Risk and return of German real estate stocks:

A simulation approach with geometric Brownian motion

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Agenda

1. Motivation
2. Hypotheses
3. Summary of key results
4. Discussion

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Research problem and aim of this study

- Listed real estate is a hybrid asset, influenced by real estate market developments, but also by general economic and stock market movements.
- There is a long and ongoing debate on what influence is dominant and how big the potential for portfolio diversification is.
- Furthermore, it remains unclear how capital allocation changes if alternative risk measures are used, especially simulation-based downside risk measures such as conditional value at risk (CVaR).

Aim: To investigate the contribution of real estate stocks to a mixed-asset portfolio in a simulation-based downside risk model of the German stock market.

The relevant literature in a nutshell

- **Risk and return of securitized real estate:** Most of the studies …
  - use data on (US-)REITs,
  - find that real estate stocks are hybrid assets that are different from both common stocks and direct real estate,
  - come to the conclusion that real estate stocks reduce risk in mixed-asset portfolios (in most market phases),
  - use volatility as the risk measure.
- **Risk measures:** Volatility is a controversial risk measure, but other risk measures too. Using alternative risk measures changes the optimal portfolio composition. There are pros and cons for the different methods to calculate VaR, e.g., historical simulation, Monte Carlo simulation, exponentially weighted moving average (EWMA) and Garch (generalized autoregressive conditional heteroskedasticity).
- **German real estate stocks:** Relatively small market compared to direct and other indirect forms of real estate investment. Overall, similar results in conventional studies, but only a few studies with downside risk measures.

Hypotheses
Conventional and new

(1) Listed real estate is largely driven by the same macroeconomic factors as unsecuritized real estate

→ real estate stocks can substitute direct real estate investments in a portfolio

(2) Real estate stock returns are not normally distributed

→ alternative risk measures should be used

(3) Real estate stocks are less risky than other stocks, when alternative risk measures are applied

→ real estate stocks can be beneficial for portfolio diversification

Main steps

(1) Sensitivity analysis (5-factor APT Model)

$$R_{LRE,t} = E(R_{LRE,t}) + b_1 R_{Stock,t} + b_2 R_{BOND,t} + b_3 R_{CPI,t} + b_4 R_{REM,t+1} + b_5 R_{GDP,t+1} + \in_t$$

$R_{LRE,t}$ = listed real estate market risk factor (applied proxy DIMAX Index),
$R_{Stock,t}$ = stock market risk factor (CDAX),
$R_{BOND,t}$ = bond market risk factor (REXP),
$R_{CPI,t}$ = inflation risk factor (Consumer Price Index),
$R_{REM,t+1}$ = real estate market risk factor (German Property Index) in period $t + 1$,
$R_{GDP,t+1}$ = economic risk factor (GDP growth) in period $t + 1$.

(2) Orthogonalization (due to multi-collinearity) and z-transformation

(3) Calculation of risk premia

(4) Risk calculation with alternative risk measures

Data

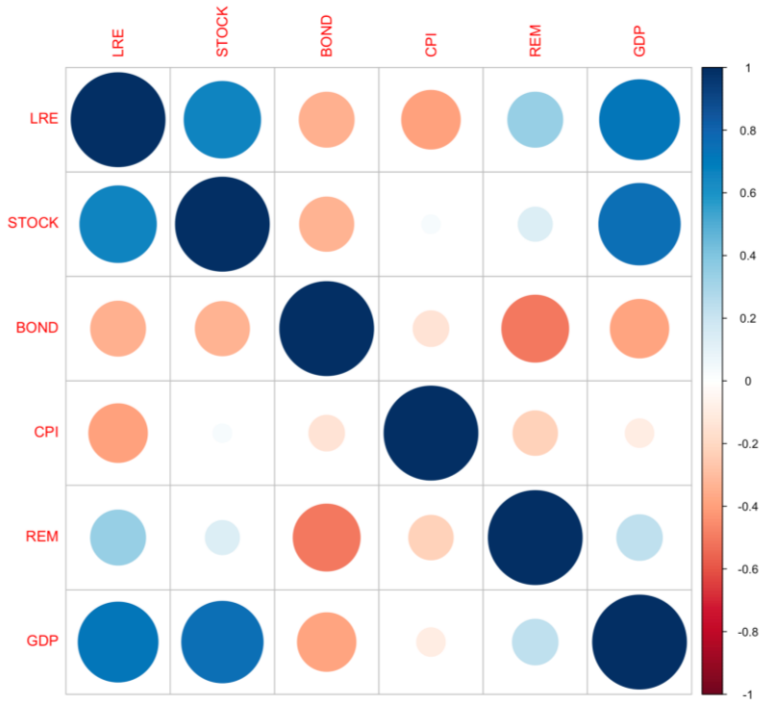
- German stock market
- 1995-2019, annual data
- Sources:
  - DIMAX by Ellwanger & Geiger
  - CDAX and REXP from Deutsche Bundesbank via Quandl.com
  - GPI from BulwienGesa
  - Risk-free rate and GDP from Deutsche Bundesbank
  - Consumer price index from Statistisches Bundesamt

Summary of key results
Descriptive statistics

Key figures

Variable	Obs.	Mean	Std. Dev.	Min.	Max.	Std. Error
LRE return	25	5.64%	0.26456	-68.70%	44.77%	0.05291
STOCK return	25	6.04%	0.23656	-55.48%	34.24%	0.04731
BOND return	25	4.55%	0.04274	-2.55%	15.44%	0.00855
Δ CPI	25	1.45%	0.00669	0.20%	3.12%	0.00134
REM return	25	8.62%	0.04530	0.04%	17.29%	0.00906
GDP return	25	1.37%	0.01938	-5.86%	4.09%	0.00388
Risk-Free Rate	25	3.51%	0.02098	0.22%	7.81%	0.00420

Correlation matrix



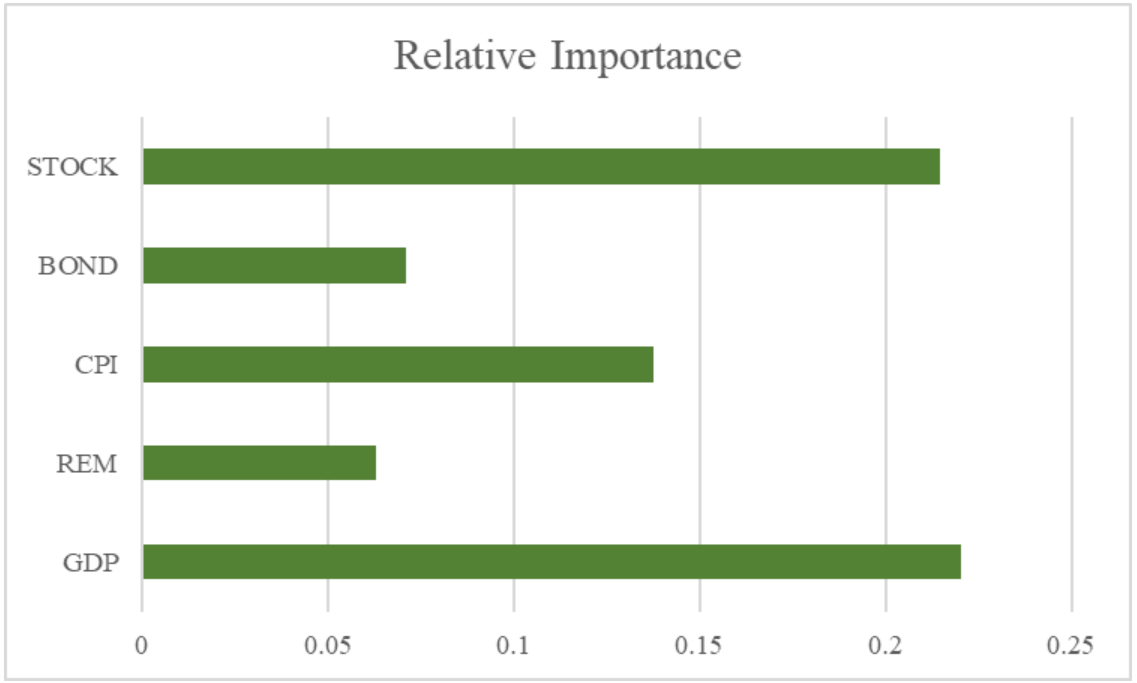
! As expected, listed real estate shows high correlation with common stocks, low correlation with the real estate market and negative correlation with bonds.

Summary of key results
Sensitivity analysis

After Orthogonalization

	Dependent variable:
	LRE
STOCKhat	1.773***
	(0.332)
BONDhat	−3.623**
	(1.314)
CPIhat	−18.690***
	(5.953)
REMhat	3.095**
	(1.121)
GDPhat	25.271***
	(4.341)
Constant	0.056*
	(0.032)
Observations	25
R^2	0.706
Adjusted R^2	0.629
Residual Std. Error	0.161 (df = 19)
F Statistic	9.137*** (df = 5; 19)
Note:	*p<0.1; **p<0.05; ***p<0.01

After Z-Transformation and R^2 decomposition



All risk factors are highly significant. Hence, real estate stock returns probably share the same underlying macroeconomic factors. Highest influence by GDP and the general stock market, lowest by the real estate market.

Summary of key results

Risk premium deduction

	Dependent variable:
	ADL Adler Real Estate AG
STOCKhat	3.710***
	(0.924)
BONDhat	−12.248***
	(3.661)
CPIhat	−33.945*
	(16.582)
REMhat	11.224***
	(3.122)
GDPhat	45.738***
	(12.093)
Constant	0.037
	(0.090)
Observations	25
R^2	0.608
Adjusted R^2	0.505
Residual Std. Error	0.449 (df = 19)
F Statistic	5.894*** (df = 5; 19)
Note:	*p<0.1; **p<0.05; ***p<0.01

	Dependent variable:
	DINK Dinkelacker AG
STOCKhat	0.045
	(0.326)
BONDhat	−0.609
	(1.291)
CPIhat	−3.667
	(5.846)
REMhat	2.148*
	(1.101)
GDPhat	0.186
	(4.264)
Constant	0.058*
	(0.032)
Observations	25
R^2	0.187
Adjusted R^2	−0.027
Residual Std. Error	0.158 (df = 19)
F Statistic	0.874 (df = 5; 19)
Note:	*p<0.1; **p<0.05; ***p<0.01

	Dependent variable:
	AGR Agrob Immobilien AG
STOCKhat	2.323***
	(0.510)
BONDhat	−6.465***
	(2.023)
CPIhat	−11.349
	(9.163)
REMhat	4.525**
	(1.725)
GDPhat	21.825***
	(6.683)
Constant	0.012
	(0.050)
Observations	25
R^2	0.600
Adjusted R^2	0.495
Residual Std. Error	0.248 (df = 19)
F Statistic	5.705*** (df = 5; 19)
Note:	*p<0.1; **p<0.05; ***p<0.01

	Dependent variable:
	HAS Hasen Immobilien AG
STOCKhat	−0.512
	(0.366)
BONDhat	−3.448**
	(1.449)
CPIhat	6.059
	(6.563)
REMhat	2.456*
	(1.236)
GDPhat	−1.783
	(4.787)
Constant	0.061
	(0.036)
Observations	25
R^2	0.401
Adjusted R^2	0.244
Residual Std. Error	0.178 (df = 19)
F Statistic	2.547* (df = 5; 19)
Note:	*p<0.1; **p<0.05; ***p<0.01

	Dependent variable:
	BAS Bastfaserkontor AG
STOCKhat	0.069
	(0.336)
BONDhat	−1.307
	(1.331)
CPIhat	0.403
	(6.029)
REMhat	2.902**
	(1.135)
GDPhat	7.512
	(4.397)
Constant	0.065*
	(0.033)
Observations	25
R^2	0.385
Adjusted R^2	0.223
Residual Std. Error	0.163 (df = 19)
F Statistic	2.376* (df = 5; 19)
Note:	*p<0.1; **p<0.05; ***p<0.01

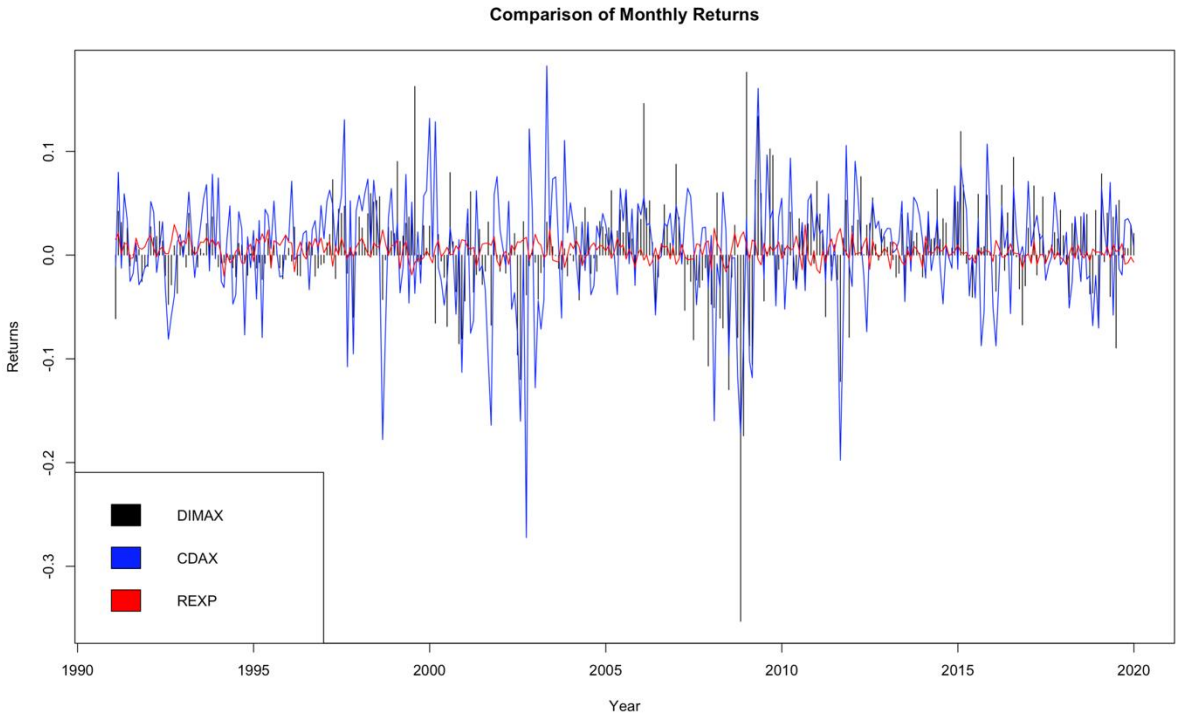
!

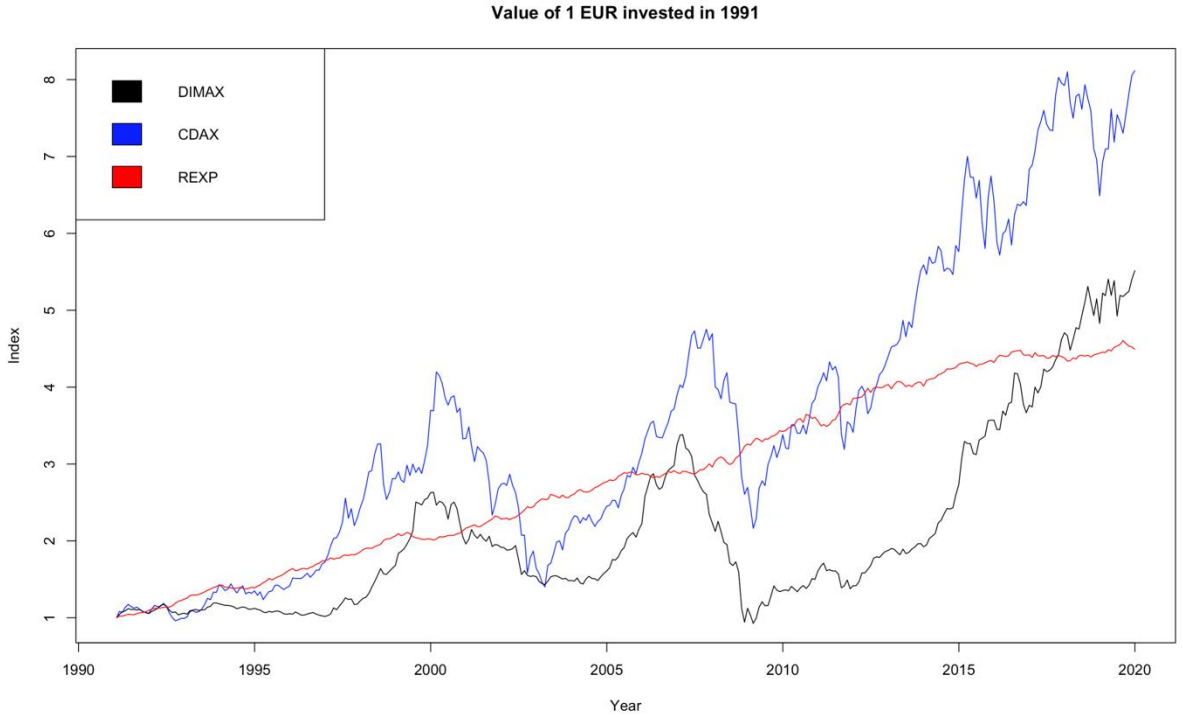
There are only five companies that have been trading since 1994 and thus can be used for the APT model. Adj. R^2 and F-statistics show that estimation of risk premia is doubtful. Not completely unexpected!

DEAD END

Summary of key results

Alternative risk measures





Real estate stocks and common stocks have followed similar return paths and have had similar volatility.
What does this say about the future and about tail risks? Not much!

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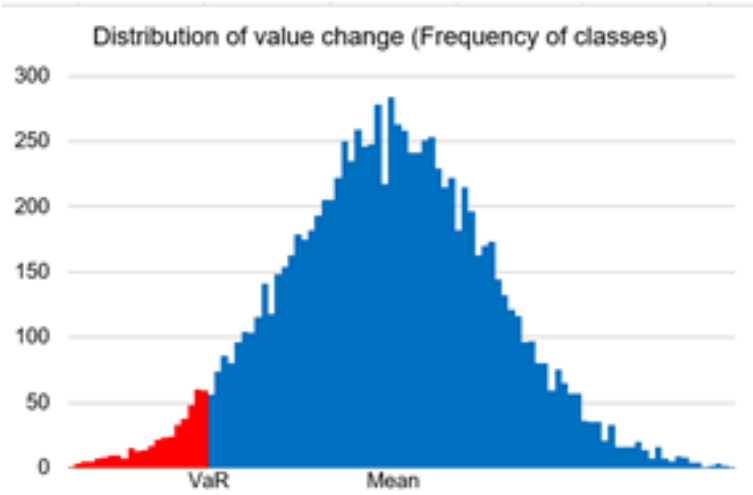
Monte Carlo simulation and Brownian motion

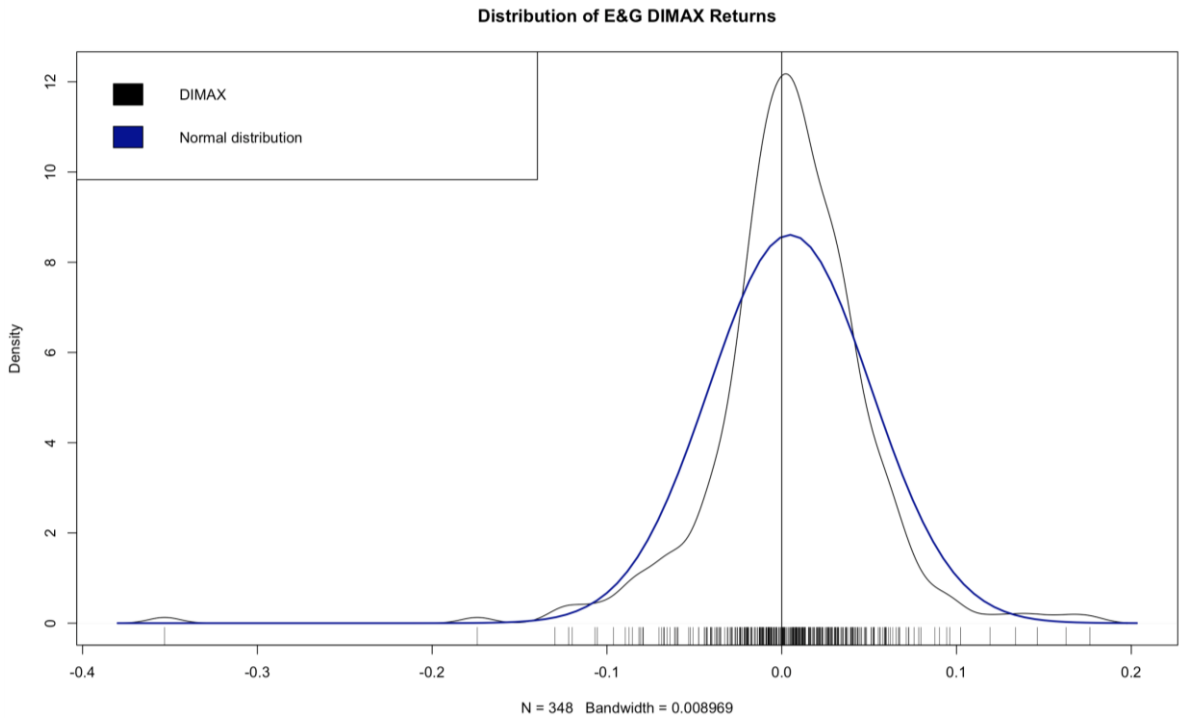
- Monte Carlo Simulation (MCS): a computer-based method that generates a large representative number of scenarios; the result is a frequency distribution, from which probabilities and risk measures such as VaR can be derived

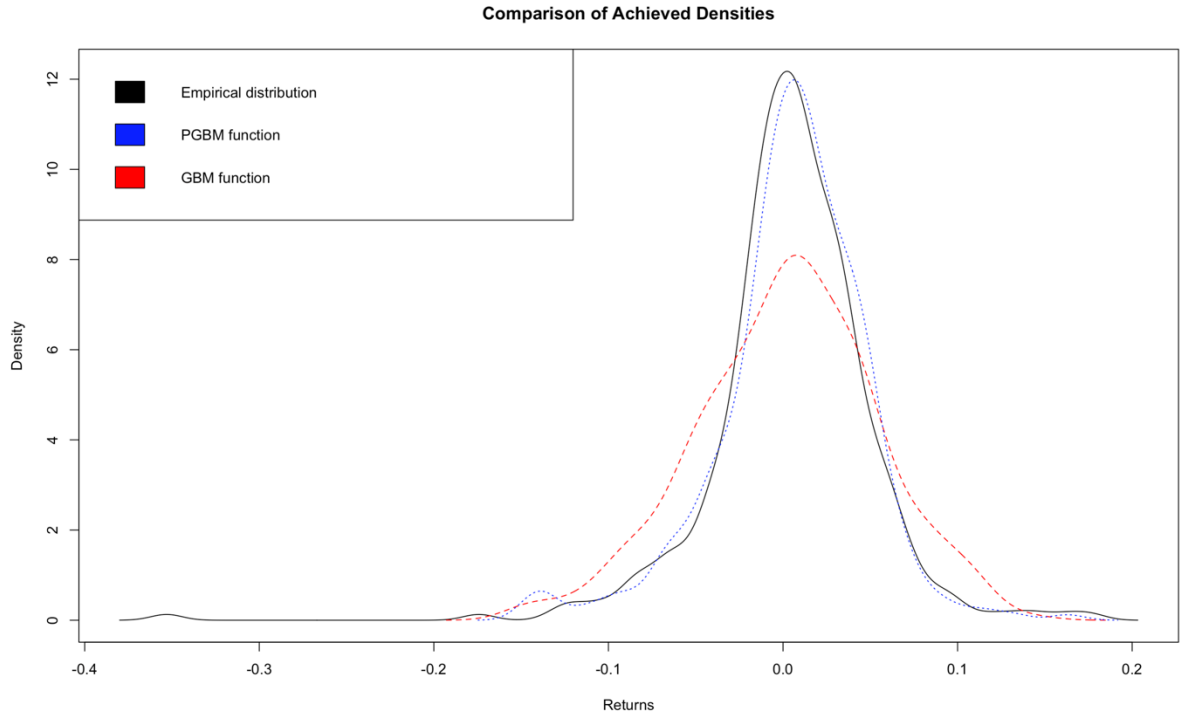


- Geometric Brownian Motion (GBM): stochastic procedure for forecasting, often used for financial instruments that entail path dependency; assumption: prices follow a random walk with “drift”
- Pseudo-Geometric Brownian Motion (PGBM): remedies GBM’s problematic condition of normal distribution by sampling the random walks from past empirical returns

Summary of key results

Non-normal distribution and GBM vs. PGBM

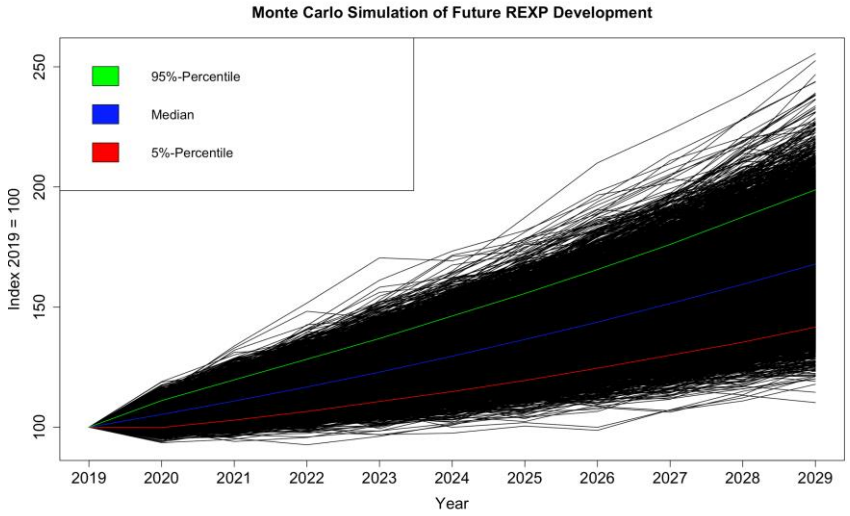


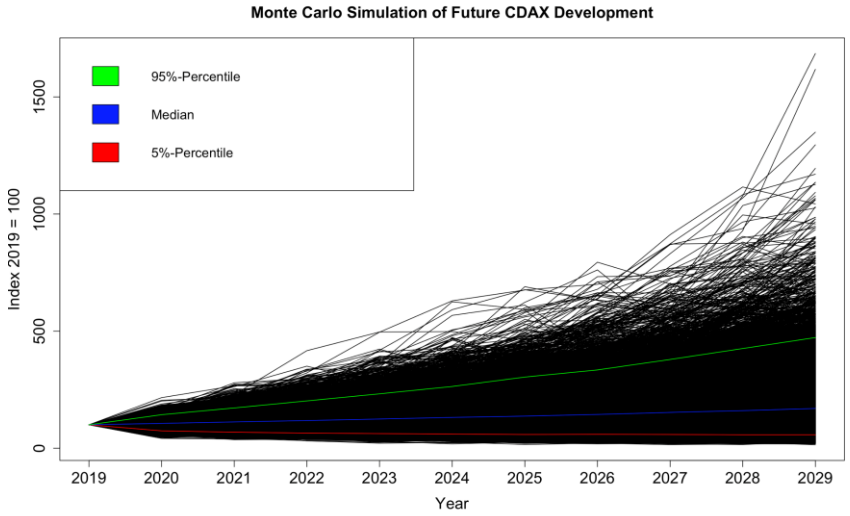


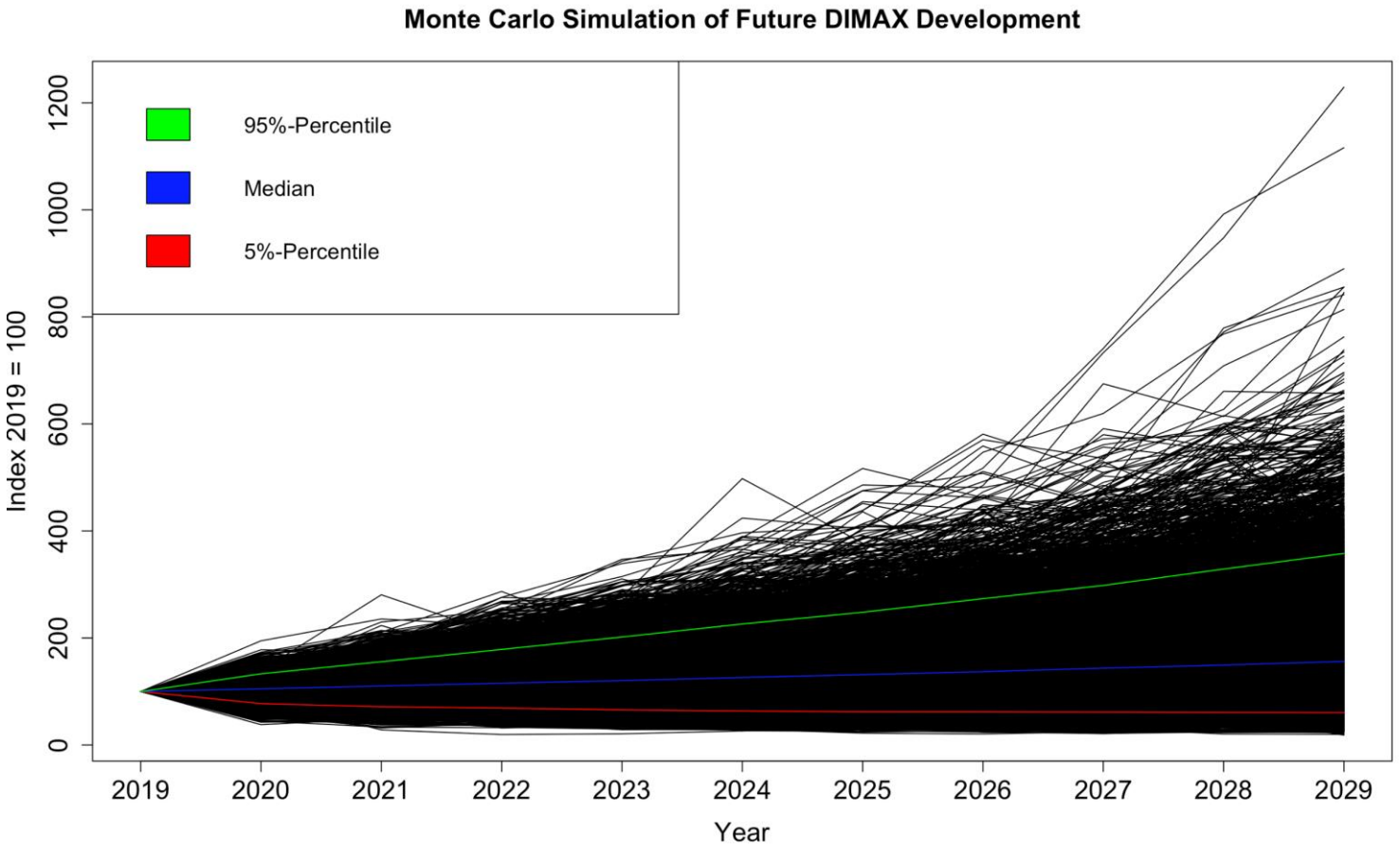
!

Listed real estate returns are skewed, leptokurtic and heavily tailed (visual impression and test results).
PGBM function renders better results.

Monte Carlo simulation of future index movements (10 years, 10,000 runs)

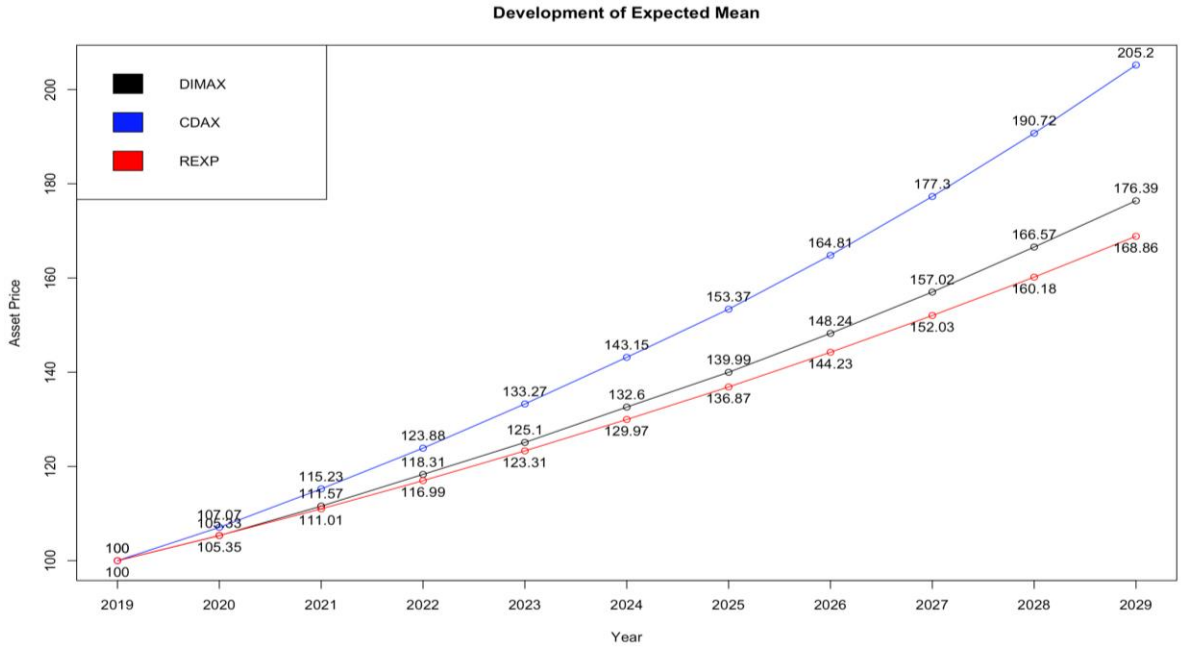


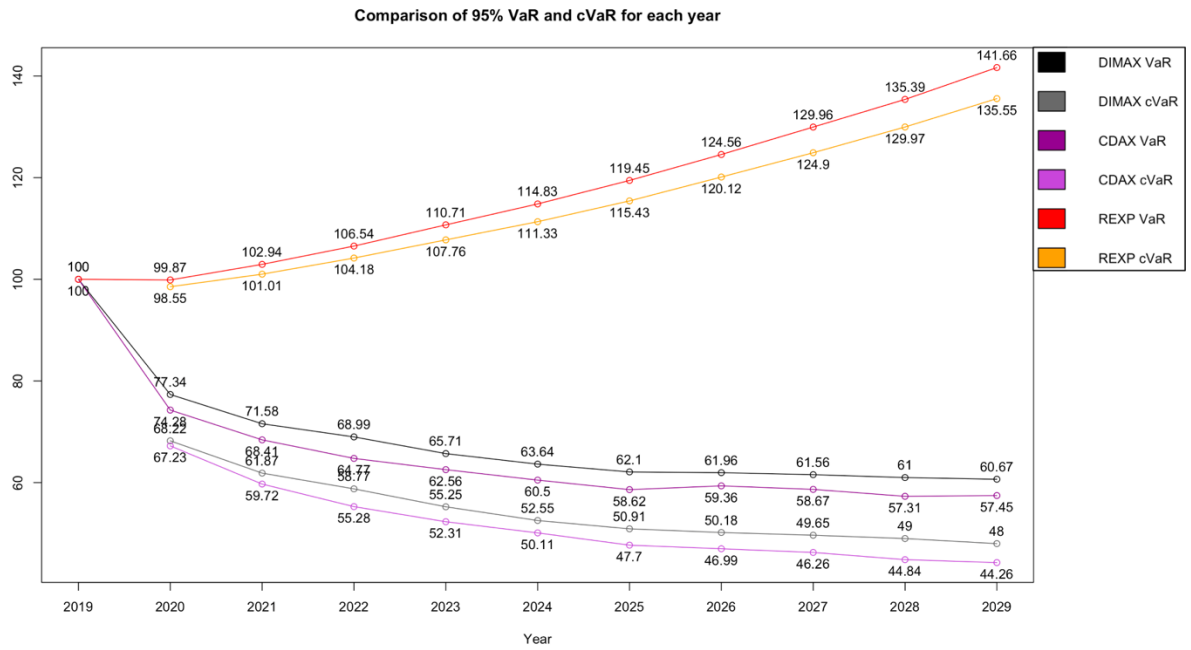




Summary of key results

Expected mean, VaR and CVaR for an investment of EUR 100

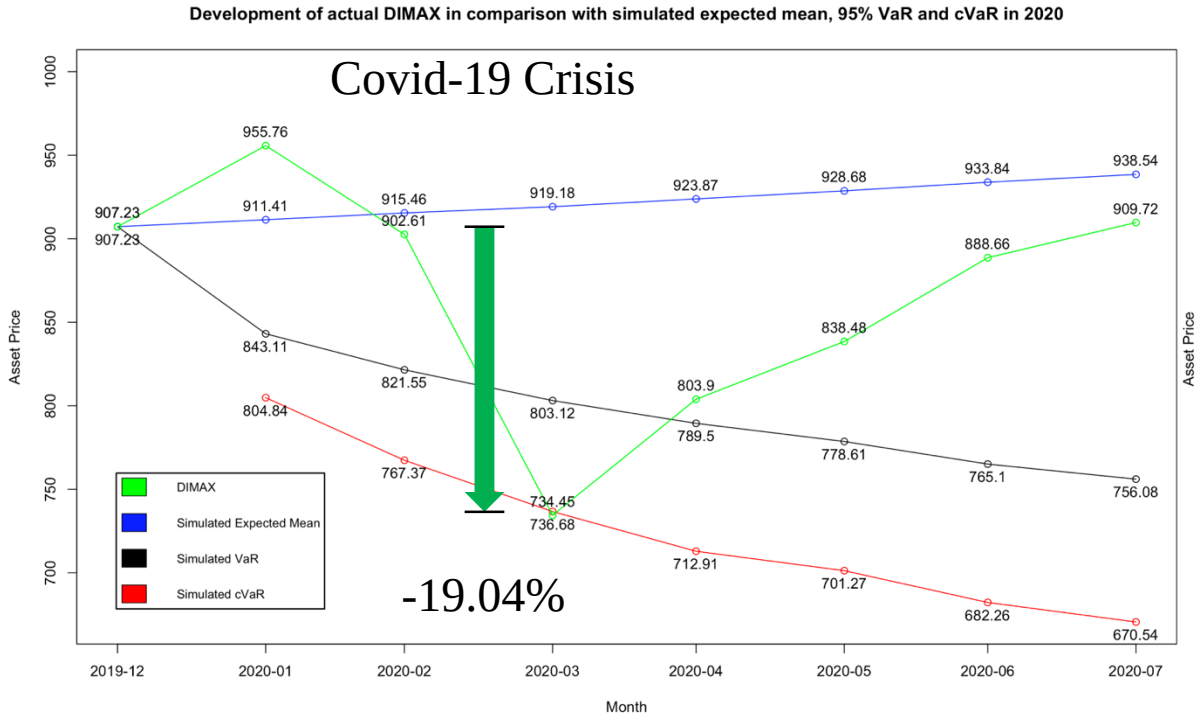


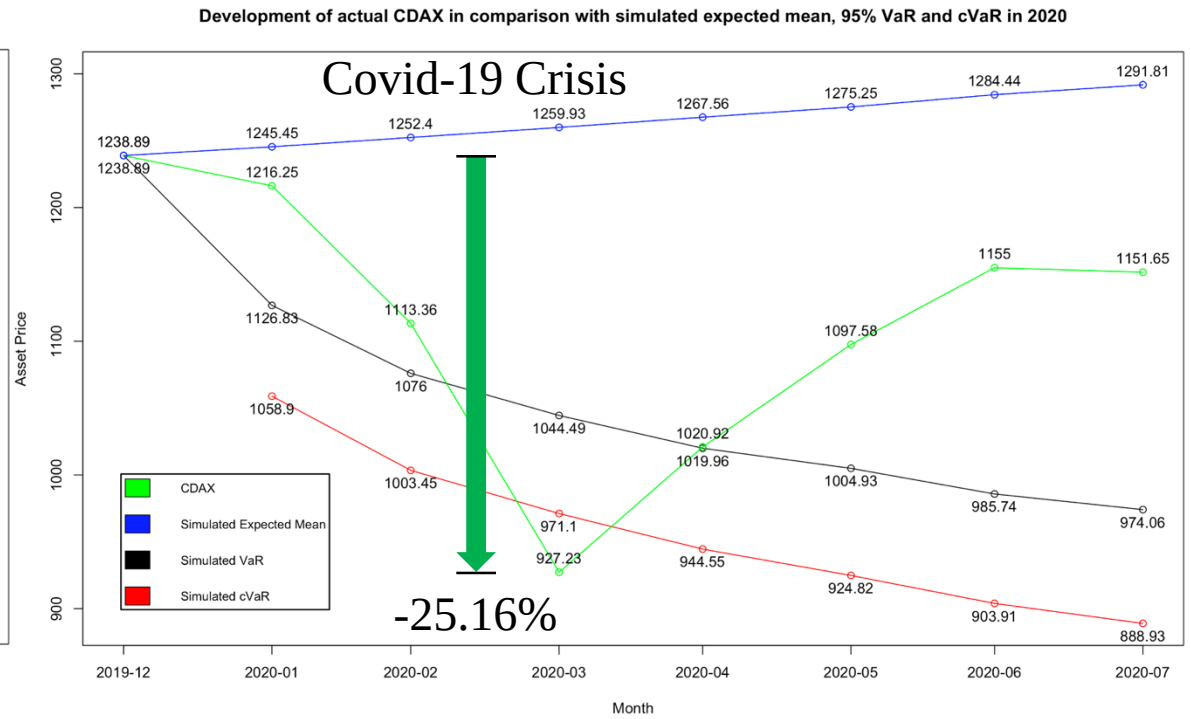


	Expected Mean (CAGR)	95%-VaR (Relative loss)	95%-CVaR (Relative loss)
DIMAX – *real estate stocks*	176.39 EUR (+5.84%)	60.67 EUR (-39.33%)	48.00 EUR (-52.00%)
CDAX – *common stocks*	205.20 EUR (+7.45%)	57.45 EUR (-42.55%)	44.26 EUR (-55.74%)
REXP – *bonds*	168.86 EUR (+5.38%)	141.66 EUR (+41.66%)	135.55 EUR (+35.55%)

! Loss indicated by the VaR is 3.22%-points higher for common stocks than for real estate stocks in Germany (3.74%-points for CVaR) → real estate stocks have less downside risk.

Validation of estimation with out-of-sample data





With new data from 2020 the model showed good applicability and estimation accuracy—even (and especially) in times of crisis.

Summary of main results

(1) German real estate stock returns are …

- significantly affected by stocks, bonds, inflation, real estate market and the overall economy,
- not very sensitive to movements in the real estate market.

(2) German real estate stock returns are …

- not normally distributed,
- better estimated with a PGBM model.

(3) German real estate stocks have …

- lower downside risk than other German stocks,
- a stabilizing influence of real estate stocks in a mixed-asset portfolio.

Limitations, practical implications and outlook

(1) Limitations:

- Very small market, which is not representative for real estate stocks. Some international studies come to adverse conclusions (e.g., Stelk et al. (2017) find that US REITs have higher downside risk during GFC).
- Problematic proxies: DIMAX (composition of index), GPI (data sources, annual data)

(2) Practical implications:

- Alternative risk measures should be applied to come to a more holistic risk view.
- Portfolio management can and should use simulation techniques and alternative risk measures.

(3) Outlook:

- Use simulation method for other data, i.e., international real estate stocks (REITs) and other kinds of indirect real estate investments (e.g., open-ended funds)
- Test other alternative risk measures, e.g., qualitative and heuristic risk measures
- Test other estimation methods instead of PGBM