Option Pricing under Stochastic Volatility of US REITs
Gianluca Marcato and Tumellano Sebehela
Agenda

• Background
• M&As within REITs Industry
• Data
• REITs Margrabe (1978) Model
• Stochastic Process
• Empirical Results
  – GARCH (1;1) Parameters
  – Call Option Values
• Conclusion
Introduction/Motivation

- During the mid-1990s the US economy was booming.
- IPOs (REOCs) or M&As (REITs).
- Sometimes NAVs ≠ Share Prices.
- Possible reasons: best valuation technique, arbitrage opportunities or extra value.
- Debate on how best to value transactions.
  - DCFs, Earnings Based Valuations (DDM or AFFO) & NAVs.
  - NAV(stable asset base), although EBVs robust but subjective.
- Unconventional Pricing: Exchange Options.
Literature Review


• REITs grew from $9 bn to $128 bn by 1997: Clayton et al. 2007:

• Direct Investment Portfolios (DIPs) vs. Stock Investment Portfolios (SIPs): Anderson et al. 2002.

• Acquisition supports REITs’ long-term growth
  - lower dividend payout and rest for acquisition,
  - external funding.

• M&A induces growth:
  - Easier to growth small-caps, but mega-caps benefited more.
Data

• SNL Financials:
  – 179 (92 on REOCs & 87 on REITs) completed US M&A deals
  – From 1994 to 2009.

• M&A deals with no specific trend
  – REOCs & REOCs, REOCs & REITs, and REITs & REITs.

• Cleaning
  – 92 on REOCs and on 47 REITs poor recorded data.

• Final sample: 40 completed M&A deals on REITs merging with REITs
  – M&As of acquirers and targets were in the same/similar line of business at different times.

• Restrictions to be relaxed to increase the sample
Model 1

- Margrabe (1978) and Sebehela (2008) illustrated that a call option of Margrabe (1978) model can be written as:

\[
C[S_1, S_2, (T - \tau)] = S_1 e^{-\gamma_1 (T - \tau)} N(d_1) - S_2 e^{-\gamma_2 (T - \tau)} N(d_2)
\]

where

\[
d_1 = \frac{\ln(S_1 / S_2) + (\gamma_1 - \gamma_2 + \frac{\sigma_p^2}{2}) \times (T - \tau)}{\sigma_p \sqrt{T - \tau}}
\]

and

\[
d_2 = \frac{\ln(S_1 / S_2) + (\gamma_1 - \gamma_2 - \frac{\sigma_p^2}{2}) \times (T - \tau)}{\sigma_p \sqrt{T - \tau}}
\]

- Lagging effect in real estate markets (i.e. IPD yearly appraisal).
Model 2

- NAVs account for the lagging effect that is not captured by share prices.
- Injected funds (external or internal) treated as “extra value” to existing project’s value: Ahnefeld and Mehler-Bicher (2002), Davis et al. 2004, Jaimungal and Lawryshyn (2009).
- Lambda ($\lambda$) will represent the “extra value”.
- REITs Margrabe (1978) model before and after taking into lambda ($\lambda$).
Model 2: Continue

\[
C[(NAV_1 + \lambda), NAV_2, (T - \tau)] = (NAV_1 - \lambda) e^{-\gamma_1 (T - \tau)} N(d_1) - NAV_2 e^{-\gamma_2 (T - \tau)} N(d_2)
\]

\[
\ln \left[ \frac{(NAV_1 + \lambda)}{NAV_2} \right] + \left( \gamma_1 - \gamma_2 + \frac{\sigma_p^2}{2} \right) \times (T - \tau)
\]

\[
d_1 = \frac{\ln \left[ \frac{(NAV_1 + \lambda)}{NAV_2} \right] + \left( \gamma_1 - \gamma_2 + \frac{\sigma_p^2}{2} \right) \times (T - \tau)}{\sigma_p \sqrt{T - \tau}}
\]

\[
\ln \left[ \frac{(NAV_1 + \lambda)}{NAV_2} \right] + \left( \gamma_1 - \gamma_2 - \frac{\sigma_p^2}{2} \right) \times (T - \tau)
\]

\[
d_2 = \frac{\ln \left[ \frac{(NAV_1 + \lambda)}{NAV_2} \right] + \left( \gamma_1 - \gamma_2 - \frac{\sigma_p^2}{2} \right) \times (T - \tau)}{\sigma_p \sqrt{T - \tau}} \quad \text{or} \quad d_2 = d_1 - \sigma_p \sqrt{T - \tau}
\]
Stochastic Process

- Stochastic process is Martingale is for all $s \leq t$ and its expectation should be represented as follows:

$$E[X_t - X_s \mid (X_s)_{t \geq s}] = 0$$

$$E[NAV_t \mid F_s] = E[NAV_s + NAV_t - NAV_s \mid F_s]$$

$$= E[NAV_s \mid F_s] + E[NAV_t - NAV_s \mid F_s]$$

$$= NAV_s + E[NAV_t - NAV_s \mid F_s]$$
## Main Results 1

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<tr>
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Source: SNL Financials
Note: Parameters were simulated using Eviews
GARCH (1;1) Parameters
## Main Results 2

<table>
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<tr>
<th>Acquirer's Ticker</th>
<th>Target</th>
<th>Acquirer's line of business</th>
<th>Target's line of business</th>
<th>Call (Share Prices)</th>
<th>Call (Share Prices &amp; Funds)</th>
<th>Funds Factor (SP)</th>
<th>Call (NAVs)</th>
<th>Call (NAVs &amp; Funds)</th>
<th>Funds Factor (NAV)</th>
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<tbody>
<tr>
<td>SPG_1</td>
<td>Chelsea Property Group</td>
<td>Region Mall</td>
<td>Outlet Center</td>
<td>-1.01</td>
<td>-0.1</td>
<td>0.897</td>
<td>0.3</td>
<td>0.34</td>
<td>0.139</td>
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<td>DeBartolo Realty Corp.</td>
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<td>Region Mall</td>
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<td>8.49</td>
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<td>Region Mall</td>
<td>Shopping Centre</td>
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<td>16.98</td>
<td>0.097</td>
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<td>Office</td>
<td>41.33</td>
<td>40.6</td>
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<td>11.43</td>
<td>11.47</td>
<td>0.003</td>
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</tbody>
</table>

Source: SNL Financials

Call Option Values
Figure 1

Source: National Council of Real Estate Investment Fiduciaries (NCREIF)
Conclusions

• When one REIT takes over another, there is “extra” occurring value from M&A.
• Although, NAV is not robust, it gives a better picture about the real situation.
• Share prices lead to misleading conclusions about M&A of REITs.
• M&A increase value without a REIT changing anything operationally.
Further Research

• Option pricing of REITs M&A under bearish market.

• When analysing REITs, when is it better to use share prices and when is it better to use NAVs.

• Option pricing of REOCs M&A using Margrabe (1978) model.
Thank You

QUESTIONS?