Search for rent and the market for apartment swap

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Abstract

This paper develops a theoretical search model of apartment swap in the rental market. Using random matching mechanism, we mimic the informal Swedish swap market, which is characterized by strong rent control and dominant ownership of the apartments by municipalities. Our proposed framework captures supply and demand dynamics of the rental market, segmented into households with rented small or big municipal apartments, searching and trying to swap apartments between each other. In the basic setup, in isolation of rental market, the value of the swap is a function of the structure of the population, probabilities of match, probability of cancelling the swap agreement and the differential in the utility gains of matching swap counter-parties. When integrated with the classical rental market, the swap market reflects not only the supply/demand conditions in the swap market, but also the tightness in the rental market. Vice-versa, a well developed swap rental market affects the rental market, so with an increase in the probability of successful swap of a particular rental apartment, the apartment rent value increases.

Keywords: Apartment swap, housing market, rent control, mismatch costs, search & match model, low supply, Sweden

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1 Introduction

The Swedish rental market is specific in that municipalities (public sector) owns a significant part of the housing stock (apartments). Furthermore, the rent system is strongly regulated, both giving possibility to the occurrence of apartment swap by the renting households. Under the Swedish rent control system, rents should be at a level determined by the principle of value-of-usage. Also, private landlords when setting their rents, closely follow (set lower or equal to) the rents for identical apartments owned by a municipal housing company. Households are allowed to swap their municipal apartment they are currently renting, with another household, also renting a municipal apartment.

Municipal apartments are allocated on the basis of the criteria of waiting time, which implies that the applicant household with the longest waiting time is given priority over a household with shorter waiting time. Given the recent shortage of housing apartments and continuously high market prices for buying an apartment, queues for rental apartments can be many years[1], and rents for attractive apartments are far below the market clearing level. One consequence is that the black market for renting apartments is burgeoning (Ellingsen, 2003).

A renting household, which due to some exogenous reason, needs to move to another location (e.g., new job, marriage, having baby, divorce, etc), is facing the situation of a long waiting time for getting another apartment. This situation creates incentives to enter into a swap agreement with another household from the desired location.

This phenomenon has been analyzed and modelled by Eriksson (2007) using a game-theoretic approach for modelling the swap market. In doing so, the paper generalizes Quinzii (1984)'s model that describes both the house-swapping game, where each player owns a house when entering the market, wishing to leave the market with a better house if possible (Shapley and Scarf, 1974), and the permutation game, where the participants make deals at a negotiated money exchange (Tijs and Rajendra, 1984). More precisely, the paper develops a more general framework of circular exchange economies for which they prove a theorem of core non-emptiness.

Igarashi (1991) develops a theoretical model with a Diamond-type random-matching mechanism, focusing on the rental housing market, while Wheaton (1990) looks at the owner-occupied housing market. Kashiwagi (2014) presents a simple model of the rental and home-ownership markets in a unified framework.

The focus of this paper is to model the rental swap market by using search models and applying matching theory instead of a strategic game. We build on previous work on price dispersion, and utilize an asymmetric information approach in the search model

[1] In Stockholm, the waiting list for an apartment is above 15 years (ref).

(Kohn and Shavell 1974; Rothschild 1974; Butters (1978); Burdett and Judd 1983). Stull (1978) and Yinger (1981) have developed the foundations of this search model by describing how uncertainty and search costs can influence the behavior of various market participants. The paper by Rosen and Smith (1983) empirically tests the importance of housing vacancies for the housing price using random matching models.

The dynamic structure of random-matching models allows to incorporate the traditional distinction between the short and the long run into a search-based model. In the short run, the total housing stock is fixed, while in the long run, supply adjusts so that profits are dissipated. The long-run vacancy rate can be interpreted as the natural vacancy rate, and the short-run vacancy rate as a deviation around the natural rate caused by slow supply adjustment in response to some shock (Igarashi, 1991).

Obviously, developments in the swap market are closely connected with developments in the rental market. Therefore, we extend our analysis to include the interdependence between classical rental market and swap rental market, because the main good that is traded in the swap market is the rental agreement. What is specific in the interplay between rental and swap market is that part of the supply of the free rental apartments which originates from the people currently renting them, which could be being diverted to the swap market. This supply stems from the perceived risk of unsuccessful transfer from one to another rental apartment in a reasonable short time. High risk aversion, especially in connection with a tight rental market, is the main reason that current renters choose to use the swap rental market instead of the classical rental market. On the other hand, part of the demand for the apartments in the rental market, that originates from people already renting apartments, is mirrored in the swap market. Therefore our model captures that conditions in the rental market affect conditions in the swap market. Frictions in rental market will affect the swap market with potentially adverse effects on the rental market itself.

Also, we maintain that a well developed swap rental market affects the rental market, through shifting and channeling some of the rental demand/supply to the swap market. Existence of the swap market enables the renters to further utilize the rental agreement, improving their chances to rent apartment according to their needs, beside searching in the classical rental market. This "option" acquired when entering rental agreement, gives them the right to participate in the swap market. Hence, it is interesting to see whether awareness of this possibility, to offer/demand apartments in the swap market, capitalize in the rental prices and increases them.

The purpose of this paper is to capture drivers and dynamics of the apartment swap market, borrowing from the housing/rental literature mentioned before. We build a theoretical model for swap rental market and have integrate it with the model of classical rental market. The model enables us to derive some stylized facts using some comparative statics. More specifically, the value of the swap is modelled as a function of the structure of the population, in order to test the changes in the modelled value of the swap, while varying for the parameters of interest rate, probabilities of match and probability of cancel of the swap agreement. The simulation results confirm the intuitive expectations from the theoretical postulation.

The existence of the Swedish swap market is unique which provides a main motivation for our research. While the existence of the swap market also attracts government attention with the aim of introducing regulatory framework, the significance of the market highlights its role as an important hedging vehicle against the risk of finding housing. Hence, there is need to understand the mechanisms of the swap market, which this paper aims to shed light on. To the best of our knowledge this is the first paper that explains the existence of apartment swap transactions using a theoretical search & match framework.

The remainder of this paper is as follows. Section 2 develops our theoretical framework of searching a new apartment and the implications for the swap market. Section 3 investigates the interaction between the swap and the rental market. In section 4 we present the conclusions.

2 Theoretical framework of search and the swap market

Swedish rental market is strongly regulated, where the rent is determined mostly by two main factors: size and age of the apartment, regardless of other characteristics (e.g., quality of the apartment, clean and quiet environment, proximity to center and other amenities, etc).

Facing long waiting time for getting a new apartment, a leitmotif for apartment switch could be inappropriate size of the apartment, which is measured in our model as a mismatch cost (for example, discomfort of living in small apartment with the big family or excessive rent obligation for singles living in big apartment).[2]

Also, we exempt from the situation where people move because of the new job. This extension could be subject of further research and could be related to the labour market conditions, but is not covered in this paper. Hence, for the purpose of studying the Swedish swap market, we assume that rental market is segmented in 2 populations: tenants with small apartments and tenants with big apartments.

[2]It would be very rare case to swap apartment because of the apartment age, as Swedish rental apartments are quite well maintained.

2.1 Model of the swap market

Tenants want to swap their apartment in the occurrence of some exogenous shock m. They would like to switch from small to big apartment for example, due to marriage or due to having child, which induces their mismatch cost x_i to increase. Similarly, tenants would like to switch from big to small apartment for example, due to divorce or because kids have grown up, making apartment inappropriate and increasing their mismatch cost.

In the model, mismatch cost is denoted with x_a for the tenant population A with small apartments and with x_b for the tenant population B with big apartments. Both mismatch costs x_a and x_b are constrained to be between "1" (highest mismatch cost, which makes tenant to wish to swap) and "0" (lowest mismatch cost, experienced by the tenant when moved to the swapped apartment).

The part of the tenant population A with small apartments, who have already swapped, is denoted with S_a, while the part of tenant population A who have not swapped so far or did not have need to swap, is denoted with R_a. Symmetrically, part of the tenant population B with small apartments, who have swapped, is denoted with S_b, while those who do not swap is denoted with R_b. All parts are considered as percentages within their respective population:

$$S_a + R_a = 1 \tag{1}$$

$$S_b + R_b = 1 \tag{2}$$

Each type of household either try to rent the apartment directly from the landlord (municipality) or to rent using the swap market. Tenants, after experiencing exogenous shock m and high mismatch cost $x_i, i \in (A, B)$, are searching for the apartments to swap. Without loss of generality we assume $m = 1$, which mean that households are very determined to find another apartment.

Probability of finding of swapped apartment is dependent on the search & matching frictions. Matching function I have assumed is Cobb-Douglass with constant returns, as in classical housing literature:

$$M(R_a, R_b) = k(R_a)^{1-\theta}(R_b)^{\theta} \tag{3}$$

Search technology is as in Diamond (1984), where these opportunities-to-match are result of the Poisson process.

Researchers from distinct population have distinct Poisson aggregate rate: $\alpha = M(R_a, R_b)/R_a$, for the searcher from population A and $\beta = M(R_a, R_b)/R_b$ for the searcher from population B.

Search is more productive for searcher A the larger the number of tenants willing to swap R_b from other population B and the smaller the number of searchers R_a from population A.

Vice-versa, search is more productive for searcher B the larger the number of tenants willing to swap R_a from other population A and the smaller the number of searchers R_b from its own population B.

One can note that it assumed that there is no swapping within population A (small for small apartment) or within population B (big for big apartment). We also ignore the possible fluctuations of aggregate variables that can be generated by Poisson processes, considering only the stationary state as in Diamond (1984).

Under these assumptions, there exists a unique stationary-state equilibrium with equilibrium level of swap premium s^* which should equate gains and loses from the swap agreement, having in consider mismatch cost differential x_a - x_b and equilibrium rent, p^*.

In general, small apartment tenant should pay positive swap premium to big apartment tenant, since larger apartment means higher rent and if gain in reducing mismatch cost is higher for the smaller apartment. Also, as mentioned, swap should reflect market friction conditions.

In the stationary state, the number of new matches (new swap agreements) that are made are equal to the number of tenants that exit (that cancel swap agreement) the swap market .

The number of swapped (matched) apartments from population A changes according to:

$$\dot{S}_a = M(R_a, R_b) - bS_a \tag{4}$$

In steady state:

$$0 = M(R_a, R_b) - bS_a$$

$$M(R_a, R_b) = bS_a \tag{5}$$

The number of swapped (matched) apartments from population B changes according to:

$$\dot{S}_b = M(R_a, R_b) - bS_b \tag{6}$$

In steady state:

$$0 = M(R_a, R_b) - bS_b$$

$$M(R_a, R_b) = bS_b \tag{7}$$

Consider the tenant of small apartment who is paying rent p and have mismatch cost of x_a. With probability α the renter get the new home (swap) or with probability $1-\alpha$ does not succeed and stays in the same apartment. Then Vr_a is the value of being a renter with small apartment, satisfying the following Bellman equation with interest rate r:

$$Vr_a r = x_a + p + \alpha\left(Vs_a - Vr_a\right) \tag{8}$$

Next, consider the tenant currently residing in swapped small apartment. Now he is paying rent p and additional swap value s, but have decreased mismatch cost of x_a to zero. Depending of the market conditions in the swap market, value of the swap can be positive $s > 0$ or negative $s < 0$. When s is positive, renter A pays $P + \Delta_P - S$, while renter B pays $P + S$. When s is negative, renter A pays $P + \Delta_P + S$, while renter B pays $P - S$. With probability b the swap is cancelled and tenant returns to its original apartment or with probability $1 - b$ stays in the swapped apartment. Let Vs_a be the value of being a swapping tenant from small to big apartment, satisfying the following Bellman equation:

$$Vs_a r = -s + p + \Delta_p + \left(Vr_a - Vs_a\right) b \tag{9}$$

On the other side of the market, consider the tenant of big apartment who is paying rent $p + s$ and have mismatch cost of x_b. With probability β the renter takes the new home (swap) or with probability $1 - \beta$ does not succeed and stays in the same big apartment. Then Vr_b is the value of being a renter with big apartment, satisfying the following Bellman equation:

$$Vr_b r = x_b + p + \Delta_p \beta\left(Vs_b - Vr_b\right) \tag{10}$$

Finally, consider the tenant moved in the swapped big apartment. Now he is paying rent p, but have decreased mismatch cost of x_b to zero. With probability b the swap is cancelled and tenant returns to its original apartment or with probability $1 - b$ stays in the swapped apartment. Let Vs_b be the value of being a swapping tenant from big to small apartment, satisfying the following Bellman equation:

$$Vs_b r = s + p + \left(Vr_b - Vs_b\right) b \tag{11}$$

When searching tenant A and B meet, swap searching tenant A accepts any swap such that $Vs_a \geqslant Vr_a$, while swap searching tenant B accepts any swap such that $Vs_b \geqslant Vr_b$. Bargaining determines the distribution of the surplus, but in this paper as in other housing literature I assume that surplus is evenly divided.

$$Vs_a - Vr_a = Vs_b - Vr_b \tag{12}$$

From the definitions in equations (8) and (9), one calculates renter's A surplus:

$$Vs_a - Vr_a = \frac{\Delta_p - x_a - s}{r + b + \alpha} \tag{13}$$

From the definitions in equations (10) and (11), renter's B surplus is given as:

$$Vs_b - Vr_b = \frac{-\Delta_p - x_b + s}{r + b + \beta} \tag{14}$$

Then, if we equalize both renter's A and renter's B surplus:

$$\frac{\Delta_p - x_a - s}{r + b + \alpha} = \frac{-\Delta_p - x_b + s}{r + b + \beta} \tag{15}$$

we can solve for equilibrium swap value:

$$s^* = \frac{(r + b + \alpha)\ x_b - (+r + b + \beta)\ x_a + \Delta_p\ (2r + 2b + \beta + \alpha)}{2r + 2b + \beta + \alpha} \tag{16}$$

One can check for the simplifying assumptions:
in the case of zero interest rate environment: $r = 0$

$$s = \frac{(b + \alpha)\ x_b - (b + \beta)\ x_a + \Delta_p\ (2b + \beta + \alpha)}{2b + \beta + \alpha} \tag{17}$$

in the case of symmetrical reduction in mismatch costs: $x_a = 1, x_b = 1$

$$s = \frac{\Delta_p\ (2r + 2b + \beta + \alpha) + \alpha - \beta}{2r + 2b + \beta + \alpha} \tag{18}$$

in the case of evenly distributed surplus: $\alpha = \beta$

$$s = \frac{x_b - x_a + 2\Delta_p}{2} \tag{19}$$

if $x_a = 1, x_b = 1, \alpha = \beta$

$$s = 0 \tag{20}$$

Next, in order to make α and β endogenous, we express in the terms of percentage of matched (swapped) population A (derivations are given in Appendix B):

$$\alpha\ (S_a) := \frac{M(R_a, R_b)}{R_a} \tag{21}$$

$$\alpha(S_a) = \frac{S_a b}{1 - S_a} \tag{22}$$

As we can see, $\alpha(S_a)$ is increasing in S_a.

$$\beta\left(S_a\right) = \frac{M(R_a, R_b)}{R_b} \tag{23}$$

$$\beta\left(S_a\right) = \left(\frac{1 - S_a}{S_a b}\right)^{\frac{1}{\lambda}-1} \tag{24}$$

Furthermore, using endogenous α and β we can express value of the swap as the function of the percentage of swapping households S_a:

$$s(S_a) = \frac{\left(r + b + \alpha\left(S_a\right)\right) x_b - \left(+r + b + \beta\left(S_a\right)\right) x_a + \Delta_p \left(2r + 2b + \beta\left(S_a\right) + \alpha\left(S_a\right)\right)}{2r + 2b + \beta\left(S_a\right) + \alpha\left(S_a\right)} \tag{25}$$

Next, we express α, β ans s in the terms of percentage of matched (swapped) population B (derivations are given in Appendix C):

$$\alpha\left(S_b\right) = \frac{M(R_a, R_r)}{R_b}, \quad \beta\left(S_b\right) = \frac{M(R_a, R_b)}{R_a} \tag{26}$$

$$\alpha(S_b) = \frac{S_b b}{1 - S_b}, \quad \beta\left(S_b\right) = \left(\frac{1 - S_b}{S_b b}\right)^{\frac{\lambda}{1-\lambda}} \tag{27}$$

$$s(S_b) = \frac{\left(r + b + \alpha\left(S_b\right)\right) x_b - \left(+r + b + \beta\left(S_b\right)\right) x_a + \Delta_p \left(2r + 2b + \beta\left(S_b\right) + \alpha\left(S_b\right)\right)}{2r + 2b + \beta\left(S_b\right) + \alpha\left(S_b\right)} \tag{28}$$

2.2 Simulation results

For the purpose of testing of the model, we have made several simulations using derived formulas in the previous section 2.1. Firstly, we simulated the value of the swap s taking the other parameters (α, β, b, r) as exogenous. Secondly, we simulated the value of the swap s as the function of the flows in swap population, now taking the parameters (α, β) as the functions of flows of populations (S_a, S_b).

2.2.1 Swap and exogenous parameters

Apartment swapping rates α and β are denoting probability (between 0 and 1) of households swapping their rented apartments for two distinct populations (A,B).

Probabilities (rates) α and β are directly comparable and relation between them reflects demand / supply condition in the swap market. Note that results in Table 2.2.1 are as expected. Assuming the same reduction in mismatch costs ($x_a = x_b = 1$), when $\alpha > \beta$, value of the swap become lower: $s < 0$.

If $\alpha > \beta$, then there is lower percentage of renters within population A, compared to higher percentage of renters within population B, which makes easier for somebody in population A to match (swap) with somebody in population B. Easier match by A, that is higher demand by B, results in lower swap to pay by A.

And vice-versa, when $\alpha < \beta$ value of the swap is increasing: $s > 0$. Lower α then β means decrease in the percentage of renter population in A against B and higher demand by A, making easier match by B and resulting in higher swap to pay by A.

Table 1: Simulation of s wrt to α, β and b

	alpha/beta	b=0.10	b=0.20	b=0.30	b=0.40
1	alpha= 0.1 beta = 0	0.331	0.199	0.143	0.111
2	alpha= 0.1 beta = 0.05	0.142	0.091	0.067	0.053
3	alpha= 0.1 beta = 0.1	0.000	0.000	0.000	0.000
4	alpha= 0.1 beta = 0.15	-0.111	-0.077	-0.059	-0.048
5	alpha= 0.1 beta = 0.2	-0.199	-0.143	-0.111	-0.091
6	alpha= 0.3 beta = 0.2	0.143	0.111	0.091	0.077
7	alpha= 0.3 beta = 0.25	0.067	0.053	0.043	0.037
8	alpha= 0.3 beta = 0.3	0.000	0.000	0.000	0.000
9	alpha= 0.3 beta = 0.35	-0.059	-0.048	-0.040	-0.034
10	alpha= 0.3 beta = 0.4	-0.111	-0.091	-0.077	-0.067
11	alpha= 0.5 beta = 0.4	0.091	0.077	0.067	0.059
12	alpha= 0.5 beta = 0.45	0.043	0.037	0.032	0.029
13	alpha= 0.5 beta = 0.5	0.000	0.000	0.000	0.000
14	alpha= 0.5 beta = 0.55	-0.040	-0.034	-0.030	-0.027
15	alpha= 0.5 beta = 0.6	-0.077	-0.067	-0.059	-0.053
16	alpha= 0.7 beta = 0.6	0.067	0.059	0.053	0.048
17	alpha= 0.7 beta = 0.65	0.032	0.029	0.026	0.023
18	alpha= 0.7 beta = 0.7	0.000	0.000	0.000	0.000
19	alpha= 0.7 beta = 0.75	-0.030	-0.027	-0.024	-0.022
20	alpha= 0.7 beta = 0.8	-0.059	-0.053	-0.048	-0.043
21	alpha= 0.9 beta = 0.8	0.053	0.048	0.043	0.040
22	alpha= 0.9 beta = 0.85	0.026	0.023	0.021	0.020
23	alpha= 0.9 beta = 0.9	0.000	0.000	0.000	0.000
24	alpha= 0.9 beta = 0.95	-0.024	-0.022	-0.020	-0.019
25	alpha= 0.9 beta = 1	-0.048	-0.043	-0.040	-0.037

When $\alpha = \beta$ and assuming the same reduction in mismatch costs ($x_a = x_b = 1$), swap value is zero.

Simulation results also show that for the same difference between α and β, with the increase in the value of both parameters, the effects on the swap are becoming smaller in magnitude. The swap range for fixed $b = 0.10$, when α is increasing from 0.1 to 0.9, is decreasing from 0.53 (= 0.331 -(-0.199)) to 0.10 (= 0.053 -(-0.048)). These might mean that in the swap market with bigger turnover, supply/demand differential have lower effect on the value of the swap.

Also, one can notice that with higher b (probability for exit from swap agreement), swap values are decreasing in magnitude (in absolute value).

The range of the swap values (given low $\alpha = 0.1$, increasing beta), when b is increasing from 0.10 to 0.40, is decreasing from 0.53 (= 0.331 -(-0.199)) to 0.10 (= 0.053 -(-0.048)). These might be interpreted that high counter-party risk, makes supply/demand differential have lower effect on the value of the swap. The range of the swap values (given high $\alpha = 0.9$, increasing beta), when b is increasing from 0.10 to 0.40, is decreasing from 0.10 (= 0.053 -(-0.048)), to 0.07 (= 0.040 -(-0.037)). It seems that in the swap market with high turnover, counter-party risk have lower effect.

2.2.2 Swap and endogenous parameters

In this section probabilities of swap for particular populations (α for A and β for B) are made dependent on the percentage of swapping households within distinct population.

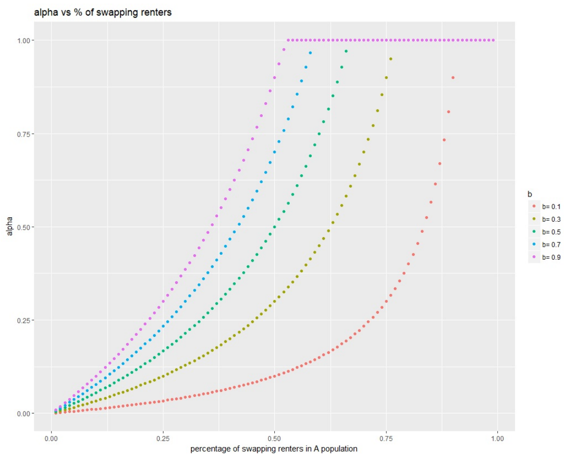

Figure 1: Alpha as a function of the % of swap households in A population

In the simulation are used already derived formulas for $\alpha(S_a)$ and $\beta(S_a)$ from part 2.1.

As expected, value of probability $\alpha(S_a)$ increase in S_a, not depending on λ (bargaining parameter), as shown in Figure 1. One can notice that after number of swapping households rise above 50%, possibility of finding swap counter-party is highly probable, as the number of renting households in population A is decreased for the same number of matched swaps.

Similarly, for the swapping households in B population, value of probability $\alpha(S_b)$ increase in S_b, not depending on λ (bargaining parameter).

(a) $\beta(S_a)$, λ=0.3 (b) $\beta(S_a)$, λ=0.7

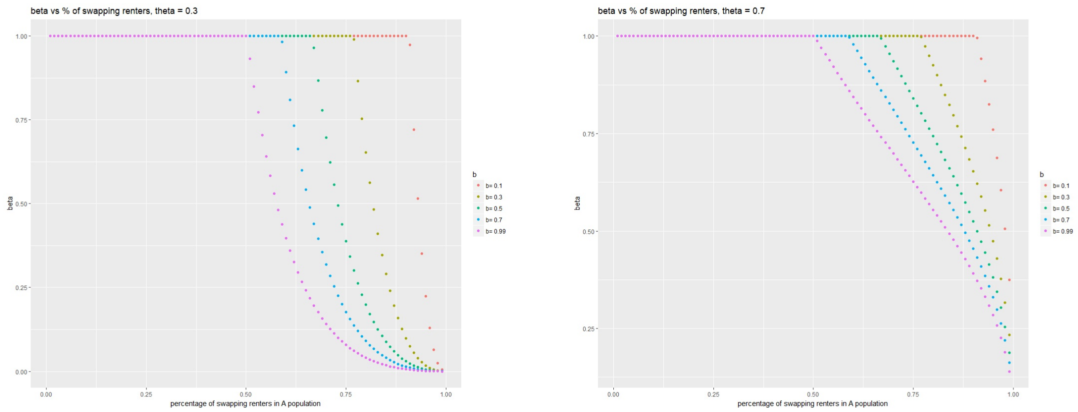

Figure 2: Beta vs % of swap households in A population

On the other hand, value of probability $\beta(S_a)$ is decreasing in S_a, depending on λ. After population of swapping households decrease below 50%, possibility of finding swap counter-party for B is almost sure, as the number of renting households in the other population B is increased (higher population, higher probability) for the same number of matched swaps.

(a) $\beta(S_b)$, λ=0.3 (b) $\beta(S_b)$, λ=0.7

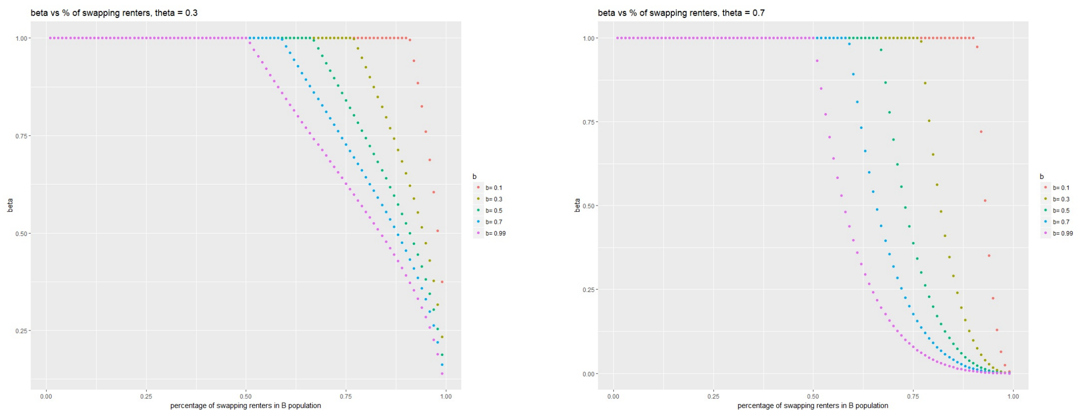

Figure 3: Beta vs % of swap households in B population

If one compares Figures 2 and 3, function of $\beta(S_b)$ is decreasing in S_b, but is inverse of the function $\beta(S_a)$ in parameter λ.

Next, I simulate the value of swap s as the function of structure of the population A and B (swapping vs renting population), denoted as $s(S_a)$ and $s(S_b)$ respectively.

As shown in Figure 4 the value of swap $s(S_a)$ is increasing in S_a as the total effect.

One can notice that increase in the probability of canceling the swap (b) is making value of the swap to be more elastic to the changes in the percentage of swapping households in population A. When b is very high, swap value s changes one for one with the

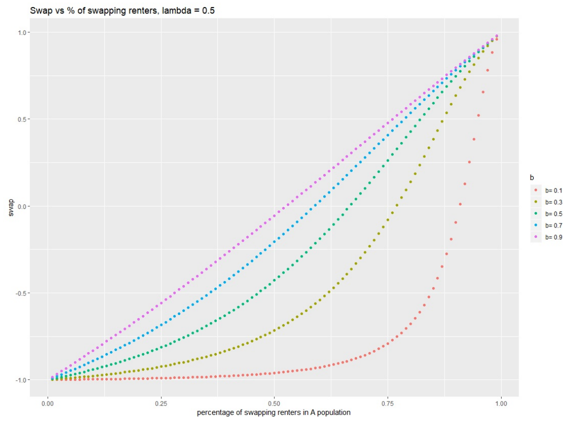

Figure 4: Swap s as a function of the % of swap households in A population

increase of the S_a (becoming becoming linear function of S_a).

For low levels of θ, increase in b is decreasing the cut-off point in S_a, after which swap value start to increase, as shown in sub-figure 9a.

For high levels of θ, increase in b is increasing the starting point (intercept) of the swap as the function of S_a, as shown in sub-figure 9b.

(a) $s(S_a)$, λ=0.1 (b) $s(S_a)$, λ=0.9

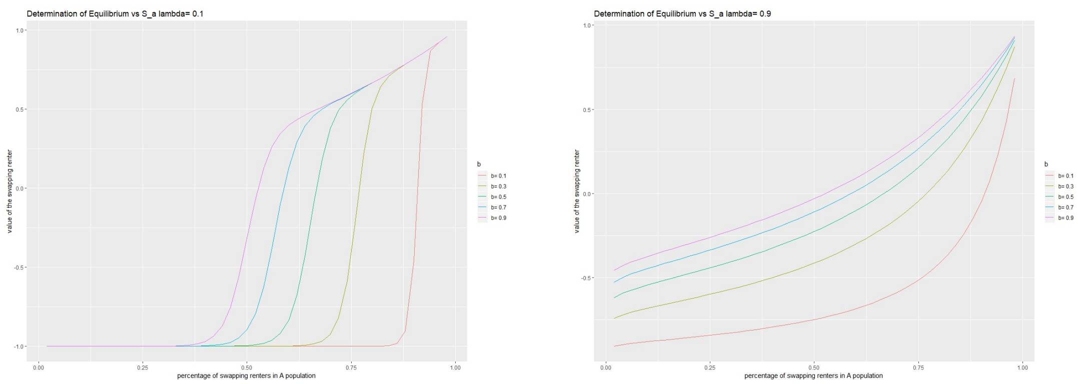

Figure 5: Swap value as a function of the % of swap households in A population

When one considers population B, the value of swap $s(S_b)$ is also increasing in S_b as the total effect.

Also, increase in the probability of canceling the swap (b) is making value of the swap to be more elastic to the changes in the percentage of swapping households in population B.

Conversely, parameter λ have inverse effects considering population B.

For low levels of λ, increase in b is increasing the starting point (intercept) of the swap as the function of S_a, as shown in sub-figure 6a.

For high levels of λ, increase in b is decreasing the cut-off point in S_a, after which swap value start to decrease, as shown in sub-figure 6b.

(a) $s(S_b)$, λ=0.1 (b) $s(S_b)$, λ=0.9

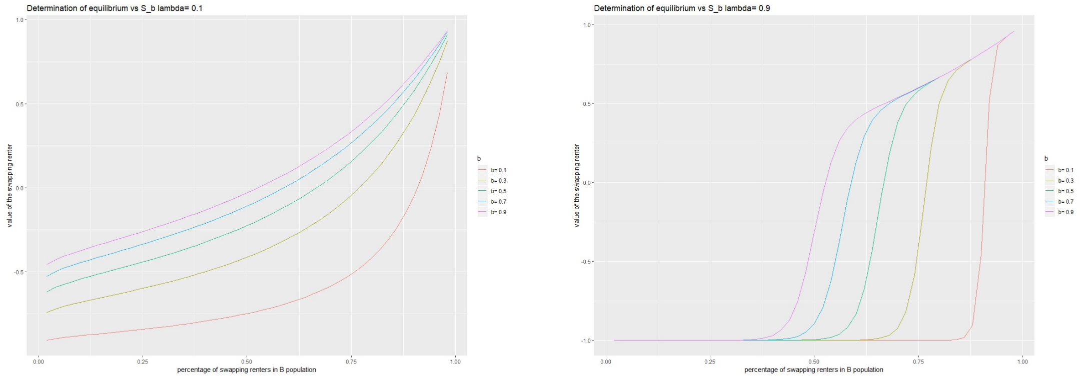

Figure 6: Swap value as a function of the % of swap households in B population

3 Swap and rental market

For the reasons of examining the influence of the rental market on the existence and functioning of the swap market, as well in order to analyze reverse effects from the swap market to the rental market, we upgrade previous toy model of the swap market and integrate it with the rental market. We add search process in the rental market along with the search process in the rental market in order to capture the effect of frictions in the whole rental market.

3.1 Effect of swap market on the rental market

Existence of the organized swap market, alongside with the classical rental market, shows that tenants are using it for satisfying their housing (rental) needs. It would be interesting to test whether the rental market is valuing (pricing) the option that rental apartment is giving to his current tenants to further exchange (swap) it for another apartment.

For this reason, we build model of search and match in the rental market, where we assume 3 transiting states for the renter (state without apartment N, state of renting apartment R, state of swapped apartment S) and two states for the landlord (state V of vacant apartment and state F of rented (filled) apartment).

Renter start from the state N without owning or renting apartment (for example living with his parents), with the value V_N, in which he experience high dis-utility x. Without loss of generality we can assume that in this state he experience the highest level of dissutility $x = 1$ and he decided definitely to rent the apartment. The renter is looking for apartment to rent with probability of search/match γ. We assume also that there is no search costs. If he is not successful in the search, the tenant is staying in the same state N with value V_N and he is sampling again with probability $1 - \gamma$. The value in state N (V_N) is described with the following Bellman equation:

$$V_N\,(r+1) = x + \gamma V_R + (1-\gamma)\; V_N \tag{29}$$

If the renter is successful in his search/match of finding apartment to rent, he move to the state R of renting the apartment. In this state, the renter pays the rent p and he lives until he experience some external shock to move to some other apartment with probability m. This shock could mean change in the needed size of the apartment. Once he experience this shock, he might either try to swap its own apartment with another apartment with probability θ or either search to directly rent other apartment with needed size from the landlord with probability μ. If the renter is not successful in the search, he need to move from the state R (from its apartment) to the state N (without rent) with probability $1-\theta-\mu$. If the renter, does not experience the shock to move, he stays with probability $1-m$ in his apartment (state N). The following Bellman equation specify the value V_R of being in state R:

$$V_R\,(r+1) = p + (\theta V_S + \mu V_R + (1-\theta-\mu)\; V_N)\, m + V_R\,(1-m) \tag{30}$$

If the renter make agreement to swap the apartment he is renting with another apartment he move to state S in which he is paying less/additional rent depending whether he is moving to less/more expensive (smaller/bigger) apartment. If the tenant is moving from less to more expensive apartment, beside the previous rent, he will pay some additional amount s, depending on the swap market conditions (differently to the model 2.1 here we simplify, so we encompass additional rent Δ_P within s). If swap is positive $s > 0$, the renter will pay in total $p + s$. If he is moving from more to less expensive apartment, swap is negative $s < 0$ and he pays less rent in total $p - s$. Renter can cancel/can be cancelled the swap agreement with probability a, after which he moves to its previous apartment (in state R). The following Bellman equation specify the value V_S of being in state S:

$$V_S\,(r+1) = s + V_R a + V_S\,(1-a) \tag{31}$$

When modeling the supply side of the rental market, we assume 2 states for the landlord: state V of vacant apartment and state F of rented (filled) apartment. Landlord has maintenance cost c in both states. He is searching/matching for new tenants with probability δ. If he is successful in finding tenant, he move to state F in which he receive the rent p ("-" sign means positive cash flow), else he is still in state V.

$$V_V\,(r+1) = c + (1-\delta)\; V_V + \delta V_F \tag{32}$$

$$V_F (r + 1) = -p + V_F (1 - (1 - \theta) m) + (1 - \theta) V_V m + c \tag{33}$$

Equating landlord and renter surplus we get:

$$\frac{p}{r - \theta m + m + \delta} = \frac{rx + \theta mx + ax - \theta ms - pr - ap}{r^2 - \mu mr + mr + ar + \gamma r - \theta am - \mu am + am + \gamma\theta m + \gamma a} \tag{34}$$

Optimal rent value p^* that solves this equilibrium:

$$\frac{(r - \theta m + m + \delta)(rx + \theta mx + ax - \theta ms)}{2r^2 - \theta mr - \mu mr + 2mr + 2ar + \delta r + \gamma r - 2\theta am - \mu am + 2am + \gamma\theta m + \delta a + \gamma a} \tag{35}$$

If we assume that probability of successful search for rental apartment, while currently being without apartment (for example living with your parents) is equal to the probability of of successful search rental apartment, while currently renting: $\gamma = \mu$, and also if we assume that there is no swap market, that is probability of swap is zero: $\theta = 0$, the solution collapses to:

$$P^*_{noswap} = \frac{r + m + \delta}{2r - \gamma m + 2m + \delta + \gamma} \tag{36}$$

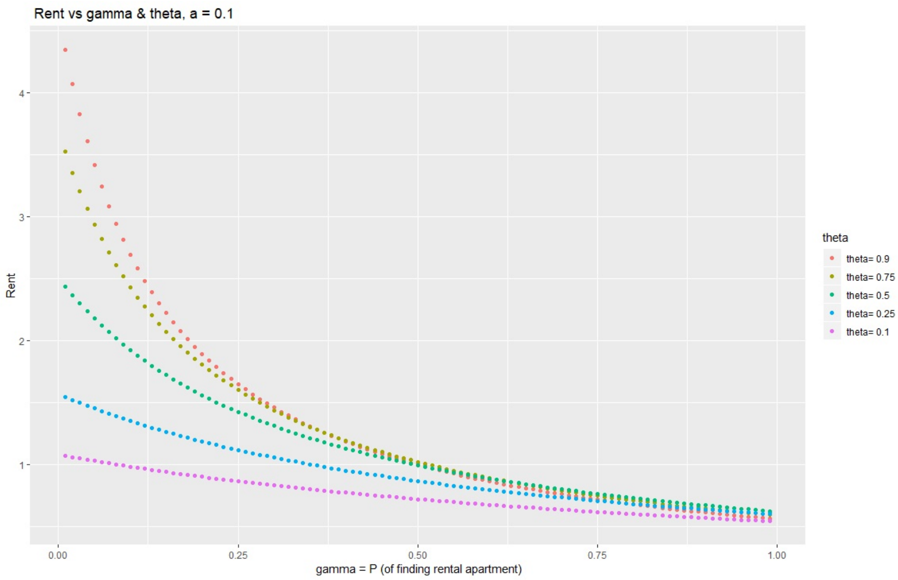

Figure 7: Rent as a function of the gamma and theta

One can prove that for $m, \gamma, \mu, \theta, a \in [0, 1]$, equation (30) $\geqslant$ equation (31), which means that with an increase in parameter θ (increase in probability of swapping the apartment), the rent level increases. Intuitively, if renter believe that rented apartment can be used as the mean for easier/faster search of eventual next apartment in the future, he is ready to pay higher rent. This swap premium, which renter include in the price of the rent is increasing in the probability of successful swap (θ), observed/assessed from the swap market.

Figure 7 helps us to visualize the increasing effect of lower friction in the swap market on the higher rent prices.

Figure 8 is similar with the previous figure, but it illustrates the effect of the change in parameter α (probability to cancel the swap agreement) on the value of the swap value. As can be seen, with the increase of the risk (probability) of canceling the agreement, the swap value is decreasing. High cancellation probability damages the value of the rental agreement as the precondition for entrance on the swap market.

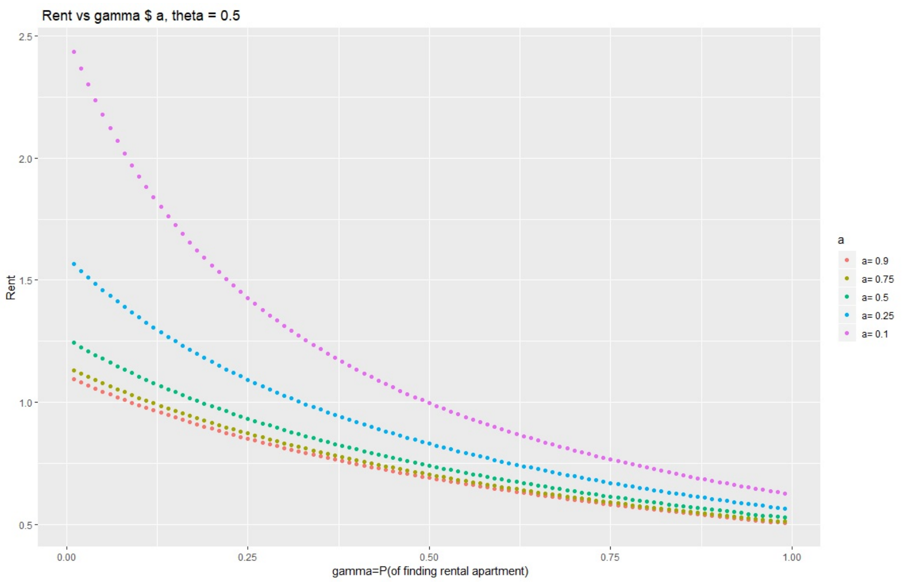

Figure 8: Rent as a function of the gamma and a

3.2 Effect of rental market on the swap market

We add another perspective on the relationship between rental and swap market. Beside the search process in the swap market we add the rental market along with its frictions. In this way, we can analyze the effect of frictions in the rental market on the value of the swap. One can argue that swap market reflect not only the supply/demand conditions in the swap market, but also the tightness in the rental market.

On one side of the search process we have renter A, who lives in the small apartment and want to change to another, but bigger apartment. On the other side of the search process we have renter who lives in the bigger apartment and want to change to smaller apartment.

Both renters have 3 states, denoted separately for both A and B renter: state of not living in the own/rented apartment (N_a / N_b), state of renting apartment (R_a / R_b) and state of living in the swapped apartment (S_a / S_b).

Renters start form the state N, where they experience dissutility (we assume to have highest level of $x_a = x_b = 1$), so they search for the apartment to rent with appropriate probabilities. Renter A search for small apartment with probability γ_a, while renter B search for big apartment with probability γ_b. The following Bellman equation specify the value V_{Na} and V_{Nb} of being in state N:

$$V_{Na}(r+1) = x_a + \gamma_a V_{Ra} + (1-\gamma_a) V_{Na} \tag{37}$$

$$V_{Nb}(r+1) = x_b + \gamma_b V_{Rb} + (1-\gamma_b) V_{Nb} \tag{38}$$

When renting the apartment (in the state R), both renters does not have dissutility anymore and they pay their rents: Renter A pays P, while renter B pays $P + \Delta_P$. Both renters with probability m can get exogenous shock that change their need. This shock switch the renter A need for small apartment to the need for big apartment. Vice versa, renter B after the "moving shock" does not need big apartment anymore, but smaller one. Hence, they might either to try to swap its own apartment with another apartment (renter A search in swap market with probability α, renter B search with probability β) or either search to directly rent other apartment (renter A search in rental market with probability γ_a, renter B search with probability γ_b).

If the renters are not successful in the search, they need to move from the state R (state of renting apartment) to the states N_a and N_b (not renting the apartment any more). If the renters, does not experience the shock to move, they stays in their apartment (state R) with probability $1-m$. The following Bellman equation specify the values

V_{Ra} and V_{Rb} of being in state R:

$$V_{Ra}\left(r+1\right)=p+\left(\alpha V_{Sa}+\gamma_{b}V_{Rb}+\left(-\gamma_{b}-\alpha+1\right)V_{Na}\right)m+V_{Ra}\left(1-m\right) \tag{39}$$

$$V_{Rb}\left(r+1\right)=p+\Delta_{p}+\left(\beta V_{Sb}+\gamma_{a}V_{Ra}+\left(-\gamma_{a}-\beta+1\right)V_{Nb}\right)m+V_{Rb}\left(1-m\right) \tag{40}$$

If the renters have swapped their apartments (they reach state S), they have swapped also the rents they have paid. Depending of the market conditions in the swap market, but also in the rental market, value of the swap can be positive $s>0$ or negative $s<0$. When s is positive, renter A pays $P+\Delta_P-S$, while renter B pays $P+S$. When s is negative, renter A pays $P+\Delta_P+S$, while renter B pays $P-S$. Renters can cancel/can be cancelled the swap agreement with probability b, after which they moves to its previous apartment (in state R). The following Bellman equation specify the values V_{Sa} and V_{Sb} of being in state S:

$$V_{Sa}\left(r+1\right)=-s+p+V_{Ra}b+V_{Sa}\left(1-b\right)+\Delta_{p} \tag{41}$$

$$V_{Sb}\left(r+1\right)=s+p+V_{Rb}b+V_{Sb}\left(1-b\right) \tag{42}$$

As the number of parameters is big enough to get intractable formula, without loss of generality, we can assume that interest rate is so low, that is zero $r=0$. Hence the optimal swap value that solves the bargain (solves the equality of the renter A and B surpluses) is:

$$s^{*}=\frac{-\gamma_{b}\left(\left(\beta m+b\right)\left(p-1\right)+\Delta_{p}b\right)-\gamma_{a}\left(\left(\alpha m+b\right)\left(p-1\right)+\Delta_{p}\alpha m\right)}{\left(\beta\gamma_{b}-\alpha\gamma_{a}\right)m} \tag{43}$$

As one can notice with the parallel decrease in the both γ parameters (rental market

(a) $s(\alpha,\gamma_a=\gamma_b)$, m=0.5 (b) $s(\beta,\gamma_a=\gamma_b)$, m=0.5

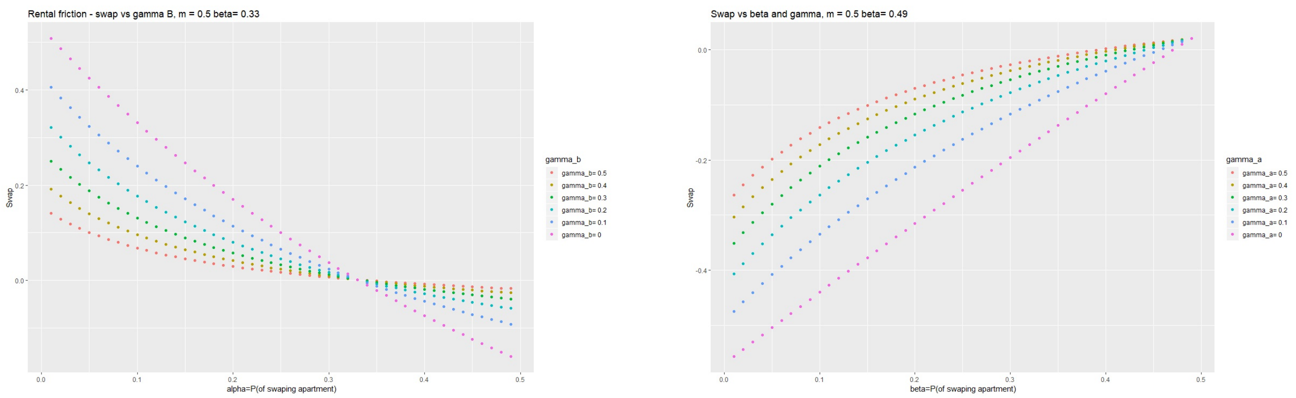

Figure 9: Swap value as a function of gamma and alpha/beta

search/match parameters), by keeping them equal so to have balanced rental market ($\gamma_a = \gamma_b$), swap value increases. Intuitively, with the increase of the friction in the rental market, swap value for renter A increase, as the swap market is increasing his chances to get desired rental apartment, when markets are tight. When rental market experience total friction ($\gamma_a = \gamma_b = 1$), relationship between the swap value and swap market search parameters (α, β) becomes linear.

4 Conclusions

This paper develops a theoretical search model of the apartment swap in the rental market. To our knowledge this is the first attempt to sublime the reality of the apartment swap transactions into a

The model is intended to mimic the Swedish swap market, characterized by strong rent controls and dominant ownership of apartments by the municipalities. In doing so, we capture supply and demand dynamics of the rental market, segmented to households with rented small or big municipal apartments, that search and try to swap between each other.

The driving mechanism of the is the motivation of the renter to use the swap to minimize mismatch cost. Similar to Arnott and Igarashi (2000), because of the low vacancy rate induced by the rent controls, a sitting tenant household may stay in the small apartment that is no longer suitable (family has extended due to new child) or may stay in large apartment that exceeds the renter needs (children have grown up and left home). These shocks to the family size that induce mismatch cost are taken to be exogenous.

The paper considers comparative statics of the apartment swap model and attempts to extract some stylized facts. For that purpose, we simulate the value of the swap itself as the function of the structure of the population, to test the changes in the modelled value of the swap, while varying the parameters of interest rate, probabilities of match and probability of cancel of the swap agreement. Simulation results confirm the intuitive expectations from the theoretical postulation. Swap value in the basic model setting is result of the swap market conditions and differential in the utility gains of matching swap counter-parties.

Furthermore, we propose that the swap market reflects not only the supply/demand conditions in the swap market, but also the tightness in the rental market. Simulation results showed that with the increase of the friction in the rental market, swap value for renter increases, which might be explained with the fact that the swap market is increasing tenant chances to get the desired rental apartment, when rental markets are tight.

Next, we test whether the rental market is valuing (pricing) the option that rental apartment is giving to his current tenants to further exchange (swap) it for another apartment. Model and simulation results show that with an increase in the probability of successful swap of the particular rental apartment, the rent level increases. Intuitively, if the renter believes that a particular apartment can be used as the mean for easier/faster way to reach another apartment in the future, he is ready to pay higher rent.

In this paper, we focus only on few aspects of the interdependence between classical rental and swap rental market.

The model that we develop is well suited for intra-city swaps, but also could be further developed to allow the tenants possibility to own the apartment or can be allowed to introduce job search perspective as the tenant "spiritus movens", thus allowing for inter-city swaps.

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Appendices

A Appendix - Bellman equations and swap

I. Bellman equations:

$$Vr_a r = x_a + p_a + \alpha\left(Vs_a - Vr_a\right)$$

$$Vs_a r = s + p_a + \left(Vr_a - Vs_a\right) b$$

$$Vr_b r = x_b + s + p_a + \beta\left(Vs_b - Vr_b\right)$$

$$Vs_b r = p_a + \left(Vr_b - Vs_b\right) b$$

II. Solving the model

- Calculate renter A surplus:

Deduct equation 1 from equation 2

$$Vs_a r - Vr_a r = -x_a + s + \left(Vr_a - Vs_a\right) b - \alpha\left(Vs_a - Vr_a\right)$$

$$\left(Vs_a - Vr_a\right) r + \left(Vs_a - Vr_a\right) b + \alpha\left(Vs_a - Vr_a\right) = s - x_a$$

$$\left(Vs_a - Vr_a\right)(r + b + \alpha) = s - x_a$$

$$Vs_a - Vr_a = \frac{s - x_a}{r + b + \alpha}$$

- Calculate renter B surplus

Deduct equation 3 from equation 4

$$Vs_b r - Vr_b r = -x_b - s + \left(Vr_b - Vs_b\right) b - \beta\left(Vs_b - Vr_b\right)$$

$$\left(Vs_b - Vr_b\right) r + \left(Vs_b - Vr_b\right) b + \beta\left(Vs_b - Vr_b\right) = -x_b - s$$

$$\left(Vs_b - Vr_b\right)(r + b + \beta) = -x_b - s$$

$$Vs_b - Vr_b = \frac{-x_b - s}{r + b + \beta}$$

Equalize both renter A and renter B surplus and solve for equilibrium swap

$$\frac{s - x_a}{r + b + \alpha} = \frac{-x_b - s}{r + b + \beta}$$

$$s = -\frac{r\, x_b + b\, x_b + \alpha x_b - r\, x_a - b\, x_a - \beta x_a}{(2r + 2b + \beta + \alpha)}$$

$$s = \frac{(r + b + \beta)\ x_a - (r + b + \alpha)\ x_b}{(2r + 2b + \beta + \alpha)}$$

B Appendix - Swap as the function of the percentage of the swapping households in population A

Express α in terms of S_a

$$\alpha\left(S_a\right) := \frac{M(R_a, R_b)}{R_a}$$

$$R_a = 1 - S_a$$

$$\alpha(S_a) = \frac{S_a b}{1 - S_a}$$

Conclusion: $\alpha(S_a)$ is increasing in S_a.

- Express β in terms of S_a

$$\beta\left(S_a\right) = \frac{M(R_a, R_b)}{R_b}$$

- Need to express R_b in terms of S_a
- Use Cob Douglas function

$$M(R_a, R_b) = {R_b}^{\theta}(R_a)^{1-\theta}$$

But we know that in equilibrium:

$$M(R_a, R_b) = S_a b$$

So,

$$S_a b = {R_b}^{\theta}(R_a)^{1-\theta}$$

$$S_a b = {R_b}^{\theta}(1 - S_a)^{1-\theta}$$

we can solve for R_b

$$R_b = (1 - S_a)^{1-\frac{1}{\theta}} {S_a}^{\frac{1}{\theta}} b^{\frac{1}{\theta}}$$

Evaluate $\beta(S_a)$ for the solved value of R_b :

$$\beta\left(S_a\right) = \frac{(1 - S_a)^{\frac{1}{\theta}-1}}{(S_a b)^{\frac{1}{\theta}-1}}$$

$$\beta\left(S_a\right) = \left(\frac{1 - S_a}{S_a b}\right)^{\frac{1}{\theta}-1}$$

Conclusion: $\beta(S_a)$ is decreasing in S_a.

- Express swap amount s in terms of S_a

$$s(S_a) = \frac{(r + b + \beta(S_a))\, x_a - (r + b + \alpha(S_a))\, x_b}{(2r + 2b + \beta(S_a) + \alpha(S_a))}$$

C Appendix - Swap as the function of the percentage of the swapping households in population B

Express α in terms of S_b

$$\alpha\left(S_b\right) = \frac{M(R_a, R_r)}{R_b}$$

but, since

$$R_b = 1 - S_b$$

and

$$M(R_a, R_b) = bS_b$$

$$\alpha(S_b) = \frac{S_b b}{1 - S_b}$$

$\alpha(S_b)$ is decreasing S_b
- Express β in terms of S_b

$$\beta\left(S_b\right) = \frac{M(R_a, R_b)}{R_a}$$

But,

$$M(R_a, R_b) = S_b b$$

and also

$$M(R_a, R_b) = {R_a}^{1-\theta} {R_b}^{\theta}$$

Hence,

$$S_b b = {R_a}^{1-\theta} {R_b}^{\theta}$$

- solve for R_a

$$R_a = \frac{(S_b b)^{\frac{1}{1-\theta}}}{{R_b}^{\frac{\theta}{1-\theta}}}$$

and having

$$R_b = 1 - S_b$$

we get

$$R_a = \frac{(S_b b)^{\frac{1}{1-\theta}}}{(1 - S_b)^{\frac{\theta}{1-\theta}}}$$

and

$$\beta(S_b) = \frac{(1 - S_b)^{\frac{\theta}{1-\theta}} S_b b}{(S_b b)^{\frac{1}{1-\theta}}}$$

with some algebra

$$\beta(S_b) = (1 - S_b)^{\frac{\theta}{1-\theta}} (S_b b)^{1-\frac{1}{1-\theta}}$$

$$\beta\left(S_b\right) = \left(1 - S_b\right)^{\frac{\theta}{1-\theta}} \left(S_b b\right)^{\frac{-\theta}{1-\theta}}$$

$$\beta\left(S_b\right) = \left(\frac{1 - S_b}{S_b b}\right)^{\frac{\theta}{1-\theta}}$$

Then we can express value of swap as the function of percentage of the swapping households in population B

$$s = \frac{(r + b + \beta(S_b))x_a - (r + b + \alpha(S_b))x_b}{2r + 2b + \beta(S_b) + \alpha(S_b)}$$