} = -e^{-\alpha(\frac{\frac{1}{2}(y+z)^2}{\alpha\sigma_{\tilde{z}}^2}+2P_1\bar{\theta}-\eta c)}. \tag{29}$$

In figure 3 we give an example of how the net gain function changes as a function of $\tilde{z}$, given different values of the cost c.

[INSERT FIGURE 3 HERE]

From the figure we notice that a seller finds profitable to sell for an interior interval of expected specific returns $\tilde{z}$, i.e. a seller finds profitable to sell when does not expected a very low or a very high specific return $\tilde{z}$. The reason for this is that more extreme expected values of $\tilde{z}$ are associated with lower probabilities and therefore the equilibrium sale price, which is a function of the average belief, is less close to the reservation price of the seller. From the figure it is also possible to see that for higher values of c the seller decides to sell the shares over a a smaller interior interval of expected specific returns $\tilde{z}$.

5.3 Comparative Statics

In order to gain a better economic intuition of the behavior of the gain function, we compute its partial derivatives with respect to the function variables.

For the buyer case, we notice from the expression of (26) that the net gain is affected by the spread between y and $\tilde{z}$, but with opposite signs, hence the overall effect is not a priori clear.

[INSERT FIGURE 4 HERE]

Figure 4 shows indifference gain curves in terms of y and $\tilde{z}$. It appears from the figure that those are almost linear. We therefore conclude that the single components of risk y and $\tilde{z}$ do not affect the net gain function per se, but it is rather the overall expected return of the fund to influence the decision to wether buy or not. We also notice from (26) that given risk aversion α, volatility σ_z decreases the net gain of participation function.

Also for the case of the seller, we notice from the expression of (28) that the net gain is affected by the spread between y and $\tilde{z}$, but with opposite signs, hence the overall effect is not a priori clear. In this case it is the indirect utility of non participating (28) to be affected because it is by non participating that the seller holds more shares and therefore is more affected by the specific risk. As for the buyer case, we plot indifference gain curves in terms of y and $\tilde{z}$.

[INSERT FIGURE 5 HERE]

Also in the case of the buyer, figure 5 shows indifference gain curves that are linear. We therefore conclude that also in the case of the seller the single components of risk y and $\tilde{z}$ do not affect the net gain function per se, but it is rather the overall expected return of the fund to influence the decision to wether sell or not. The indifference curves have opposite slope than in the case of the buyer because the smaller the value of $\tilde{z}$ and the higher is the utility from selling.

6 Participation to the market

In this section the joint behavior of both buyers and sellers is analyzed. In previous sections the function that describes the behavior of a single investor has been determined, in a buyer and seller case. Moreover, participation conditions have been fulfilled. In this section the number of participants for both classes will be calculated and the interaction between them will be simulated in a artificial case scenario.
By definition $z \sim N(\mu_z, \sigma z)$. In our case $\mu_z = \hat{z}$ and therefore: $z \sim N(\hat{z}, \sigma z)$. For this reason we can apply the density function in order to compute the number of participants into the market. Data are drawn from previous examples. The confidence interval could be implemented simply with the standard deviation of the distribution and the distance of the event from the average of the distribution. From definition we know that:

$$erf(\frac{n}{\sqrt{2}}) \tag{30}$$

where erf is the error function and n is the number of σ_z used for the interval of confidence. Moreover we know that $\bar{\theta}$ represents the average percentage of units owned by agents in the market and therefore we can obtain the number of the participants as:

$$erf(\frac{n}{\sqrt{2}}) * \frac{1}{\bar{\theta}} = \#participants$$

we can substitute n and obtain

$$erf(\frac{\gimel \pm (\hat{z} - x)/\sigma_z}{\sqrt{2}}) * \frac{1}{\bar{\theta}} = \#participants \tag{31}$$

where x is

$$g(.) = 0|\tilde{z}$$

and $\gimel$ is the value which represents the 50% of the fund. The sign of $\gimel$ is given by the position of x in relation to $\hat{z}$: particularly if $x < \hat{z}$ therefore $\gimel$ is positive. When x does not exist the number of participants in the market is 0%[10].

Below we present an example of this application in the buyer case.

[INSERT FIGURE 6 HERE]

[10] algebraically the participation could also be 100% but we impose a specific value of the cost c that exclude this possibility

The grey area represents the area of investors in the real case scenario. Investors participate for a value of $g(.) \geq 0$. In this case the price at which REMF units are bought is $P_1 = 7.5$ and the participants in the market are the 61.3% of the whole population.

6.1 Liquidity black hole definition

A liquidity black hole is a consequence of participation in the market, it is originated by the investors' behavior and it is driven by further specific reasons. The scope of this paper is to define a LBH and to design some actions to limit the phenomenon. This research is not aimed to understand the specific reasons originating a LBH: in our model we allow for extreme events (i.e. extreme negative values of $\hat{z}$ or y) but the source of this situation is not explored.

Hameed, Kang, and Viswanathan (2010) identify two different but complementary sources contributing to the origination of a LBH: demand and supply. The demand effect is driven by a sudden sale of units due to panic in the market, as a consequence of an extreme negative event, connected with price and performance. Moreover this panic sale is characterized by heavily one-sided order flows (usually outflows), rapid price changes, and financial distress on the part of many traders as pointed out by Morris and Shin (2004).

The supply effect instead is originated by the financial intermediaries which reduce their provision of services to the market, withdrawing their injection of liquidity. Particularly Bernardo and Welch (2004) affirm that the market maker sector can absorb the demand effect, but it has a limited capacity. Therefore in the case of a LBH, the ability of the market maker to prevent the effect is overwhelmed.

Consequently, a liquidity black hole could be characterized by a strong participation of sellers with a limited number of available buyers. The extreme situation in the market occurs when buyers participate at 100% and sellers at 0%, but there could also be some other weaker phases leading to a LBH. At the same time, the market maker or fund manager can hedge and absorb the effect of heavily one-sided flows. Let us define the capacity of the market maker (i.e. cash-like cushion for a specific fund) as ζ. Following this definition, we include all short-term investments which can be liquidated in a similar way as cash. The return of these investments is not important for the empirical analysis, but we are supposed to be close to a cash payoff. The limits for the origination of a LBH come into

play when net flows are higher than the capacity of the market maker:

$$Outflows - Inflows > \zeta$$

Given the (31) therefore:

$$erf(\frac{n_S}{\sqrt{2}}) * \frac{1}{\bar{\theta}} - erf(\frac{n_B}{\sqrt{2}}) * \frac{1}{\bar{\theta}} > \zeta$$

Substituting and simplifying

$$erf(\frac{\gimel \pm (\hat{z} - x_S)/\sigma}{\sqrt{2}}) - erf(\frac{\gimel \pm (\hat{z} - x_B)/\sigma}{\sqrt{2}}) > \zeta \tag{32}$$

where $\gimel$ follows the process reported above. Marcato and Tira (2010) demonstrate that outflows are driving inflows in REMFs. More specifically INREV (2010) show that the most common practice to redeem units is trading. So we expect that net flows occurred in the first term will unlikely be greater than the capacity ζ of the fund. This implies that in a real case scenario the event of a LBH is an unlikely event, and that it cannot be inferred instantly by the regulatory forces of the market, as demonstrated by Bernardo and Welch (2004). However, in our model we do not impose scenarios with different likelihood, and therefore a LBH could be as common as other events. We have decided to follow this assumption in order to have a greater possibility to observe the effect of a LBH on the performance of REMFs.

While the first term of the inequalities can be obtained following the procedures expressed in the main economic model, the second term is not well defined. In a further development of our research agenda we will also analyze the capacity of a market maker to absorb fund flows.

In the specific real estate context, the capacity ζ of the market maker also represents the capacity of the manager and can be defined as cash. In fact it can be expressed as the reserve which can be used by the market maker without moving the price of the fund units. Therefore in REMFs where the price is determined by $P = \frac{NAV}{\#stocks} + spread$, if the manager decides to sell assets to liquidate redemptions, she will modify the price of units and therefore the investors' behavior. The portion of value not used to determine the NAV of a REMF is exactly represented by cash (i.e. fund's capacity). In fact, during the period $2007 - 2009$ we observe that a significant drop of performance for some funds (i.e. origination of a LBH), has occurred after a sudden change in NAV figures. The price change has been driven by asset sales, as a consequence of the penetration of the fund's capacity

ζ. In our research, evidences on the instruments used by managers to increase the fund's capacity will also be provided.

In our model we have not defined the capacity of the market maker to absorb the demand effect, but it is possible to identify some areas of absolute LBH. These areas correspond to situations where net flows are so negative that no capacity ζ can absorb the effect. We have already observed the case when sellers participate at 100% and buyers at 0%. In this case, instead, the manager cannot absorb the request of sellers fully and she is obliged to reduce the NAV in order to redeem units. We can then argue that in such areas - which we call areas of absolute LBH - the origination of a LBH is very likely. Moreover, there are some funds (e.g. German REMF) that have a "cash-like" reserve up to 50% of the total dimension of the fund. For this reason we can assume the presence of absolute LBH for values of Netflows higher than this boundary. Other instruments to face a LBH (such as winding up the fund) exist, but they are not treated in this paper. However, a manager sensitive to the creation of a LBH can stop its origination by applying one of these instruments.

6.1.1 LBH effect: economic failure

In this section we give an insight of the impact a LBH can produce. An economic failure is a similar concept to the default. It does not lead necessarily to the annihilation of the fund, but at most to a significant reduction in fund performances. According to Duffie, Eckner, Horel, and Saita (2009), a company is considered an economic failure if their assets are significantly dropping in value compared to their liabilities. Moreover Black and Scholes (1973), Merton (1974) and others have structured this process as a geometric Brownian process. They define this drop as the distance to the default state, expressed as number of standard deviations included in the yearly asset growth in excess of the fund's liabilities. This approach is used by the main rating agencies such as Moody's and Fitch. From this approach, we can understand the proliferation of LBHs in real estate markets during the crisis period 2007-2009. In particular REMFs were characterized by a significant percentage of leverage, which contributed to increase funds' liabilities and therefore to diminish the limit for an economic failure.
Other studies such as Klaus and Rzepkowski (2009) provide a more naive definition of economic failure: it is when a fund is forced to liquidate its assets in an illiquid market

and therefore is under-performing the expected return. The fund is forced by redemptions of investors or by failures of other funds. However, the first impact prevails on the second. As a consequence, the contagion effect is not considered in this paper and it does not influence the optimal solution of the model. Another definition in line with the latter is the one of **?**) that define economic failure as the fund under-performing the average of its fund family (style).
Finally the economic failure is described by Diamond and Rajan (2005) as the insolvency of an instrument. This insolvency is originated by the different term - short and long - of investments inside the portfolio, the expectations of investors (specific risk) and the interest inside the economy (idiosyncratic risk). The financial instrument then becomes insolvent when resources it can raise do not exceed required redemptions.

6.2 Joint participation and absolute LBH

In this section the contemporaneous participation in the market for buyers and sellers is analyzed. A table of choices made by agents is created by applying the definition to both buyer and seller. It is important to remember that the percentage of participants refers to the class of agents only and, if considered on the whole population, it has to be divided by two.

[INSERT FIGURE 7 HERE]

The table shows the percentage of market participants for both buyers and sellers. The mid price is obtained following the definition of P^*.Therefore the price has to be interpreted as a function of P_0. In fact if $\hat{z}$ is the expected return at the liquidation of the fund, therefore P_1 is the present value of the price of the fund, and results are consistent.
In the next section, a reduction of the model will be analyzed by imposing the fund's unit price in order to obtain a better representation of the functioning of a REMF. Moreover, we will also present an analysis using the two methods with a real case scenario (Falcon Properties Fund).
By looking at inflows we can observe that they grow with the increase of returns. When future fund's performances are positive, they attract more investors revealing a return chasing behavior, as reported by Marcato and Tira (2010). Moreover, as explained in the comparative stats, both y and $\hat{z}$ have a direct positive impact on $g(.)$. Therefore inflows

increase with the incremental value of these two variables. We could have expected a significant increase between different states of nature, but the growing effect connected with y and $\hat{z}$ is mitigated by the increasing price. On the other hand, outflows are growing with increasing value of the fund's unit price. This result demonstrates that risk averse investors prefer to achieve the certain payoff connected with cash given by the increased price, rather than being exposed to the risk of expected positive returns. As a result of these behavior, the last column of the table reports net flows, obtained as a difference between inflows and outflows. We can observe that Net flows are close to zero but the seller side prevails for positive values. Moreover this column allows us to check for the origination of absolute LBHs. From our figures we clearly infer that there are no situations in which a LBH could be originated, because all net flows are small in magnitude. We can then conclude that this particular REMF will not experience a LBH as far as our data sample is concerned.

6.3 Imposed price

In this section we analyze the model using a price imposed by an external source rather than an optimal combination of sellers' and buyers' preferences. In the mutual fund industry, a transaction price-based approach is the most common: funds are listed in the stock market and available to retail investors. However the case of unlisted mutual fund is different. In this market, the fund manager decides herself a price for each unit (normally by following the theoretical definition: $\frac{NAV}{\#of units}$ plus a spread). Hence, in this market context (well represented by REMFs), a model with an imposed price rather than an equilibrium price is more appropriate. In order to isolate the manager's solution to imbalances in order flows, in this paper we consider the mid price of a fund rather than the bid-ask spread.

The price for the unlisted market is generally higher than the transaction-based price. Firstly, there are less units in a fund, by comparing funds of comparable size. Secondly, unlisted funds are less subjected to frequent withdrawals, mainly for two reasons: frequent modification of the structure of the fund are not cost efficient, and the manager tries to avoid them with appropriate pricing policy. Thirdly, investors in unlisted funds are mainly institutional investors with a long-term investment horizon. The price for the transaction-based model varies between 1 and 20, therefore we decide to use a price of 8 for the imposed price model.

Given the definition of $g(.)$ for both the buyer and the seller, we substitute (21) with $P = 8$ and we can then obtain the new net gain function for both buyer and seller:

$$g_B(y, \tilde{z}) = -\frac{1}{\alpha} ln(\frac{exp[-\alpha(8\theta^*(y + \tilde{z}) + P_0\bar{\theta} - 32\alpha\sigma\theta^{*2} - c)]}{exp[-\alpha P_0\bar{\theta}]} \tag{33}$$

$$g_S(y, \tilde{z}, \hat{z}, \bar{\theta}) = -\frac{1}{\alpha} ln(\frac{exp[-\alpha(8\theta^*(y + \tilde{z}) + 16\bar{\theta} - 32\alpha\sigma\theta^{*2} - c)]}{exp[-\alpha(16\bar{\theta}(y + \tilde{z}) + P_0\bar{\theta} - 128\alpha\sigma\bar{\theta})]}) \tag{34}$$

We then calculate participants in the market given the two net gain functions and we report the main results in the table below, which shows the participation choice for both buyer and seller, and the spread between the two.

[INSERT FIGURE 8 HERE]

In this case we have imposed a price $P_1 = 8$, which represents a positive future expectation (i.e. $y = 1\%, \hat{z} = 5\%$). On one hand the table above shows that in a situation in which this spread is high, inflows increase while outflows decrease compared to the example with the equilibrium price. Buyers will buy more units of a REMF when the price is below the fair value because they are accessing units at a discount. Sellers, instead, will not achieve the positive payoff connected to the increasing price, and would then prefer to hold their units in order to achieve the expected final payoff of the fund. On the other hand, in a situation where the level is below the spread, the opposite outcome happens: inflows decrease while outflows increase. Particularly, in a situation where $y + \hat{z} \leq 0$, outflows are equal to 0. This result could be explained by the fact that sellers are confused by the fact that they expect a negative return from the fund, while the market is pricing a high positive return. Moreover, the variable $\hat{z}$ represents the future expected value at the time of liquidation, and therefore they will not expect a negative return of the fund; probably they wouldn't even enter, and the fund is empty.

6.4 Real case scenario: equilibrium price vs. imposed price

As an empirical application of our theoretical model, we apply it to a real case scenario, using a specific REMF established in the United Kingdom. We have chosen to apply the model to the AXA institutional property fund, taking data from the IPD property pooled fund database. The data sample is collected quarterly between March 1995 and March 2009. The large sample allows us to set up and tune the model, although the latter is applied only on the period 2005-2009. Every quarter is to be considered as an economy, and

therefore investors will liquidate their position in the following instant. The idea is that the characteristics of the previous period define the values of variables in the following period.
The main variables of the model are obtained following the assumptions of Marcato and Tira (2010). In particular y, the risk of the market, is defined with an Autoregressive model with 4 lags, on the IPD property pooled fund index. Moreover the return of the fund N is obtained trough a CAPM approach with moving Beta: the return of the market is given by y while the beta is rolling on a sample period of 10 years[11]. The variable $\hat{z}$ is obtained as a difference of the two previous variables. The costs for both buyers and sellers include a cost for brokerage and for fees and taxes, on top of the value declared by the fund. The cost is expressed as percentage. The price for each quarter is obtained from the mid price (average between bid and ask spread), as incremental difference with the previous quarter.For this reason the first period of the sample is omitted.

First of all we have applied the model with the price obtained as an equilibrium price. We applied the model for the whole sample and results are illustrated in the following table:

[INSERT FIGURE 9 HERE]

We can clearly observe both the number of participants in the market and netflows. The behavior of both buyers and sellers is similar to what observed in our previous simulation: increasing values of expected return and prices lead to an increase of flows, and the effect is dominated by outflows. However, in this case the number of participants in the market is higher than in the previous example. In particular for extreme negative value of returns, i.e. March 2009, the spread between out- and inflows widen. Finally, net flows are varying around the value of -9% and therefore the fund experiences no LBH.
Furthermore, we have applied the model using the price imposed by the market maker. In this case the P_0 is equal to the price of the previous period. The following table summarizes our results:

[INSERT FIGURE 10 HERE]

First of all the magnitude of flows is sensibly reduced, when compared to the equilibrium price model. In particular we observe inflows are close to real values of the fund

[11]These approaches has been chosen after analyzing many different approach, with different sample period, or lag structure. The one used in the model is the best approximation to real values.

and follow the same pattern of real flows. In the second part of the sample, when returns begin to turn negative, no investor wants to enter in the fund. Outflows, instead, assumes higher values than inflows and they are sensibly increasing when expected returns turn to negative or null values. As a result the net flows are negative for the whole sample, as it happened in the previous example, but they are highly negative in the second part of the sample. These conditions seem to suggest the possibility of origination of an LBH, with returns in line with the benchmark. Both inflows and outflows experience an anomalies during the year 2007. This effect is to be re-conduced to the limit of our predictions. Finally we compared the results of the two model with the flows information that AXA has experienced during the same sample period. Results are reported in the following table:

[INSERT FIGURE 11 HERE]

The model using the equilibrium price has both inflows and outflows very different from real ones. However, The transaction based price is not used in the REMF market, and this result is predictable. As we evidence before, flows follow the change in price. These changes are very different in transaction based and in real case scenario. On the other hand, inflows of the imposed price model empirically find a confirmation in our real case: numbers do not significantly differ from the real ones, but could have been induced by exogenous parameters used in the model, such as α. Moreover, outflows follow the same pattern of the real case scenario. We now take into account the distinction between internal and external investor, because our model explore external investor only. We then weigh outflows on the external ownership of fund units and we recalculate net flows. Results significantly change and are reported in the following table:

[INSERT FIGURE 12 HERE]

In this case outflows, along with inflows, are very loyal to the real case scenario and the peak of the second part of the sample is achieved in the simulation. However in the negative part of the sample, outflows are significantly smaller than the real ones compared to the previous part. This spread is driven by internal investors decisions, not considered in the model. Therefore internal investors prefer not to move during positive state of the economy, but adjust their positions in negative times. The periods that are significantly distant from reality are driven by external reasons not included in the model, such as time deferral in redemption outstanding. In fact, in these periods a high number of outstanding redemptions was registered and this phenomenon reduced the investors' willingness to

queue for redemptions, resulting in outflows smaller than expected. As a result, net flows are very similar to the actual ones.

In conclusion our model is able to forecast the net flows of a REMF. The imposed price model is to be preferred to the equilibrium price model in this case. This result is driven by the fact that a REMF is closer to the former model. In fact the price of a fund unit is imposed by the market maker which is the fund manager for REMFs. However, the imposed price model works on a price given by an external agent, but it is not proven to be fair. Therefore, the equilibrium price is the fairest of the economy, even if it is not the one realized. From this empirical work we can conclude that if the equilibrium price were achieved in the economy, the fund would never experience a LBH. On the other hand, the imposition of a price on the fund can generate significant imbalances in flows and lead to the creation of a LBH. This result suggests that flow imbalances are not driven only by transaction costs but by pricing as well.

7 Conclusions

We presented a model of optimal behavior for mutual fund investors, and we explicitly modeled the origination of a LBH. The investor's choice (i.e. buy-sell-hold) is based on CARA preferences and is a function of the idiosyncratic risk and the expected return, given a signal on the specific risk. The result of the model is based on expectations of both market and investors. The analysis of investors' behavior is crucial to understand irrational consequences connected to liquidity dry-ups in the economy. Particularly, an attentive study on the reasons behind movements in the decision function allows the manager and the market maker to limit the change in fund pricing. We define the conditions for the creation of a LBH focusing on the participation of investors in the market and the capacity of the market maker to hedge the problem of redemptions.

Starting from the liquidity effect on asset pricing and following an intuition in Marcato and Tira (2010), we discover that illiquidity is originated at a micro-structural level rather than in a macro-economic context. Specifically, looking at our current state of the economy, we discover that the optimal price and allocation are the same for both buyer and seller, in line with Easley and O'Hara (2010). Furthermore, we discover that the cost to participate in the market does not influence the optimal allocation but has a significant effect on the investor's decision, preventing the full participation, as illustrated by Huang and J.Wang (2010). The investor's decision is influenced by the unit price, especially for

the seller. We create a net gain function which models investors' choices. We then apply it to a randomly generated scenario. The result is a map of investor's behaviors, given different levels of risk and expectations. In addition, participation rules are applied to the function, and the dimension of REMF flows is derived. A close approximation of the number of participants in the market (expressed both as a percentage and number of agents) is obtained. More specifically, inflows are driven by the fund's expected return, while outflows are driven by the level of prices. However the outflows effect prevails.

Furthermore, we created a more general case, in which we impose the price of the fund to be an exogenous information. Comparing our results with the previous model, the participation of buyers increases when there is a higher price per unit, and decrease otherwise. On the other hand, the seller is subjected to a fixed price and therefore she experiences a different behavior driven by similar preferences. We prove that areas of absolute LBH can be originated in this scenario: these areas correspond to significant negative values for both components of risk.

Finally we applied the model to a real case scenario finding similar pricing patterns as in our theoretical model and predictions. Particularly, both our models are able to predicts flows. When the nature of the investor(i.e. internal or external) is taking into account the confidence of our results increases. The imposed price model is more suitable to a REMF, while the use of an equilibrium price model undermines the possibility of a LBH origination. Moreover the LBH could be originated not only for the presence of a participation cost but also for an unfair pricing policy.

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Appendix 1

Proof of proposition 1

The certainty equivalent is defined as the wealth W^* such that an investor is indifferent between participating in a gamble and receiving W^* with certainty. Given the chosen CARA utility function the certainty equivalent from participation can be found solving the following equation:

$$e^{-\alpha W_p^*} = J_p.$$

Therefore we find that $W_p^* = -\frac{1}{\alpha}\ln(J_p)$. Symmetrically we find that the certainty equivalent from non participation is $W_{np}^* = -\frac{1}{\alpha}\ln(J_{np})$. Subtracting we find that $W_p^* - W_{np}^* = -\frac{1}{\alpha}\ln(\frac{J_p}{J_{np}})$.

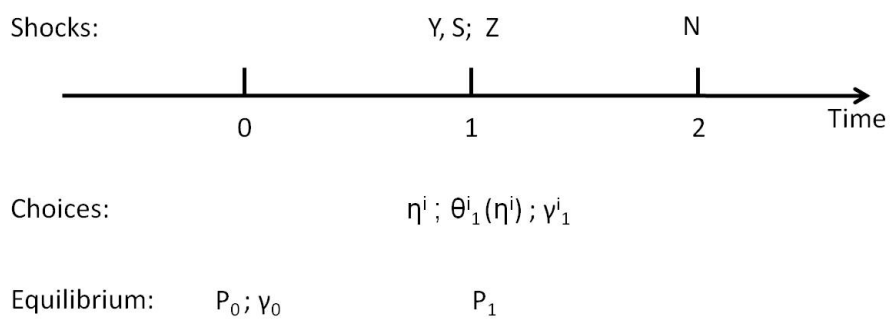

Figure 1: ***Timeline of the economy. Shocks are endogenous to actors, Choices are decided by actors, and Equilibrium is a consequence of actors' choices***

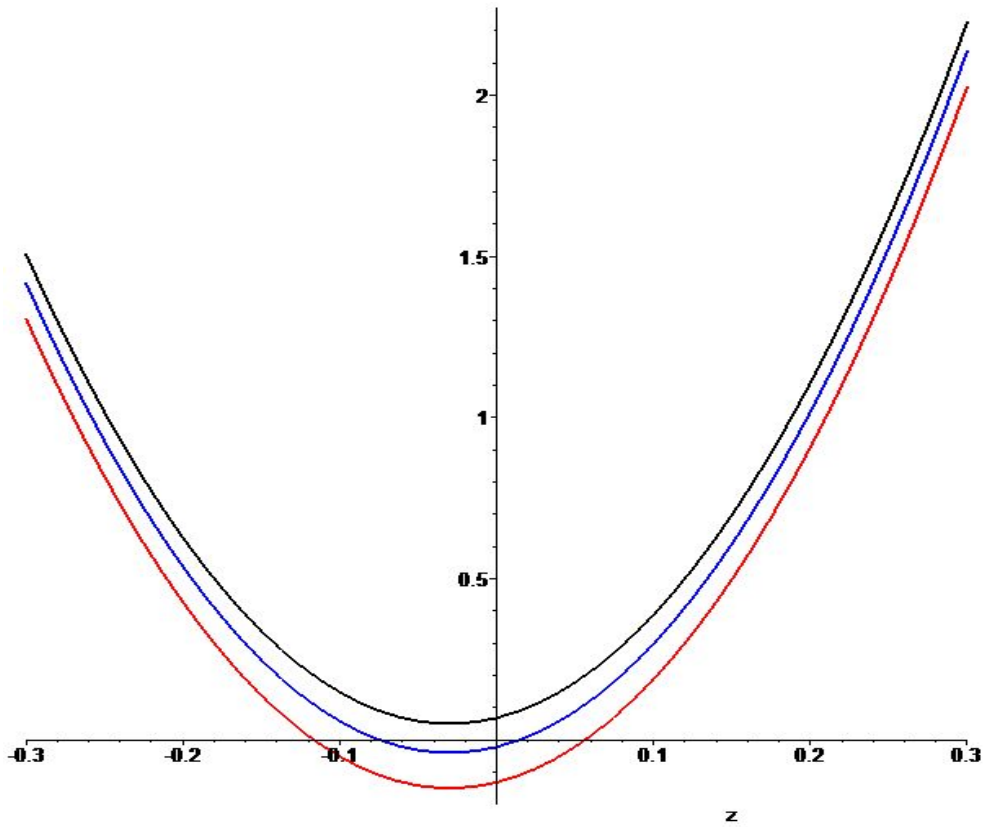

Figure 2: ***Buyer net gain function with cost to participate in the market.Variable used for the graph:*** $\alpha = 2.5, \sigma = 0.1, \theta = 0.005, y = 0.03, P_0 = 10, c_g = 0.09, c_b = 0.2$

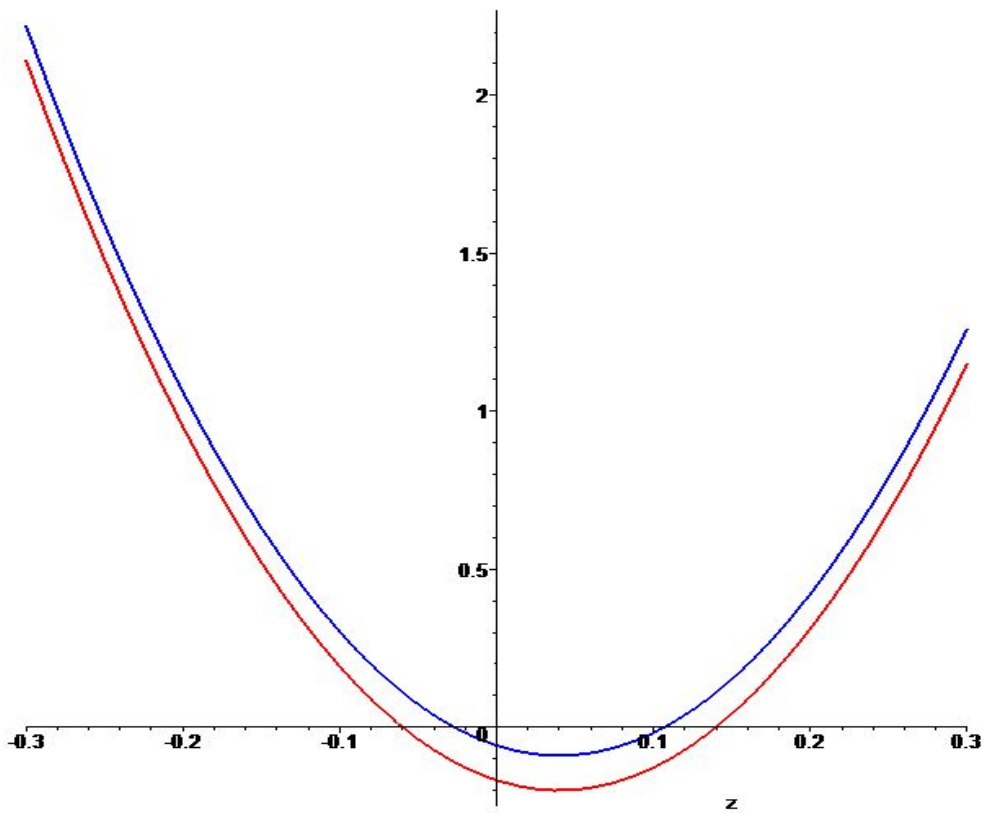

Figure 3: ***Seller net gain function with cost to participate in the market.Variable used for the graph:*** $\alpha = 2.5, \sigma = 0.1, th\bar{e}ta = 0.005, P_0 = 30, c_r = 0.09, c_b = 0.2, \hat{z} = 0.05, y = 0.03$

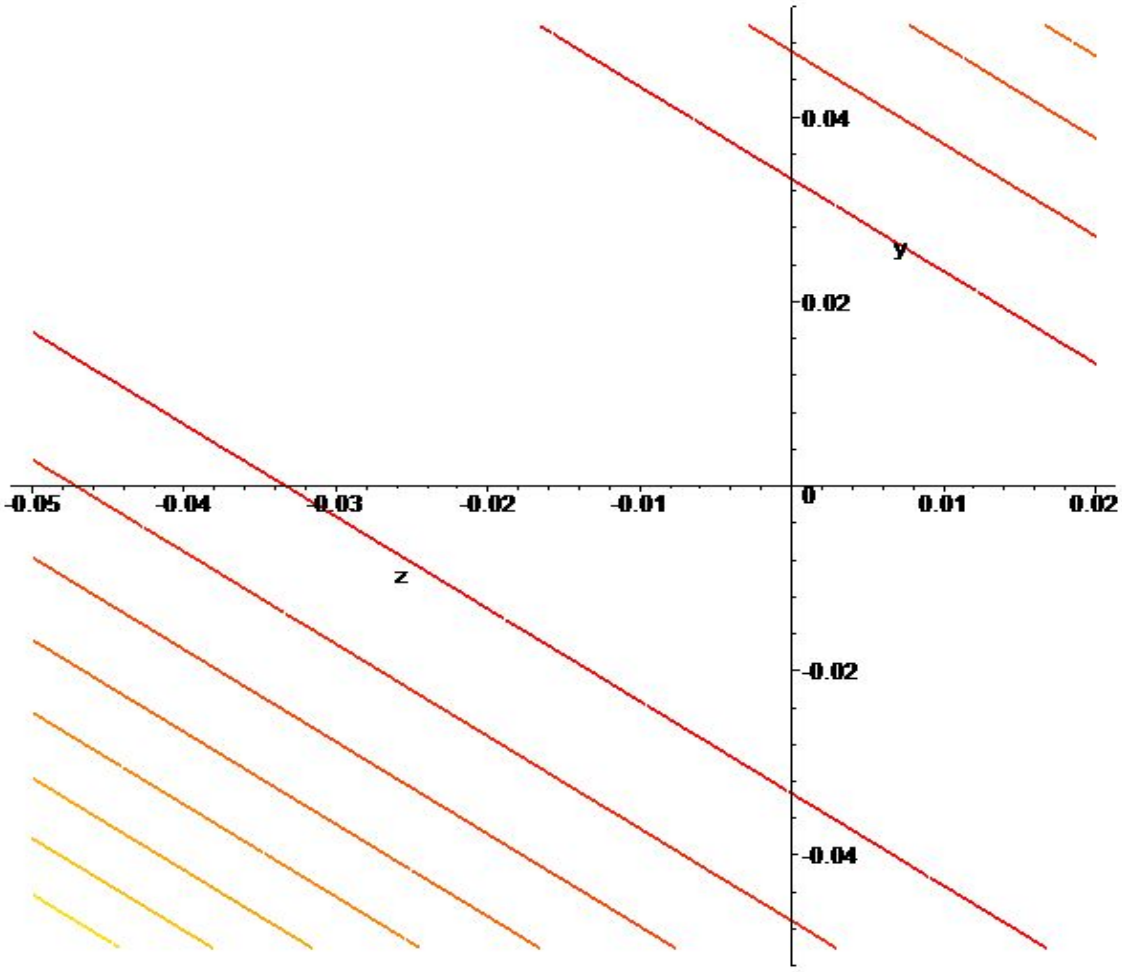

Figure 4: ***Buyer net gain function given value of y and z. Indifference curve grow from red to yellow.The zero indifference curve passes in the point*** $[y = -0.03, \tilde{z} = 0.032]$ ***Variable used for the graph:*** $\alpha = 2.5, \sigma = 0.1, c = 0.09$

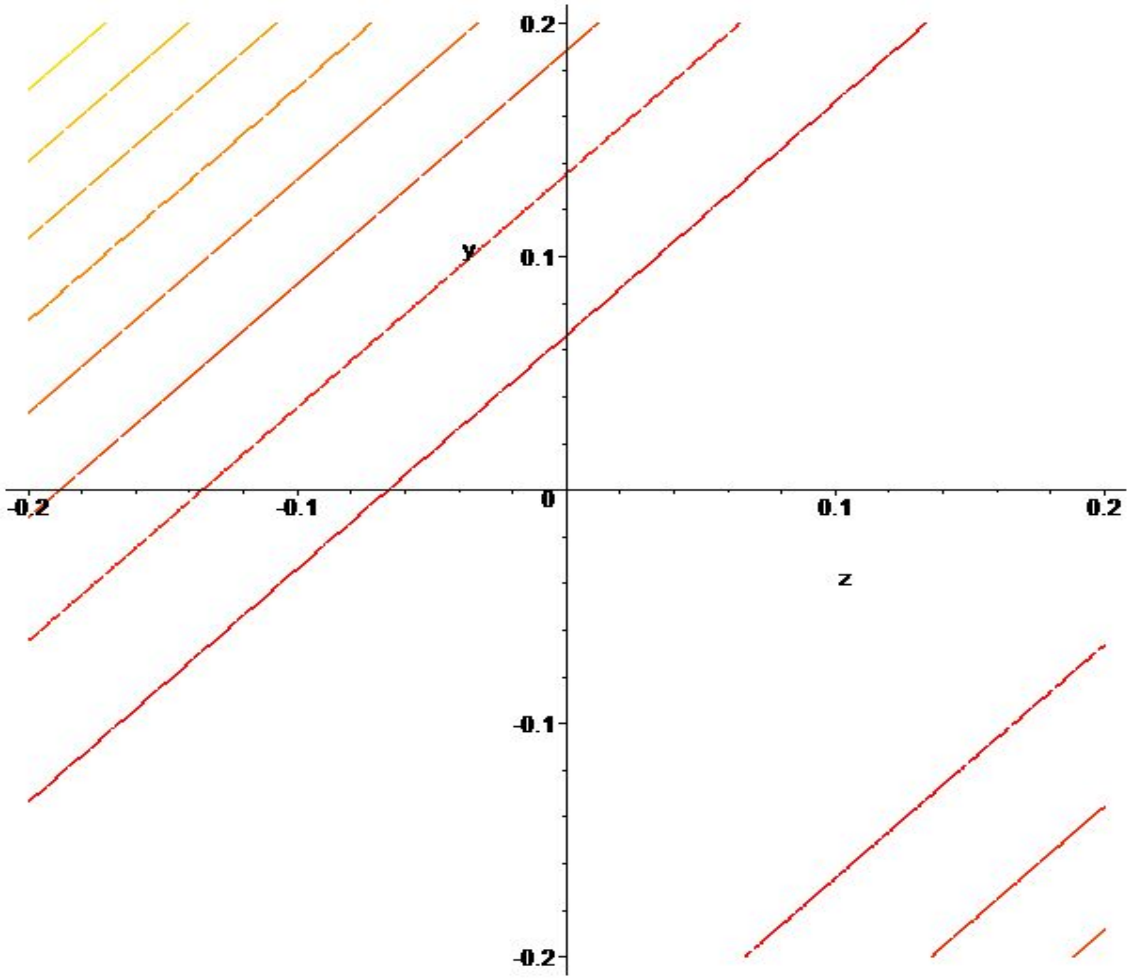

Figure 5: ***Seller net gain function given value of y and z. Indifference curve grow from red to yellow.The zero indifference curve passes in the point*** $[y = 0.094, \tilde{z} = 0.05]$ ***Variable used for the graph:*** $\alpha = 2.5, \sigma = 0.1, \hat{z} = -0.1, c = 0.09$

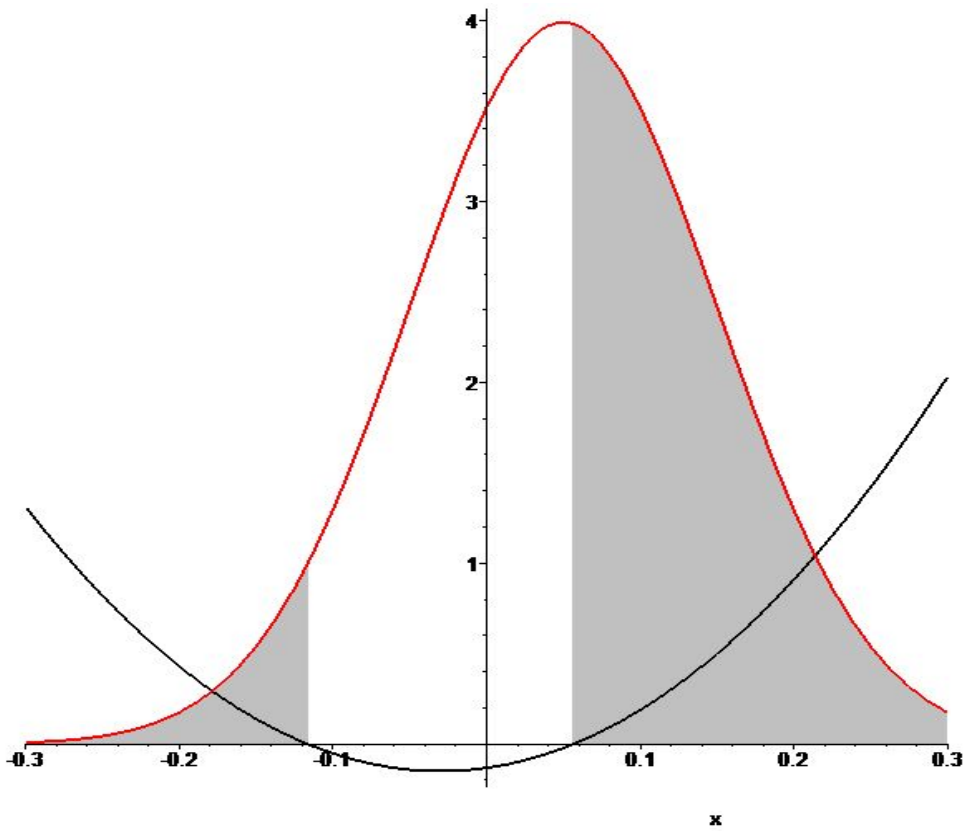

Figure 6: ***Buyer Participation function. The grey area represent the area of participation for buyers.The x axe represent the expectance z of investors. Variable used for the graph:*** $\alpha = 2.5, \sigma = 0.1, \bar{\theta} = 0.005, \hat{z} = 0.05, y = 0.03, c = 0.2, P_0 = 1$

y	z^	P_1	Buyer	Actors	Seller	Actors	Net flow
5.0%	10%	18.75	25.6%	26	28.0%	28	-2.5%
2.5%	10%	15.63	24.8%	25	26.9%	27	-2.1%
0.0%	10%	12.50	24.1%	24	26.4%	26	-2.3%
5.0%	5%	12.50	23.0%	23	24.4%	24	-1.4%
2.5%	5%	9.38	22.6%	23	23.2%	23	-0.6%
0.0%	5%	6.25	22.1%	22	22.6%	23	-0.5%
5.0%	0%	6.25	21.0%	21	21.0%	21	0.0%
2.5%	0%	3.13	20.8%	21	20.8%	21	0.0%
0.0%	0%	-	20.6%	21	20.6%	21	0.0%
5.0%	-2.5%	3.13	20.1%	20	20.0%	20	0.1%
2.5%	-2.5%	-	20.0%	20	20.0%	20	0.0%
0.0%	-2.5%	- 3.13	20.0%	20	20.1%	20	-0.1%

Figure 7: ***Joint participation. Variable used for the table:*** $\alpha = 4, \sigma = 0.4, c = 0.2, \theta = 0.005, P_0 = 1$

y	z^	Buyer	Actors	Seller	Actors	Net flow
5.0%	10%	25.6%	26	6.5%	7	19.1%
2.5%	10%	24.8%	25	8.9%	9	15.9%
0.0%	10%	24.1%	24	12.4%	12	11.7%
5.0%	5%	23.0%	23	12.4%	12	10.6%
2.5%	5%	22.6%	23	17.0%	17	5.6%
0.0%	5%	22.1%	22	24.3%	24	-2.2%
5.0%	0%	21.0%	21	24.2%	24	-3.2%
2.5%	0%	20.8%	21	30.9%	31	-10.1%
0.0%	0%	20.6%	21	0.0%	0	20.6%
5.0%	-2.5%	20.1%	20	30.9%	31	-10.8%
2.5%	-2.5%	20.0%	20	0.0%	0	20.0%
0.0%	-2.5%	20.0%	20	0.0%	0	20.0%

Figure 8: ***Joint participation with imposed price. Variable used for the table:*** $\alpha = 4, \sigma = 0.4, c = 0.2, \theta = 0.005, P_0 = 1, P = 8$

Date	Zhat	Y	Buyer	Actors	Seller	Actors	Net flow	Theta
Jun-05	-4.73%	10.96%	32.48%	9	41.16%	12	-9%	0.0172
Sep-05	-3.95%	9.42%	32.19%	9	41.15%	12	-9%	0.0179
Dec-05	-3.52%	9.83%	32.25%	9	41.54%	12	-9%	0.0179
Mar-06	-4.89%	12.19%	32.75%	9	41.74%	12	-9%	0.0179
Jun-06	-7.08%	13.25%	32.30%	9	41.52%	12	-9%	0.0179
Sep-06	-6.95%	12.89%	32.27%	9	41.48%	12	-9%	0.0172
Dec-06	-6.98%	12.83%	32.25%	10	41.46%	13	-9%	0.0161
Mar-07	-5.75%	10.61%	32.09%	9	41.31%	12	-9%	0.0179
Jun-07	-4.69%	8.81%	31.99%	9	41.21%	12	-9%	0.0172
Sep-07	-3.47%	6.90%	31.93%	9	41.14%	12	-9%	0.0172
Dec-07	-1.52%	3.76%	31.82%	9	41.04%	12	-9%	0.0172
Mar-08	5.11%	-8.96%	31.97%	9	41.18%	12	-9%	0.0172
Jun-08	5.32%	-11.70%	32.35%	9	41.56%	12	-9%	0.0172
Sep-08	6.14%	-15.18%	32.97%	10	42.15%	12	-9%	0.0172
Dec-08	8.99%	-21.03%	33.90%	10	43.05%	12	-9%	0.0172
Mar-09	15.75%	-42.08%	37.65%	10	50.30%	14	-13%	0.0185

Figure 9: ***Joint participation with optimal price. Variable used for the table:*** $\alpha = 4, \sigma_z = 0.4, c = 0.05, P_0 = 1$

Date	P_1	Zhat	Y	Buyer	Actors	Seller	Actors	Net flow	Theta
Jun-05	1.04	-4.73%	10.96%	8.72%	3	15.49%	4	-7%	0.0172
Sep-05	1.03	-3.95%	9.42%	11.01%	3	18.49%	5	-7%	0.0179
Dec-05	1.04	-3.52%	9.83%	9.07%	3	16.00%	4	-7%	0.0179
Mar-06	1.03	-4.89%	12.19%	7.24%	2	13.05%	4	-6%	0.0179
Jun-06	1.03	-7.08%	13.25%	9.77%	3	16.27%	5	-7%	0.0179
Sep-06	1.03	-6.95%	12.89%	9.68%	3	16.13%	5	-6%	0.0172
Dec-06	1.03	-6.98%	12.83%	8.38%	3	15.18%	5	-7%	0.0161
Mar-07	1.02	-5.75%	10.61%	12.64%	4	20.54%	6	-8%	0.0179
Jun-07	1.01	-4.69%	8.81%	14.06%	4	22.77%	7	-9%	0.0172
Sep-07	0.99	-3.47%	6.90%	17.14%	5	25.78%	7	-9%	0.0172
Dec-07	0.95	-1.52%	3.76%	23.88%	7	32.46%	9	-9%	0.0172
Mar-08	0.96	5.11%	-8.96%	0.00%	0	26.82%	8	-27%	0.0172
Jun-08	0.98	5.32%	-11.70%	0.00%	0	10.91%	3	-11%	0.0172
Sep-08	0.94	6.14%	-15.18%	0.00%	0	31.18%	9	-31%	0.0172
Dec-08	0.89	8.99%	-21.03%	0.00%	0	39.43%	11	-39%	0.0172
Mar-09	0.95	15.75%	-42.08%	0.00%	0	38.06%	10	-38%	0.0185

Figure 10: ***Joint participation with imposed price. Variable used for the table:*** $\alpha = 4, \sigma_z = 0.4, c = 0.05, P_0 = 1$

Date	Optimal price			Imposed Price			Reality		
	Buyer	Seller	Net flow	Buyer	Seller	Net flow	Buyer	Seller	Net flow
Jun-05	32.48%	41.16%	-8.68%	8.72%	15.49%	-6.77%	0.00%	0.00%	0.00%
Sep-05	32.19%	41.15%	-8.96%	11.01%	18.49%	-7.48%	5.82%	-0.05%	5.77%
Dec-05	32.25%	41.54%	-9.29%	9.07%	16.00%	-6.93%	4.78%	-0.12%	4.66%
Mar-06	32.75%	41.74%	-8.99%	7.24%	13.05%	-5.81%	3.75%	-0.27%	3.48%
Jun-06	32.30%	41.52%	-9.22%	9.77%	16.27%	-6.50%	0.03%	-0.03%	0.01%
Sep-06	32.27%	41.48%	-9.21%	9.68%	16.13%	-6.45%	3.09%	-0.02%	3.07%
Dec-06	32.25%	41.46%	-9.21%	8.38%	15.18%	-6.80%	8.39%	-0.40%	7.99%
Mar-07	32.09%	41.31%	-9.22%	12.64%	20.54%	-7.90%	7.38%	-0.38%	7.00%
Jun-07	31.99%	41.21%	-9.22%	14.06%	22.77%	-8.71%	2.83%	-0.34%	2.49%
Sep-07	31.93%	41.14%	-9.21%	17.14%	25.78%	-8.64%	0.00%	-2.11%	-2.11%
Dec-07	31.82%	41.04%	-9.22%	23.88%	32.46%	-8.58%	0.10%	-7.26%	-7.16%
Mar-08	31.97%	41.18%	-9.21%	0.00%	26.82%	-26.82%	-0.01%	-6.12%	-6.13%
Jun-08	32.35%	41.56%	-9.21%	0.00%	10.91%	-10.91%	0.00%	-1.32%	-1.31%
Sep-08	32.97%	42.15%	-9.18%	0.00%	31.18%	-31.18%	0.00%	-3.85%	-3.85%
Dec-08	33.90%	43.05%	-9.15%	0.00%	39.43%	-39.43%	0.00%	-4.48%	-4.48%
Mar-09	37.65%	50.30%	-12.65%	0.00%	38.06%	-38.06%	0.05%	-7.98%	-7.93%

Figure 11: ***Comparison between the model applying the optimal and an imposed price, with the effective values of the fund***

Date	Optimal price			Imposed Price			Reality		
	Buyer	Seller	Net flow	Buyer	Seller	Net flow	Buyer	Seller	Net flow
Jun-05	32.48%	2.63%	29.85%	8.72%	0.99%	7.73%	0.00%	0.00%	0.00%
Sep-05	32.19%	2.22%	29.97%	11.01%	1.00%	10.01%	5.82%	-0.05%	5.77%
Dec-05	32.25%	1.99%	30.26%	9.07%	0.77%	8.30%	4.78%	-0.12%	4.66%
Mar-06	32.75%	1.91%	30.84%	7.24%	0.60%	6.64%	3.75%	-0.27%	3.48%
Jun-06	32.30%	1.94%	30.36%	9.77%	0.76%	9.01%	0.03%	-0.03%	0.01%
Sep-06	32.27%	1.70%	30.57%	9.68%	0.66%	9.02%	3.09%	-0.02%	3.07%
Dec-06	32.25%	1.88%	30.37%	8.38%	0.69%	7.69%	8.39%	-0.40%	7.99%
Mar-07	32.09%	1.57%	30.52%	12.64%	0.78%	11.86%	7.38%	-0.38%	7.00%
Jun-07	31.99%	1.32%	30.67%	14.06%	0.73%	13.33%	2.83%	-0.34%	2.49%
Sep-07	31.93%	1.29%	30.64%	17.14%	0.81%	16.33%	0.00%	-2.11%	-2.11%
Dec-07	31.82%	1.25%	30.57%	23.88%	0.99%	22.89%	0.10%	-7.26%	-7.16%
Mar-08	31.97%	1.26%	30.71%	0.00%	0.82%	-0.82%	-0.01%	-6.12%	-6.13%
Jun-08	32.35%	1.27%	31.08%	0.00%	0.33%	-0.33%	0.00%	-1.32%	-1.31%
Sep-08	32.97%	1.29%	31.68%	0.00%	0.95%	-0.95%	0.00%	-3.85%	-3.85%
Dec-08	33.90%	1.31%	32.59%	0.00%	1.20%	-1.20%	0.00%	-4.48%	-4.48%
Mar-09	37.65%	1.78%	35.87%	0.00%	1.34%	-1.34%	0.05%	-7.98%	-7.93%

Figure 12: ***Comparison between the model applying the optimal and an imposed price, with the effective values of the fund, using the outflows weighted on the role of the investor (i.e. internal vs external)***