Optimal Selling Mechanism, Auction Discounts, and Time on Market

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Literature

- Adams, Kluger, and Wyatt (JREFE, 1992)
  - Slow Dutch auction v.s. search
  - Positive auction discounts
  - Slow Dutch auction is never optimal
- Mayer (JUE, 1995)
  - English-style auction v.s. search
  - Positive auction discounts
- Quan (REE, 2002)
  - First-price sealed-bid multiple object auction v.s. search
  - Negative auction discounts
Common Features of Previous Studies

- **Risk neutral agents**
  - Consistent with the mainstream auction literature’s maximizing expected revenue assumption
  - Is this assumption realistic for individuals?

- **Begin from a search model, then augment to obtain an auction model**
  - Selling without recall model
  - The seller cannot recall previous offers
  - How about a selling with recall model?
This Paper’s Position

- Risk averse seller
  - Mean-variance utility or
  - Downside risk focus, loss aversion

- Selling with recall model
  - The seller can recall all or part of previous offers
  - A variant of Cheng, Lin and Liu (REE, 2008)

- Portfolio theory approach
  - All possible strategies (e.g. different reserve prices/different stopping time) in one selling mechanism form an opportunity set
  - Compare opportunity sets and efficient sets
SRTM and SRNB

- Consider two alternative stopping rules in selling with recall framework:
  - SRTM – the stopping rule of choosing an optimal time on market
  - SRNB – the stopping rule of choosing an optimal number of bidders (analysed by Cheng et al. 2008)

- Both rules choose the highest available price among previous offers.
Duality of the SRTM

- **SRTM is a valid search rule**
  - “a rational seller will try to plan for an optimal marketing period. (Cheng et al. 2008, page 821)”
  - Sellers plan to move, change jobs, or face financial distress tend to have a fixed deadline but not necessary go for auctions

- **SRTM can be treated as a private valuation, no reserve, first-price sealed bid auction**
  - Remaining buyers send in their offers in sealed envelopes and the seller chooses the highest offered price
  - Can also be treated as an English auction if the seller chooses the second highest offer
The Model

- Uniform bid price distribution
- Exogenous and homogeneous Poisson arrivals
- Constant holding cost $c$ per unit of time
- $\Theta$ - Retention rate
  - $\Theta = 1$, perfect recall
  - $0 < \Theta < 1$, partial recall
- Closed-form means and variances available for the SRNB and the SRTM.
Seller’s Optimization Problem

- **SRTM**
  - $K(T) = Y_{N(T)} - cT$
  - $\max E(U(K(T))), T \in (0, +\infty)$
  - T is fixed, N is random

- **SRNB**
  - $K(N) = Y_N - cT (N)$
  - $\max E(U(K(N))), N \in \{1, 2, \ldots, +\infty\}$
  - N is fixed, T is random
Main Result 1 – (mean-variance analysis)
Auction Discounts and Risk Reductions

- There are many stopping strategies in the SRNB and the SRTM.
- Calculating auction discounts on the selling mechanism level is meaningless.
- Need to define comparable strategies.
- Auction discounts can be defined on comparable strategies.
Definition 2 For each stopping strategy $N$ (waiting for $N$ buyers) of the SRNB, its waiting equivalent stopping strategy is the stopping strategy of the SRTM which satisfies $T_{we}(N) = N/\lambda$ (waiting a fixed time $T_{we}(N)$). $T_{we}$ is the waiting equivalent TOM.

Definition 5 For each stopping strategy $N$ in SRNB, its certainty equivalent stopping strategy is the strategy of the SRTM which satisfies $E(K(N, \theta)) = E(K(T_{ce}(N), \theta))$. $T_{ce}(N)$ is the certainty equivalent TOM.
Main Result 2 – (auction discounts, Theorem 1)
Main Result 3 – (Holding Cost, Risk Aversion and TOM, if the seller chooses a fixed TOM)
Downside Risk

- Few real estate researches analysed downside risk
- Loss Aversion - Genesove and Mayer (2001)
- This paper use Value at Risk and expected shortfall to quantify downside risk.
- Downside risk is important to consider when TOM is uncertain and holding cost is significantly high.
Main Result 4 – (Downside Risk)

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Conclusion

- This paper uses modern finance theory to solve a conventional microeconomic problem.

- Major findings:
  - More risk averse sellers choose auctions
  - Less risk averse sellers choose an optimal number of buyers and wait for a random time
  - Positive auction discounts are compensated by decreased risk
  - Sellers’ choices are impacted by holding cost, risk aversion and downside risk
  - A unique and universal optimal selling mechanism in real estate market does not exist

- Extension: results on English auction is straightforward to obtain by simulation.