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FUZZY NUMBERS FOR THE ASSESSMENT OF REAL ESTATE MARKET AND PROPERTY VALUATION

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Abstact

The paper is focused on the use of fuzzy numbers in some different spheres – real estate researches, investments and valuation.

At the first part of paper it is shown that fuzzy numbers allows us to get exact model for the property valuation. The probability of this model is 100% in contradistinction of traditional models which has inaccuracy about 10%.

The real estate market is not perfect and it is subject of many external influences, such as: an economic situation in the country, seasonal activity, availability of financial assets. Therefore it doesn’t have precise logic connections and definitions. That is why it is necessary to use fuzzy numbers when we research a real estate market. The process of estimation of real estate investments of any country considerably becomes simpler. In another part of paper is analyzed the real estate market of Minsk – the capital of Belarus. It was used fuzzy numbers.

THE USE OF FUZZY NUMBERS IN THE PROCESS OF THE PROPERTY VALUATION

APPROACHES AND METHODS OF ENTERPRISE COST VALUATION

The specific feature of cost valuation process is its market aspect. It means that the valuation is not limited by production cost or purchase cost of the estimated object. It also takes into account the aggregate market factors – time factor, risk factor, market condition, level and type of competition, economic features of the estimated object as well as macro- and microeconomic environment.

The basic factors are time and risk. Market economy is characterized by dynamism. Time factor is likely to be the major factor influencing on all market processes, on price, cost and decision making.

Time during which the money should be paid or received is of prior importance in the process of evaluation. Market price varies in time due to numerous factors, so it can be calculated for a certain moment in time. Tomorrow market price might be different. As a result, regular estimate and re-estimate of property objects in general is a must for market economy.

There is a number of methods of enterprise cost valuation that are commonly divided into property approach, income approach and comparative approach.

Property approach is based on the analysis of the enterprise assets and include the below methods:
- enterprise assets accumulation
- corrected financial statement cost (or net assets method)
- calculation of replacement cost
- calculation of liquidation cost

All methods of property approach are applied as a rule to evaluate the controlling block of the enterprise shares.
The basic advantage of property approach – it is based on real assets and cost. At the same time these methods are ‘static’ and do not consider the prospects of the enterprise development, its future profitability.

The methods of comparative approach of enterprise valuation are based on the comparison of the present enterprise cost with the cost of the similar enterprises. Comparative approach includes the below methods:
- capital market
- transactions (method of comparative analysis of sales)
- branch coefficients

The method of capital market is based on market prices of the shares of enterprise similar to the estimated enterprise. The advantage of the approach is the use of information available at the stock market. It is not based on the predicted data as in case of income approach application.

Transaction method (or method of comparative analysis of sales) is a private case of capital market method, it is based on the price analyses of purchase of controlling block of the shares of the similar enterprises. In this case stock market information is not analyzed, but the information of merger market. The major advantage of the method is its reflection of the current

real economic activity. Its drawback – it relies on the past events and does not consider the future terms of economic activity.

Methods of income approach of the enterprise cost valuation based on the definition of the current cost of future income include:

- method of income capitalization
- method of money flows discounting

Method of income capitalization is a variety of income approach of the operating enterprise valuation. As other varieties of income approach it is based on the notion that the cost of enterprise share property is equal to the current cost of future income that this property will bring. The essence of the method is delivered in the below formula:

Valuation cost = Net profit / Capitalization rate

Profit capitalization method is applicable in cases when it is expected that the enterprise will receive the approximately equal profit (or its growth speed will be constant) during a long time period.

If it is expected that future income will be different from prior income and unstable over the predicted years the valuation is carried out with the method of money flows discounting.

The most complicated point of income approach methods is the process of future income forecast. In case of income capitalization – income level definition for the 1st predicted year (and it is accepted that the income will be of the same amount over all predicted years); in case of money flows discounting – it is the income definition for every future year of the entire predicted period as well as for the remaining period. Added to that, it is difficult to define capitalization rates and discount rates of future enterprise income due to incomplete and unstable market data.

In terms of economy, discount rate is the income rate required by the investors for the invested capital into the similar (from risk point of view) objects. In other words, it is the required income rate that can be obtained from other investment projects of the approximately same risk at the date of valuation.

There are various methods of discount rate calculation. The most popular are:

- evaluation of capital assets
- cumulative method
- model of average-weighted capital cost

In accordance with the model of capital assets evaluation the discount rate is calculated:

$$R ? R_f ? ? (R_m ? r_f) ? S_1 ? S_2 ? C,$$

R – required by the investor income rate (for his own capital)

R_f - risk free income rate

$?$ - beta coefficient (a system risk related to the macroeconomic and political processes that take place in the country)

R_m - general market profitability (average market portfolio of securities)

S_1 - small enterprise bonus

S_2 - bonus for the risk particular for a separate company

C - country risk.

When the discount rate is defined the capitalization rate is calculated in this way:

$$R = d - g$$

R – capitalization rate
d – discount rate
g – long-term speed of profit growth or money flow

The last steps of profit capitalization method application are simple.

The preliminary cost is calculated:

$$I / R = V$$

To make corrections for non-functioning assets, it is required the valuation of their market cost in conformity with the accepted methods for a particular type of assets (real estate, cars, equipment and etc.).

An important step forward in considering future types of interest was the theory of fuzzy numbers.

Recently there was suggested a number of methods of economic activity research taking into account uncertain terms when the data is incomplete and models are projected on the basis of logic.

Our basic statement is that qualitative methods of business-system analysis can't be applied to the real enterprise systems because the business communication language in its essence has no distinct logic connections and definitions. This statement is also applicable to the property valuation.

BASIC NOTIONS OF FUZZY NUMBERS

The notion of fuzzy numbers was initially introduced by Mr. L. Sade in 1965.
The approach has three features:

1. So called 'linguistic' variables are applied.
2. Simple relations between variables are described with the help of fuzzy expressions.
3. Complicated relations are described with the help of fuzzy algorithms. For instance:
   a. Fuzzy variable income: high, not high, very high, low and etc. The words 'very, not, and, but' are used.
   b. If x is small than y is large
   c. If y is large then reduce x

Let's define certain notions of fuzzy numbers.

Fuzzy number A in the entire multitude area of U reasoning is characterized by the function of F_A belonging: U belongs to [0,1] segment that puts in accordance number F_A (y) from segment [0,1] (that describes the level of element belonging to A multitude) to every element from U multitude.

Now let's define the notion of 'fuzzy numbers'. We will consider a certain kind of so called 'triangular' fuzzy numbers. Triangular fuzzy number N – three numbers (a, ?$_l$, c_r), where a – center, c_l – the amount of fuzziness from left, c_r – the amount of fuzziness from right.

MONEY FLOW OPTIMIZATION WITH THE HELP OF FUZZY NUMBERS

The decision to purchase 'investment object' when the interest is known and constant is applicable if the value prior to the investment start is positive. It is a preliminary condition when there is a choice from a number of alternate variants that are in accordance with several objects of possible investments.

If we assume that for the object there are n+1 payments a_0, a_1,…, a_n at the moments $_{0,1,2,\ldots,n}$ and n returns b_1,b_2,…,b_n at the moments $_{1,2,\ldots,n}$, investment will be profitable if the received value with the constant i interest rate will be more or equal to the amount of payments. It means if:

$$\sum_{j=1}^{n} b_j (1+i)^{-j} \geq a_0 + \sum_{j=1}^{n} a_j (1+i)^{-j},$$

or in another way:

$$V_n = \sum_{j=1}^{n} (b_j - a_j)(1+i)^{-j} - a_0 \geq 0,$$

Where V_n - the actual value of summary income in relation with moment 0. If we indicate the difference b_j – a_j as A_j, then:

$$V_n = \sum_{j=1}^{n} A_j (1+i)^{-j} - A_0,$$

Where A_0=a_0

In the conditions of investments A_j is understood as the difference between income and payments. Their adding up considering the volume growth of money supply for the fixed i interest is actualized profit. Actualization method enables to compare series of capitals (returns as well as expenditures) which can't be classified by other methods due to the difference in the pace of expiry dates. This method arranges 'complete order' among all possible series of capitals.

When the investment object that is profitable from economic point of view is defined the next step is its comparison with other objects. The following selection criterion is applied. If we assume that it is possible to get any amount of money with the calculated type of interest it is more profitable to select the investments where the actualized value of the profit will be the largest.

These conditions for A and B can be written in this way:

If $V_n^A - V_n^B > 0$, then A is applicable,

If $V_n^A - V_n^B < 0$, then B is applicable.

If the variable but known in advance type of interest is considered the actualized profit value looks like this:

$$V_n = -A_0 + \frac{A_1}{(1+i_1)} + \frac{A_2}{(1+i_1)(1+i_2)} + \cdots + \frac{A_n}{(1+i_1)(1+i_2)\cdots(1+i_n)}.$$

If the question of investments is regarded in fuzzy terms then the interest type is not just variable in time, but takes a vague form. Then it is possible to apply fuzzy actualization. The above expression for a certain level can be written:

$$V_n^{(\alpha)} = -A_0 + \frac{A_1}{\left(1+\left[r_1^{(\alpha)}, s_1^{(\alpha)}\right]\right)} + \frac{A_2}{\left(1+\left[r_1^{(\alpha)}, s_1^{(\alpha)}\right]\right)\left(1+\left[r_2^{(\alpha)}, s_2^{(\alpha)}\right]\right)} + \cdots$$

$$+ \frac{A_n}{\left(1+\left[r_1^{(\alpha)}, s_1^{(\alpha)}\right]\right)\left(1+\left[r_2^{(\alpha)}, s_2^{(\alpha)}\right]\right)\cdots\left(1+\left[r_n^{(\alpha)}, s_n^{(\alpha)}\right]\right)}.$$

Considering the formula we have got above:

$$\frac{1}{1+\left[r_k^{(\alpha)}, s_k^{(\alpha)}\right]} = \left[\frac{1}{1+s_k^{(\alpha)}}, \frac{1}{1+r_k^{(\alpha)}}\right]$$

We get:

$$V_n^{(?)} ? ? A_0 ? A_1 \left[\frac{1}{1 ? s_1^{(?)}}, \frac{1}{1 ? r_1^{(?)}} \right] ?$$

$$? A_2 \left[\frac{1}{(1 ? s_1^{(?)})(1 ? s_2^{(?)})}, \frac{1}{(1 ? r_1^{(?)})(1 ? r_2^{(?)})} \right] ? \square ?$$

$$? A_n \left[\frac{1}{(1 ? s_1^{(?)})(1 ? s_2^{(?)}) \square (1 ? s_n^{(?)})}, \frac{1}{(1 ? r_1^{(?)})(1 ? r_2^{(?)}) \square (1 ? r_n^{(?)})} \right].$$

This expression for every level enables to find the range of opportunities among which as we hope is the real result. Added to that, varying the possession level and the programmed step (determined or fuzzy) it will enable us to define at what ***a*** level it would be possible to provide selection A_j (j=0,1,2…,n) and assumption in relation with the intervals $\left[r_k^{(?)}, s_k^{(?)} \right]$.

Let's illustrate the above by a number example. In case of investments into an object life duration of which is evaluated by 3 years, it means 1 = 0, 1, 2, 3, it is possible to assume that interest kind for this economic object is fuzzy and expressed (in %) by the following triangular numbers:

First year: $(r_1, m_1, s_1) ? (8,10,13)$;

Second year: $(r_2, m_2, s_2) ? (9,12,15)$;

Third year: $(r_3, m_3, s_3) ? (7,10,12)$.

On the other hand, if A_0 =7000, A_1 = 5000, A_2 = 4000, A_3 = 2000 money units then it is possible to calculate $V_n^{(?)}$ successively for eleven ***a*** values*:* 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1. For that purpose, the intervals of authenticity are defined (in %) for each level:

$\left[r_1^{(?)}, s_1^{(?)} \right] ? ? [8 ? 2?, 13 ? 3?]$;

$\left[r_2^{(?)}, s_2^{(?)} \right] ? ? [9 ? 3?, 15 ? 3?]$;

$\left[r_3^{(?)}, s_3^{(?)} \right] ? ? [7 ? 3?, 12 ? 2?]$.

Considering these data, $V_3^{(?)}$ is calculated with the help of below formula:

$$V_3^{(?)} ? \; ?7000 ? 5000 \left[\frac{100}{100 ? 13 ? 3?}, \frac{100}{100 ? 8 ? 2?} \right] ?$$

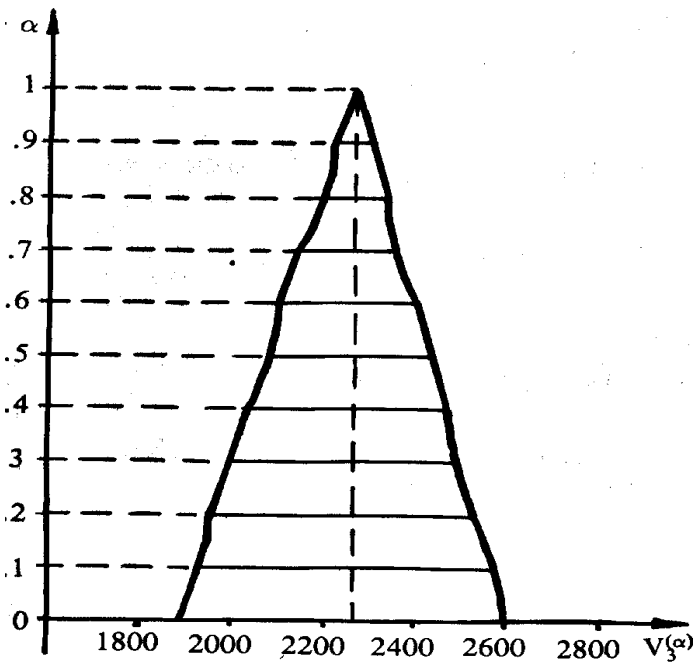
$$?4000 \left[\frac{100}{100 ? 13 ? 3?} \frac{100}{100 ? 15 ? 3?}, \frac{100}{100 ? 8 ? 2?} \frac{100}{100 ? 9 ? 3?} \right] ?$$

$$?2000 \left[\frac{100}{100 ? 13 ? 3?} \frac{100}{100 ? 15 ? 3?} \frac{100}{100 ? 12 ? 2?}, \frac{100}{100 ? 8 ? 2?} \frac{100}{100 ? 9 ? 3?} \frac{100}{100 ? 7 ? 3?} \right] ?.$$

We get for different levels:

?	$V_3^{(?)}$
0	[1875.9,2614.5]
0.1	[1914.1,2578.8]
0.2	[1951.2,2542.9]
0.3	[1990.0,2507.8]
0.4	[2028.8,2472.3]
0.5	[2067.4,2437.4]
0.6	[2106.2,2403.1]
0.7	[2146.9,2368.2]
0.8	[2185.7,2334.4]
0.9	[2226.6,2300.3]
1	[2266.2,2266.2]

These results can be depicted in a graphic way:



One can notice from the above that the final curve is not entirely triangular, but close to that.

The above simplified reasoning about fuzzy percent of triangular kind can be easily shown for arbitrary fuzzy numbers.

The indicated scheme can be easily combined with the method of fuzzy initial basic budget if there are any financial limitations (determined or fuzzy).

Thus, if the prediction of mid-term and long-term consequences of investment at the enterprise is difficult or impossible, the assumption about the fuzzy kinds of percent is fruitful.

VALUATION OF FREE MONEY FLOW WHEN THE DATA IS FUZZY

No need to talk about the importance of the methods that enable to predict the future enterprise behavior in terms of uncertainty. Of course, they should be based on classic instruments. I mean to use in classic models the recent mathematics achievements that consider the uncertainty of data. Thus, to analyze the free money flows and conclusions about the predicted flows, the application of the known methods with regard to the new situations with fuzziness to evaluate the intervals over which the movement of money flows will take part might be useful. In this way we get the information about mid-term alterations in solvency.

Studying free money flows in terms of uncertainty, it is possible, in particular, to use intervals to evaluate the future balance sheet as the enterprise is unaware about the exact cost of different property though it is possible to state its upper and lower limits. The indicators of the initial financial statement will be exact as we talk about the certain data.

Let's have a look at the balance sheet of an enterprise:

Assets		Liabilities	
Money assets			
Cash and bank	50	Short-term credits	50
Customers and money to be received	50	Long-term credits	100
Stock	250	Ownership capital	600
Basic capital	400		
Total	750	Total	750

Settlement transactions that are related with the money assets and short-term credits and that should be carried out next year look in this way:

Cash transactions.....……..…..... [650, 700]
Cash payments for production supplies…….. [650, 675]
Fluctuation 'cash – bank' .....…….. [-25, 50]
Sales…….....…..... [125, 150]
Fluctuation 'customers and money to be received'..... [125, 150]
Production supplies…….....…..... [75, 75]
Dividends to be paid out…….....…..... [75, 175]
Fluctuation 'short-term credits' ……..... [150, 250]

As a result of the above transactions the balance sheet will look at the end of the period like this:

Cash and banks .....……..... 50 (+)[-25, 50] =[25, 100]

Customers and money to be received..... 50 (+) [125, 150] = [175, 200]
Money assets..... [200, 300]
Short-term credits ……..... 50 (+) [150, 250] = [200, 300]

The above changes influence, on the one hand, on money resources and, on the other hand – on short-term loans. To get the report about the source condition and money use, it is sufficient to arrange them into groups in a proper way. To cut deductions in the above instance, we have considered just the transactions related to production activity. But in reality in the forecast it is necessary to take into account other kinds of returns.

EXAMPLE OF ENTERPRISE COST VALUATION IN FUZZY TERMS

Let's consider an example that will reflect the process of enterprise cost valuation in fuzzy terms. Several predicted values are calculated with the help of fuzzy numbers that are considered as triangular for simplification. Let's regard the below balance sheet:

ASSETS (in thousands $)		LIABILITIES (in thousands $)	
Money assets	(20,22,30)	Internal funds	(100,116,130)
Stock	(30,36,40)		
Net intangible assets	(50,58,60)		
Total	(100,116,130)	Total	(100,116,130)

In this case the enterprise has no debts. If the profit is valued as B=(14000,16000,20000), we get:

Assets profitability = Capital profitability =B/A=(14000,16000,20000)/ /(100000,116000,130000).

To get these coefficients let's apply fuzzy triangular numbers in the form of a-sections:

B=[14000+2000a,20000-4000a], A=[100000+16000a,130000-14000a].

Assets profitability and capital profitability will be expressed in the form of the below interval:

D=[14000+2000a,20000-4000a]/[1000000+16000a,130000-14000a]=
=[(14000+2000a)/(130000-14000a),(20000-4000a)/(100000+16000a)].

To get a fuzzy number, let's make a table of upper and lower limits of this interval when a=0;0,1;0,2;…;0,9;1,0.

Lower and upper limits of fuzzy number (in thousands $)

A	14+2a	20-4a	100+16a	130-14a	14+2a/ 130-14a	20-4a/ 100+16a
0	14,0	20,0	100	130,0	0,1076	0,2000
0,1	14,2	19,6	101,6	128,6	0,1104	0,1929
0,2	14,4	19,2	103,2	127,2	0,1132	0,1860
0,3	14,6	18,8	104,8	125,8	0,1160	0,1793
0,4	14,8	18,4	106,4	124,4	0,1189	0,1729
0,5	15,0	18,0	108,0	123,0	0,1219	0,1666
0,6	15,2	17,6	109,6	121,6	0,1250	0,1605
0,7	15,4	17,2	111,2	120,3	0,1281	0,1546
0,8	15,6	16,8	112,8	118,8	0,1313	0,1489
0,9	15,8	16,4	114,4	117,4	0,1345	0,1433
1,0	16,0	16,0	116,0	116,0	0,1379	0,1379

It is obvious that profitability will not exceed 20% and not drop below 10,76%. The most probable value in this case is 13,79%.

If to take assumption level, for instance, a = 0,6, then we get profitability that will not exceed 16,05% and not drop below 12,5%.

Thus, the sought ready triangular number in a certain approximation will be equal to:

D=(0,1076;0,1379;0,2000).

Presenting it in the form of interval, we get:

D=[0,1076+0,0303a;0,2000-0,0621a].

Average value of this triangular number will be:

$D_{average}$ =(0,1076+2*0,1379+0,2000)/4=0,1458.

From the above, for a certain intermediate level, for instance, a = 0,6, we get the interval [0,1250;0,1605] from the formula that determines the interval for D. And from its triangular approximation we get [0,1257;0,1627]. In this case the difference is insignificant. Particularly, if we consider the fuzzy terms and high degree of subjective estimates.

On the basis of this example let's estimate the enterprise cost. As we know, the enterprise cost is defined with the help of the below formula:

$$\mathbf{V = I / R}$$

To determine the discount rate, let's apply the above-mentioned formula:

$$R ? R_f ? ? (R_m ? r_f) ? S_1 ? S_2 ? C,$$

The below values are characteristic for the Republic of Belarus:

Rf=20%
B=1
Rm=30%

S2=4%
C=6%.

Discount rate is 40%.

If to present the predicted values with the help of fuzzy numbers the above formula will look:

R=(23,20,18)+1((35,30,29)-(23,20,18))+(6,5,4)-(6,4,3)=(35,31,30).

Then taking into consideration the profit from assets and liabilities sheet the enterprise valuation cost = (14000,16000,20000)/(0,30;0,31;0,35).

A	14+2a	20-4a	0,3+0,01a	0,35-0,04a	14+2a/ 0,3+0,01a	20-4a/ 0,35-0,04a
0	14,0	20,0	0,3	0,35	46,66667	57,14286
0,1	14,2	19,6	0,301	0,346	47,17608	56,6474
0,2	14,4	19,2	0,302	0,342	47,68212	56,14035
0,3	14,6	18,8	0,303	0,338	48,18482	55,6213
0,4	14,8	18,4	0,304	0,334	48,68421	55,08982
0,5	15,0	18,0	0,305	0,330	49,18033	54,54545
0,6	15,2	17,6	0,306	0,326	49,6732	53,98773
0,7	15,4	17,2	0,307	0,322	50,16287	53,41615
0,8	15,6	16,8	0,308	0,318	50,64935	52,83019
0,9	15,8	16,4	0,309	0,314	51,13269	52,2293
1,0	16,0	16,0	0,310	0,310	51,6129	51,6129

It is evident that the property cost is not lower **$ 46 687** and not above **$ 57 143.**
With **100%** certainty, we can state that the cost of the estimated property is **$ 51 613**.

Conclusion: from the above analysis we can state that the application of fuzzy numbers in the process of property evaluation enables to determine property value with much higher probability (100%) in comparison with the traditional approaches of evaluation according to which the cost valuation precision is up to 10%.

THE USE OF FUZZY NUMBERS IN THE PROCESS OF THE REAL ESTATE ANALYSIS

It if possible to use fuzzy numbers in real estate analysis and research. Lets see how it is possible to do using example with real estate market of Belarus. We will consider two main elements for the purpose of real estate market estimation: the level of citizen's incomes and of knowledge about the goods (the real estate).

Lets define fuzzy numbers in (0,1) as a triangle fuzzy numbers (a, c_l, c_r), where a – center, c_l and c_r - sizes of illegibility's from the left and right hand.

The fuzzy function is expressed in the below form:

$$\mathbf{V = f\ (U\ \text{“income”},\ U\ \text{“level of knowledge”}) = 0{,}2U_{income} + 0{,}6U_{know},}$$

Where V – complex estimation of real estate market development,

U_{income} – fuzzy estimation of citizen's income level,

U_{know} - – fuzzy estimation level of knowledge about the goods (the real estate),

0,2 and 0,6 – fuzzy numbers (0,2; 0,1; 0,1) and (0,6; 0,1; 0,1).

It means that the level of citizen's incomes and of knowledge about the goods (the real estate) is estimated as "low" and "very high"

For the purpose of estimation it is necessary to set vocabulary:

Description	Fuzzy number
Very high	(1,00; 0,2; 0,00)
High	(0,75; 0,15; 0,15)
Middle	(0,5; 0,15; 0,15)
Low	(0,25; 0,15; 0,15)
Very low	(0,00; 0,00; 0,20)

$$a = 0{,}2*1{,}00 + 0{,}6*\ 0{,}25 = 0{,}35$$

$$c_l = 0{,}35 – (0{,}1*0{,}8 + 0{,}5*0{,}1) = 0{,}22$$

$$c_r = (0{,}3*1{,}00 + 0{,}7*0{,}4) – 0{,}35 = 0{,}23$$

Using these dates we've got fuzzy number of complex estimation (0,35; 0,22; 0,23). It means that we can estimate belarusian real estate market as "low", therefore it is necessary to rise level of citizens incomes and to give more knowledge about the goods (real estate).

Using this technique and more criteria we can estimate real estate market of any country. It is very important information for the banks, which tries to find the country for investments. Let's emerging that the bank is searching several countries for investments and ready to invest in

country, which has very high, high and middle level. The only question is how much to invest in particular country. If use fuzzy numbers the answer is very simple. Lets consider the example. Let fuzzy function is expressed in the below form:

V = f (U "income from real estate investments", U "level of security", U "level of real estate development") = $0,5U_{income}+0,3U_{sec}+0,2U_{dev}$,

Let the income from real estate investments of one country (for example, Germany) is very high, level of security is high, level of real estate development if middle.

In another country (for example, England) the same parameters are middle, middle, very high.

In the third country (for example, France) – high, middle, middle.

Description	Fuzzy number	Volume of investments, bill. $
Very high	(1,00; 0,2; 0,00)	2000
High	(0,75; 0,15; 0,15)	1500
Middle	(0,5; 0,15; 0,15)	1000
Low	(0,25; 0,15; 0,15)	500
Very low	(0,00; 0,00; 0,20)	0

Germany

$$a = 0,5*1,00 + 0,3* 0,75 + 0,2*0,5= 0,825$$

$$c_l = 0,825 – (0,4*0,8 +0,2*0,6 + 0,0*0,35) = 0,35$$

$$c_r = (0,6*1,00 + 0,4*0,9 + 0,3*0,65) – 0,825 = 0,33$$

England

$$a = 0,5*0,5 + 0,3*1 + 0,2*0,5 = 0,65$$

$$c_l = 0,65 – (0,4*0,35 +0,2*0,35+ 0,1*0,8) = 0,36$$

$$c_r = (0,6*0,65 + 0,4*0,65 + 0,3*1) – 0,65 = 0,3$$

France

$$a = 0,5*0,75 + 0,3* 0,5 + 0,2*0,5 = 0,625$$

$$c_l = 0,625 – (0,4*0,6 +0,2*0,35+0,1*0,35) = 0,28$$

$$c_r = (0,6*0,9 + 0,4*0,65+ 0,3*0,65) – 0,625 = 0,37$$

Thus volume of investments in Germany can be (1650; 700; 660) bill. $, in England - (1300; 720; 600) bill. $, in Frane - (1250; 560; 740). Common volume of investments is 4200 bill. $

Using fuzzy logic technique it is possible to find mathematical definition situation in real estate researches, investments and valuation.

Fuzzy numbers logics lets make dialog about real estate using traditional language of business world. This circumstance can be real contribution to making real estate computer communication.