Time to Homeownership and Mortgage Design

Gianluca Marcato[1] Rafal Wojakowski[2]

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[1]Department of Real Estate and Planning, Henley Business School, University of Reading, Reading RG6 6UD, United Kingdom, Tel: +44 118 3788178, Email: g.marcato@henley.reading.ac.uk.
[2]Faculty of Business, Economics and Law, University of Surrey, Guildford GU2 7XH, United Kingdom, Tel: +441483683477, Email: r.wojakowski@surrey.ac.uk.

Abstract

Accessibility to homeownership in western countries, especially for middle to low income earners has decreased over time due to several factors such as stringent covenants, pressure of rental growth on household income expenditure and a negative gap between wage and house price growth. Moreover, young households face higher accumulated student loans and, in a steadily rising and strong rental market, they are not able to generate enough savings to cover the initial deposit necessary to become homeowners. We design an income sharing mortgage product where borrowers accept to pledge a portion of their future income to anticipate the time necessary to become homeowners by obtaining a higher LTV (up to 100%). Our analysis finds that this mortgage may be useful for lower income households in periods of higher uncertainty and that it may become less expensive in a high interest rate environment. Finally, this product also embeds an incentive to save, with potential benefits for the overall systemic risk of the banking sector. As a consequence we find that the default risk is not higher than a plain vanilla mortgage with lower LTV.

Keywords: Mortgage Product, Risk Transfer, Affordability, Economic Shocks, Borrowing constraints

JEL Codes: G11, G13, G21, R33

1 Introduction

Over the recent years housing markets in western countries have experienced an accelerated growth thanks to a steady increase in both domestic and foreign demand and constrained supply. Recent government policies, especially in the UK, tried to respond to a situation of under-supply and unavailability of affordable products for a vast proportion of the population but without succeeding to tackle the issue of unaffordability.

Furthermore, the increasing number of investors trying to access the property ladder made the UK buy-to-let market grow from some million to more than 200 billion pounds over the last two decades. Since the beginning of this century the proportion of buy-to-let properties also increased and reached a value of approximately 20% of the overall stock - Figure 1 Panel A. This situation has created a reduction in homeownership rates and, most importantly a pressure on the rental market, which affects the saving ability of households, especially during the first part of their lifecycle.

More recently, young households started to battle against increasing amounts of student loans whose outstanding balance is now almost 80 billion pounds in England only (and expected to grow to more than 250 billion by 2025). Hence, the income available for savings is decreasing in relative terms, also considering the increasing price-to-rent ratio and the average cumulative rental payments of 52,900 pounds (source: www.arla.co.uk) borrowers make before becoming homeowners. Figure 1 Panel B shows a constant and significant decline in saving rates in the UK from 1970s, but also reports a downward trend - even if less steep - in the US.

[Insert Figure 1]

The lack of accumulated savings is a clear deterrent for people wanting to become homeowners in the early stage of their household lifecycle. This phenomenon is not only referred to a minute proportion of the population with low education entry and weak prospects of income growth. In fact, even if income growth prospects are good and the default risk decays over a mortgage product life cycle, young households cannot become homeowners.

Consequently, we advocate for the need to develop mortgage products that transfer the risk between the two phases of the lifecycle using an incentive mechanism to make borrowers save. The structure of our mortgage product includes the ability to obtain a higher LTV (hence reducing the time of entry in the homeownership market) in exchange of a proportion of residual income after non-discretionary expenditure to be shared with the lender and another portion cumulated in a saving account. This fund would act in two ways: firstly it impacts on the attitude people have towards consumption and their propensity to save (favouring the latter); secondly, it may be used for temporary needs of cash flows in case of income loss.

More specifically, our work develops some new ideas combining the features of mortgages, student loans and insurance/retirement products as follows:

1. Tranche "m" is the plain vanilla mortgage component with cash flow assimilated to a fixed income product (in case of fixed rate product), which may potentially accumulate arrears and thus negative equity / liability (bullet lump sum due) at time t. By design, however, our *constant-payment* product does not allow for expected negative equity at time T as all "m" amounts will be paid out by the end. The modelling can also be extended to model *interest-only* products and a *partial balloon payment* at maturtity.

2. Tranche "α" is the proportion of disposable income available for discretionary expenses the lender periodically receives over the life of the mortgage. In particular, this disposable income is computed as gross income minus fixed mortgage component "m" and non-discretionary expenses. Clearly our product could also model a product where the borrower decides to share a higher proportion "α" for a shorter period of time (e.g. 5

or 10 years). This situation may be preferable if the lender's and borrower's expected income growth prospects over the mortgage life differ. The "α" component could be assimilated to a student loan, which acts as a tax *"here and now"* and does not accumulate any arrears (i.e. it is paid if income is above a threshold). After maturity (or chosen shorter period) the borrower is no more liable for this component.

3. Tranche "γ" represents the saving component, which also works as a *"cash flow"* guarantee in case there are periods of income loss due to job loss or salary reduction below levels allowing the borrower to pay non-discretionary expenses and the "m" tranche. This component is used to further guarantee the lender, so that early default due to *"liquidity"* issues is reduced to zero in expectation. In other words, we allow the borrower to pay the "m" tranche *"if he can"* and the "γ" tranche balances arrears over the mortgage life. Clearly, if no income loss before the set threshold happens before maturity, the component γ will be fully cumulated and retained as saving account that could be either spent or transferred to a pension fund at time T.

The paper is structured as follows: the next section introduces a concise literature review. Section 3 presents the model derivation. Before presenting the modelling of our mortgage product, we introduce the set up of a rent v.s. buy decision in a competitive cost setting and determine the optimal time of entry for homeowners. We then analyse the pricing of our mortgage product in a certainty case and subsequently add a stochastic domain where we obtain the optimal share of income (partly paid to the lender and partly accumulated in a saving account) necessary to increase the accessible LTV. Sections 4 and 5 respectively describe the data and main economic assumptions of our model, and discuss the main results. Finally section 6 concludes the paper.

2 Literature Review

In the 1970s first nonstandard mortgages appeared. Authors were concerned about high levels of inflation impacting affordability. See e.g.: **?**, **?**, **?** who advocated Price Level Adjusted Mortgages (PLAM) i.e. where the payment would be indexed by inflation. Products such as Shared Appreciation Mortgages and Shared Income Mortgages (the former in the residential sector, the latter in the commercial sector) made their appearance, see e.g.: **?**, **?**, **?**, **?**, **?**, **?**, **?**. Their purpose was to allow on the real estate ladder buyers who otherwise couldn't afford the double digit mortgage rates.

By the end of 80's, beginning of 90's double index mortgages were proposed for high inflation, developing countries such as those in Eastern Europe or South America. Proposals included making the payment *doubly* indexed, i.e. contingent on future evolution of personal incomes *and* the consumer price index. See e.g.: **?**, **?**, **?**, **?** who discuss applications and provide numerical examples crafted to such countries as e.g. Poland.

During the 2007 subprime bubble, researchers linked increased expenditures on housing by young, constrained households, to using newly designed mortgages, see **?**, **?**. Concerns were raised about borrowing against overvalued real estate collateral, see **?**.

After the burst of the house price bubble in the US, rather than widening participation, academics suggested non standard products with embedded insurance features against house price declines. **?** and **?** advocate Continuous Workout Mortgage which automatically decreases monthly payment as house prices go down. Alternatively, **?** suggest Adjustable Balance Mortgage where the outstanding balance decreases in line with a target loan to value ratio when a house price index decreases. Finally, **?** present foreclosure implications for Amortizing Participation Mortgages (APMs), which reduce agency costs by improving affordability (credit facilities suggested bu the Dodd-Frank Wall STreet Reform and Consumer Protection Act 2010).

The life-cycle nature of mortgage markets/products lends itself to a slow innovation highlighted by **?** and due to several constraints: cost of experimentation due to a spread public customer base, time length to prove experimentations, liquidity issues, general mistrust in not like-for-like products, myopic view of potential risks, selection bias compromising the testing phase, lack of key data for testing implementation, lack of incentives for financial institutions to innovate. The literature on the subject of optimally choosing the best mortgage loan product to a given economic environment includes a series of works exploring new developments in mortgage market/product design. For example **?** shows how demographics (population density) affect house price responses and hence the optimal structure of housing finance. **?** explores cross-country variation in mortgage market structures, while **?** provides an analysis of alternative mortgage products with lower initial payments to smooth consumption which provides several benefits such as portfolio diversification, tax savings and market efficiency (reducing transaction costs). The choice between fixed- and adjustable-rate mortgages (respectively ARM and FRM) is studied by **?**. They find economic gains for inflation-indexed FRM products and show how risky income, low moving probability, high default costs and risk aversion in large mortgages reduce the attraction of ARM products. Extending their previous work, **?** solve a dynamic model of households' mortgage decisions determining the impact of loan characteristics on premia and default. They also find that mortgage selection by heterogeneous borrowers partly explains pricing differences between market segments. **?** extend the analysis looking at the international evidence on mortgage choice: borrowers try to minimize costs over the life cycle paying attention to the differential between ARM and FRM rates, and expected short- (one year ahead) ARM movements affect this choice more than medium-term (three) forecasts. Embedding several other mortgage characteristics, **?** show that the combination of LTV limits (which reduce default probabilities with virtually no effect on housing demand) and recourse mortgages (which increase LTVs, payments and hence the cost of default) leads to default rate reduction, sustained demand and a mitigation of spikes in default after house pricing shocks.

Finally, researchers argue that home ownership is an efficient risk management device against such risks as moving job (see e.g. ?), regulation (see ? and ?), or uncertainties linked to retirement age (see e.g. ?). The latter contribution encourages for a more innovative approach to mortgage lending, where reverse mortgages - advocated by ? - are also part of the provision - see ? for a calibrated life-cycle model of retirement granting significant welfare gains. These products refer to a different point of the life-cycle, but they represent financial engineering applications for savings which share similar pricing function with standard mortgage products. The loan design we present in the next section combines a normal mortgage product with a saving component to allow for an augmented proportion of income saved into a retirement fund, which may allow the borrower to retain homeownership throughout the life-cycle and hence open to intergenerational transfers of wealth (with no need of underwriting a reverse mortgage).

3 Model Derivation

3.1 Certainty case

Before introducing the model in the stochastic domain, we take a look at the life cycle implications of saving versus borrowing for buying a house under certainty. The lifecycle model of a household is presented in Figure 2 Panel A. A finite lived ($T_c > 0$) representative household is endowed with initial wealth W_0 which can be negative (e.g. student debt, etc.). The household is initially a renter and has a low annual expendable income P_0 growing at the rate μ. We define expendable income as gross income after the payment of all basic expenses such as food, clothing, gas, electricity, communications, TV license, council tax, education of children, transports, health and pension provisions, etc.; but before any (a) payment for rent or mortgage; (b) decision to spend the "surplus income" in goods and services related to non-basic needs (e.g. luxury goods, entertainment, alcohol, restaurants, etc.) rather than saving.

The initial rent is R_0 and it grows at a rate ν. We assume that a household can save a fraction β of the annual expendable income $P_t = P_0 e^{\mu t}$ net of rental payments $R_t = R_0 e^{\nu t}$ and accumulate it at the prevailing interest rate r. The maximal allowed loan-to-value ratio is positive $0 \leq L$ so that we expect the household to be able to buy the house at some point in time $\tau_L \geq 0$ when the forward value of savings will be equal to the required deposit:

$$\int_0^{\tau_L} \beta \left(P_0 e^{\mu t} - R_0 e^{\nu t} \right) e^{r(\tau_L - t)} dt + W_0 e^{r\tau_L} = (1-L)\, H_0 e^{g\tau_L} \ . \tag{1}$$

The left hand side represents the accumulated expendable income net of rental payments and any spending on non-basic goods and services. The right hand side is the payment required at time τ_L: (a) the lump sum of cash to buy the house (when $L = 0$); or (b) the deposit equal to the difference between the house price $H_{\tau_L} = H_0 e^{g\tau_L}$ and the outstanding loan balance[1] LH_{τ_L}.

At time τ_L the household becomes a homeowner[2] because they have accumulated enough income to either buy or pay the deposit. To compute τ_L equation (8) must be solved numerically. However, in the special case $\mu = \nu = g = r$ and when $\beta > 0$ we get a very insightful and intuitive expression for the time to buy τ_L, depending on which way the home ownership is approached:

$$\tau_L = \frac{H_0 (1-L) - W_0}{\beta (P_0 - R_0)} \ . \tag{2}$$

[1]We sometimes refer to the house price as HP and to the outstanding loan balance as OLB.

[2]With stochastic house prices H_t, rent R_t, income P_t and/or interest rate r this occurs at some random time $\tilde{\tau}_L > 0$ in future. In reality, savings capacity β and the maximal loan to value requirement L change unpredictably in time too. We relax these constraints from section 3.2 onwards to consider stochastic income.

3.1.1 Cash buying v.s. taking a mortgage

Time to buy τ_L (expressed in years) is equal to the required dollar payment net of current wealth, divided by the net savings rate (dollars per year). Time to buy τ_L decreases if the savings capacity $\beta \leq 1$ improves ($\beta \to 1$), income rate P_0 increases, rental rate R_0 decreases, house H_0 is cheaper and, finally, if the loan to value ratio L can be increased.

Depending on the two fundamentally different ways of acquiring the house (cash purchase or mortgage) we have two interpretations:

1. $L = 0$: *Saving to buy.* In this case deposit is not required. No borrowing is necessary or allowed and the quantity τ_0 reflects the time it takes to collect the whole amount to buy cash. Immediate purchase ($\tau_0 = 0$) is only feasible if the initial wealth W_0 is high enough ($W_0 \geqslant H_0$). Otherwise, $\tau_0 > 0$ is the earliest time cash buying can occur. The household will never have opportunity to buy cash ($\tau_0 > T_c$), if the income rate P_0 or the saved proportion β are too low, the rental rate R_0 too high or the starting house value aimed at, H_0, too big relative to the current wealth W_0.

2. $L > 0$: *Borrowing to buy.* In this case the deposit required is strictly positive. The individual takes a mortgage either: (a) immediately ($\tau_L = 0$) if W_0 is high enough, i.e. $H_0\,(1-L) \leq W_0 < H_0$; or (b) at time $\tau_L > 0$ which comes as soon as they can collect the deposit $H_0\,(1-L)$. Since $W_0 < H_0\,(1-L) < H_0$ in the latter case, the net amount to be collected is $H_0\,(1-L) - W_0 > 0$. Saving can only happen if accumulation is positive ($P_0 - R_0 > 0$) and strong enough. A household is currently priced out of the housing market if $\tau_L > T_c$, in which case they must rent.

It is interesting to compare the time to buy τ_0 to the time to borrow τ_L. The difference $\tau_0 - \tau_L$ measures the *time reduction* achieved thanks to taking a mortgage. In practice it can be substantively huge and may even exceed the lifetime T_c. It is proportional to the allowed loan amount $H_0 L$ and inversely proportional to the achievable saving rate $\beta(P_0 - R_0)$.

In practice $L < 1$ is required because of the presence of default risk. However, the borrower may obtain a loan to value ratio L closer to 1, if the default risk of the homeowner is transferred from the first part of the lifecycle $\{0, \tau_L\}$ to the second $\{\tau_L, \tau_r\}$. Time τ_r represents the beginning of the retirement age when the income drops to a lower level represented by a pension (see Figure 2 Panel A).

3.1.2 Substituting the deposit for a pledged fraction of disposable income

The idea behind this paper is to propose a financial product which generates exactly this. That is, our product enables an effective loan to value ratio which is high enough. The product requires the household to demonstrate higher income prospects and adequate ability to accumulate enough income in future. This reduces the gap $L \to 1$ and therefore reduces the time to buy $\tau_L \to 0$. That is, as a result, we expect time to buy τ_L to shift to a much earlier time. Ideally, the house could be purchased (almost) immediately. As a consequence, increased numbers of homeowners should also reduce the pressure on the rental market, making it less attractive to buy-to-let investors.

Consider the following situation. At time τ_L a household has no wealth or savings ($W_0 = 0$, $\beta = 0$). Mortgages currently require a loan-to-value strictly lower than 100%: $L < 1$ and command a contract rate $r_c \geq r$. Therefore, fixed rate T-year repayment mortgages require the following annual repayment

$$m = \frac{r_c H_{\tau_L} L}{1 - e^{-r_c T}} \tag{3}$$

In order to pledge a fraction α of their current disposable income P_{τ_L}, net of mortgage payments m to substitute for the deposit $(1 - L) H_{\tau_L}$, the household solves the following problem

$$\alpha \int_{\tau_L}^{\tau_L + T} \left(P_0 e^{\mu t} - m \right) e^{-r(t - \tau_L)} dt = (1 - L) H_{\tau_L} \tag{4}$$

When $\mu = r_c = r$ and $\tau_L = 0$ the above can be solved simply as

$$\alpha = \frac{(1-L)H_0}{PT - LH_0} = \frac{\text{required deposit}}{\text{disposable income net of loan repayments}} \tag{5}$$

For example, for a house which costs $H_0 = 500$ (think of thousands of dollars or pounds), an available surplus income of $P = 20$, a standard time to maturity of $T = 30$ years and a loan-to-value $L = 80\%$, the required deposit is 100 and the fraction to be pledged is $\alpha = \frac{1}{2}$. However, if the less stringent loan-to-value of $L = 90\%$ is allowed, the required deposit decreases to 50 and the fraction to be pledged reduces to $\alpha = \frac{1}{3}$. Obviously, when lenders grant $L = 100\%$ loans, mortgaging a house can be done straight away and no pledging of any income is necessary ($\alpha = 0$).

What pledging future disposable future income does is to allow for 100% loans straight away, in an economic environment when $L < 100\%$. This effectively acts as an extra tax on future income and accelerates homeownership. From lender's perspective there is increased risk of lending to a prospective homeowner who does have a deposit as opposed to one who does not. Therefore, it is to be expected lenders will require extra guarantees. These can be provided from the fraction of disposable income, $1 - \alpha$, still remaining after the payment of the α "tranche," substituting for initial deposit. Such extra guarantee would not normally be a yet another expense. Instead, it would take a form of depositing a fraction γ (from the $1 - \alpha$ "tranche" available) into a special fund. Money would have to remain until $\tau_L + T$, when the mortgage is fully amortized and repaid, thus encouraging saving. This fund would compensate for any shortfalls should any mortgage arrears accumulate. Moreover, upon maturity of the mortgage, the γ "tranche" could act as an extra contribution channel (lump sum) into a specific professional pension plan.

The design described here fulfills a role similar to that in the Graduated Payment Mortgage (GPM), where repayments, rather than remaining at the fixed rate of m per year, increase at the same rate as inflation, thus making the initial m lower, preventing negative "tilt" and improving affordability, especially in times of high inflation. Here, it is the deposit which is shifted forward to be gradually repaid along the standard mortgage, the latter being amortized in the usual fashion.

The design is very flexible. The simplest design is when the α and γ "tranches" extend from the moment the mortgage is taken, that is $\tau_L = 0$ in our example above, until the loan is repaid, that is until T. However this needs not to be the case. Both these tranches, depending on the social policy targets aimed at, might begin at some later stage and not necessarily simultaneously. Likewise, they may terminate before, as well as after the maturity time T, when the core home loan is repaid in full, and not necessarily simultaneously either. Finally, the mortgage component could be designed as a graduated repayment too,[3] further improving affordability.

3.1.3 Income pledge in a lifecycle perspective

In a lifecycle perspective the household can be thought of as maximizing the net present value of the initial wealth W_0 plus the lifetime income net of any rent and mortgage, plus any income obtained from reversing the mortgage or bequesting the house

$$NPV = W_0 + \underbrace{\int_0^{T_c} P_0 e^{(\mu - r)t} dt}_{\text{lifetime income}} - \underbrace{\int_0^{\tau_L} R_0 e^{(\nu - r)t} dt}_{\text{rent}} \underbrace{- e^{-r\tau_L} \int_{\tau_L}^{\tau_L + T} m(\tau_L) e^{-rt} dt}_{\text{mortgage}} + \underbrace{H_{T_c} e^{-rT_c}}_{\text{bequest orreverse mortgage}} \tag{6}$$

The NPV is either consumed over the lifetime $[0, T_c]$ in non-basic goods or bequested. Obviously this is an over simplification. For example we assumed that the income grows at the rate μ indefinitely until the end of lifecycle T_c, that there is no uncertainty in the system, that the term structure of interest rates is flat struck at r, etc. Since W_0 and the lifetime income are exogenously given, such maximization would amount to, essentially, (a) minimizing the cost of renting; and (b) minimizing the cost of mortgage net of any reverse

[3]However, the negative amortization feature of a GPM might increase default risk substantially.

mortgage income or bequest extracted from the house at the end of lifetime.

$$\min_{\tau}\left\{\underbrace{\int_{0}^{\tau}R_0e^{(\nu-r)t}dt}_{\text{rental cost}}+\underbrace{e^{-r\tau}\int_{\tau}^{\tau+T}m\left(\tau\right)e^{-rt}dt-H_0e^{(g-r)T_c}}_{\text{mortgage cost}}\right\} \tag{7}$$

Where a strictly positive loan-to-value ratio $L>0$ is imposed, the choice of optimal τ is constrained strictly above zero, i.e. $\tau\geq\tau_L$ where τ_L is typically *strictly* greater than zero. However, where a product such as ours is offered, the starting time can be set to zero, $\tau=0$, which is the obvious candidate for minimum. It has for effect to completely eliminate the lifetime rental (a sunk cost), and only leaving the net mortgage cost. In our setup here the mortgage cost is zero because there is no uncertainty. However, under uncertainty, if mortgages are fairly priced i.e. when the mortgage contract rate r_c involves a market risk premium above the riskless rate r, the household only pays for the risks involved, typically the default and prepayment risks which are inherent in a mortgage. That is, lenders will require a premium when offering a financial product which is exposed to such risks.

In this lifetime perspective it is easy to see that pledging a fraction α of the lifetime income net of mortgage costs is nothing else but delaying the consumption to a later date, away from $t=0$, towards t between T and T_c, assuming that it is possible to take the mortgage an buy a house from $\tau=0$.

3.2 Shared Income Model

We model income as a stochastic variable. We restrict our model to one only variable to obtain closed form solutions.[4] In the second part of this section we also model a sudden disruption in income flows, pledgable income, security fund and statistical cover.

We estimate that the house price is H_0 at time 0 and it grows at the rate g, with the

[4] A possible further development of this research may extend the model to include other stochastic variables (e.g. housing price, interest rate) and obtain numerical solutions.

house price at time $t > 0$ being equivalent to H_t or $H_0 e^{gt}$.

Initially, a household has access to an annual expendable income P_0 growing at the rate μ. We define expendable income as gross income after the payment of all basic expenses such as food, clothing, gas, electricity, communications, TV license, council tax, education of children, transports, health and pension provisions, etc.; but before any:

- Payment for rent or mortgage;
- Decision to spend the "surplus income" in goods and services related to non-basic needs (e.g. luxury goods, entertainment, alcohol, restaurants, etc.) rather than saving.

We assume that a household can save a fraction β of the annual expendable income P_0 net of rental payments and accumulate it at the prevailing interest rate r. The initial rent is R_0 and it grows at some rate ν. The maximal allowed loan-to-value ratio is $L < 1$ so that we expect the household to be able to save for the deposit at time $\tau > 0$, when the forward value of savings (accumulated expendable income net of rental payments) will be equal to the required deposit:[5]

$$\int_0^{\tau} \beta \left(P_0 e^{\mu t} - R_0 e^{\nu t}\right) e^{r(\tau - t)} dt = (1 - L)\, H_0 e^{g\tau} \ . \tag{8}$$

At this point we note that, when $\mu = \nu = r$ and $P_0 > R_0$, condition (8) can be solved in closed form for the expected entry time τ

$$\tau_L = \begin{cases} \frac{W\left(\frac{H_0(1-L)(r-g)}{\beta(P_0-R_0)}\right)}{r-g} & r \neq g \\ \frac{H_0(1-L)}{\beta(P_0-R_0)} & r = g \end{cases} , \tag{9}$$

where W is the *Lambert W* function. We use the subscripted notation τ_L because the household accumulates a deposit imposed by the maximum loan to value ratio L. It then jumps onto the housing ladder by borrowing the remainder LH_{τ_L} with a fixed maturity e.g.

[5] In reality this occurs at some random time $\tilde{\tau} > 0$ in future.

$T = 30$-year Fixed Rate Mortgage (FRM)

$$LH_0 e^{g\tau_L} = \int_{\tau_L}^{\tau_L+T} m_{\tau_L} e^{-r(t-\tau_L)} dt = \underbrace{\frac{m_{\tau_L}}{r}\left(1-e^{-rT}\right)}_{\text{annuity with } r, T, m_{\tau_L}} \tag{10}$$

where m_{τ_L} is the annual mortgage payment[6] charged from time τ_L to time $\tau_L + T$.

If the household is able to save a security amount equal to a fraction θ of the annual mortgage payment (for example an entire payment, with θ equal to 1), the lender may offer an innovative 100% mortgage product, effectively substituting the need to save for a large deposit with the requirement to save for a much smaller amount to cover temporary liquidity issues in case of temporary loss of income. A lump sum equal to the initial security amount plus any accumulated extra savings net of any arrears will be paid back to the household at maturity.

This innovative mortgage is composed of two elements. The first element is an "accelerated" lending at the fixed rate r of the proportion L of the value of the house. To make up for the remaining part, $1 - L$, a fraction α of the future income P (net of mortgage payment m_{τ_θ}) is pledged in exchange of accessibility to a higher LTV, so that:

$$H_{\tau_\theta} = \underbrace{\frac{m_{\tau_\theta}}{r}\left(1-e^{-rT}\right)}_{\text{modified annuity with } r, T, m_{\tau_\theta}} + \underbrace{\alpha E\left[\int_{\tau_\theta}^{\tau_\theta+T} e^{-r(t-\tau_\theta)}\left(P_t - m_{\tau_\theta}\right)^+ dt\right]}_{\text{income cap } C} \tag{11}$$

Note that, at time τ_θ the household is effectively being granted a 100% LTV mortgage loan ($L = 1$). For that to happen the household must pledge a fraction α of every penny available after paying off the mortgage and basic expenses. The house can be acquired at time τ_θ when the security amount θm_{τ_θ} has been accumulated. As the deposit $(1 - L) H_0 e^{g\tau}$ is typically much higher than the security amount θm_{τ_θ}, we expect $0 < \tau_\theta \ll \tau_L$, i.e. acquisition

[6] Clearly a long-term FRM product is not yet available in the UK, but we start from this case to obtain some initial intuitions on the economic implications of the mortgage features. In a future draft of the paper we intend to extend this work to Variable Rate Mortgage (VRM) contracts including the stochasticity of interest rates.

occurring at τ_θ being close to zero and $\tau_\theta \to 0$ for $\theta \to 0$. If the first element of the loan is granted at time τ_θ, by analogy to (10) it would have to verify

$$LH_0 e^{g\tau_\theta} = \frac{m_{\tau_\theta}}{r}\left(1 - e^{-rT}\right) \tag{12}$$

Solving for m_{τ_θ} gives

$$m_{\tau_\theta} = \frac{LrH_0 e^{g\tau_\theta}}{1 - e^{-rT}} \tag{13}$$

The security amount θm_{τ_θ} (worth one year of mortgage payment m_{τ_θ} when $\theta = 1$) is obtained when τ_θ satisfies

$$\int_0^{\tau_\theta} \beta\left(P_0 e^{\mu t} - R_0 e^{\nu t}\right) e^{r(\tau_\theta - t)} dt = \theta m_{\tau_\theta} \tag{14}$$

which gives

$$\frac{P_0\left(e^{\mu\tau_\theta} - e^{r\tau_\theta}\right)}{\mu - r} - \frac{R_0\left(e^{\nu\tau_\theta} - e^{r\tau_\theta}\right)}{\nu - r} = \frac{\theta}{\beta}\frac{LrH_0 e^{g\tau_\theta}}{1 - e^{-rT}} \tag{15}$$

This condition must be solved for τ_θ numerically unless $\theta = 0$ (no guarantee required) and hence $\tau_\theta = 0$, or $\mu = \nu = r$ when, similarly to (9), we have

$$\tau_\theta = \begin{cases} \frac{W\left(\frac{\theta LrH_0(r-g)}{\beta(P_0 - R_0)\left(1 - e^{-rT}\right)}\right)}{r-g} & r \neq g \\ \frac{\theta LrH_0}{\beta(P_0 - R_0)(1 - e^{-rT})} & r = g \end{cases} \tag{16}$$

To specify the model further, we assume that the available funds, P_t, follow a growth rate μ and are subject to a randomness characterized by a volatility parameter σ

$$dP_t = P_t\left(\mu dt + \sigma dz_t\right) \tag{17}$$

where z_t is a standard Brownian motion.[7] If borrowing starts at time τ, we also require $P_\tau > m_\tau$ as a condition for the mortgage to be granted and homeownership becoming

[7]When a market for trading claims on future income exists (such as a futures or options market on relevant occupancy index, see Shiller **?**), we can set $\mu = r$, the riskless rate and z_t can be thought of as evolving under the pricing measure.

available.

Under relevant parameter values we are interested in:

- Evaluating the fraction α of future income a household should pledge to allow immediate or earlier purchase, hence eliminating or shortening time to save and enter homeownership.

- Assessing what makes the lender indifferent between granting the two mortgages: a traditional mortgage with deposit paid at time τ_L and the innovative mortgage granted at a much earlier time τ_θ. [8]

- Computing the required saving fraction γ of the remaining net income to accumulate savings which would cover the bank (on average) against possible losses on arrears. The following condition can be used:

$$\begin{aligned} \theta m_{\tau_\theta} + \gamma (1-\alpha) E \left[\int_{\tau_\theta}^{\tau_\theta + T} e^{-r(t-\tau_\theta)} (P_t - m_{\tau_\theta})^+ \, dt \right] \\ = E \int_{\tau_\theta}^{\tau_\theta + T} e^{-r(t-\tau_\theta)} (m_{\tau_\theta} - P_t)^+ \, dt \end{aligned} \tag{18}$$

  where the initial margin plus a fraction γ of the total capacity to save (left hand side) is equal to the expected defaulted value (right hand side). For $\theta = 0$ i.e. no initial margin deposit but further saving *is* required, this can be simplified to:

$$\gamma (1-\alpha) E \left[\int_0^T e^{-rt} (P_t - m_0)^+ \, dt \right] = E \int_0^T e^{-rt} (m_0 - P_t)^+ \, dt \tag{19}$$

  Parameter γ is part of the innovative mortgage contract and must be evaluated and imposed beforehand at time $t = 0$, when the loan is originated.

This paper focuses on the first two issues and we leave the third one to be discussed in a follow up paper where the possibility of default is modelled assuming both negative equity

[8] This situation requires that the two mortgages offer the same present value of future cash flow.

and accumulated payments in arrears higher than accumulated savings.

3.3 Sudden Disruptions in Income Flow

Most of residential mortgage defaults post 2008 crisis in the U.S. can be associated with sudden declines in income flow. In this section we augment our approach to incorporate the source of uncertain income discontinuity. A Poisson jump process, as in **?** (with a risk-neutral jump compensator λ) represents a large income decline. This is consistent with modelling defaults via the reduced form (hazard rate) approach of **?** to credit risk who extend **?** results to include unpredictable and undiversifiable shocks. Such approach seems most appropriate in our case as it allows separating the occupational income fluctuations from individual circumstances. For a specific "class" of occupational income such individual shocks can be diversified away. This corresponds to a situation where the occupational risk is tradable via "macro markets," as in **?**, with the moral hazard component removed.

We make no assumptions concerning why negative shocks occur. Rather, we specify the dynamics of default via the default rate (or intensity) λ of the Poisson event triggering temporary personal income disruption. In our analysis we focus on the most simple case, where there is just one such adverse event arriving with intensity λ over the life span $[0, T]$. The random default time is then a Poisson event independent of state variables governing the diffusion processes for income and house prices.

In what follows we focus on the special case of $\theta = 0$ i.e. there is no initial margin deposit but further saving into security fund *is* required.

The advantage, in practice, of our approach is that for the lender is able to price the credit sensitive, modified loan as if it were default-free, using the risk free rate adjusted by the level of intensity estimated from historical data. In the simplest case this will only require replacing the riskless interest rate parameter r by $r + \lambda$. Whether the income level will take a relatively long time to be restored will primarily depend on other parameters of the income process which are already present in our analysis, i.e. the growth rate (equal to the riskless rate r under the pricing measure) and the income volatility σ.

3.4 Pledgable Income

Let λ denote the intensity of income disruption. We assume that if such disruption occurs at some random time t_d, it will also trigger a default on mortgage repayment. Thus we use the terms income disruption and payment default interchangeably. This implies that the cumulative probability to preserve the original income source beyond time t is

$$e^{-\lambda t} = \Pr\left(t_d > t\right) < 1 \tag{20}$$

and is decreasing with t.

Furthermore, we argue that if such disruption and payment default happens, a delay η must occur before the household recovers a fraction π of the previous level of income. Therefore, the pledgable portion of income becomes

$$I = I\left(T\right) = E\int_0^T e^{-\lambda t} e^{-rt}\left(P_t - m_0\right)^+ dt \tag{21}$$

$$+\,E\int_{\eta\wedge T}^T \left(1 - e^{-\lambda(t-\eta)}\right) e^{-rt}\left(\pi P_t - m_0\right)^+ dt \tag{22}$$

where the time lag η reflects the fact that upon occurrence of the sudden drop in personal income to zero the pledgable income potential will temporarily cease to grow. The first expectation (21) means that the initial level of income, P_t, "survives" beyond time t with probability $e^{-\lambda t}$. After observing the time lag $\eta > 0$, the second expectation (22) assumes a decreased level of income, πP_t, where $\pi < 1$.

It is straightforward to compute the first integral as

$$C\left(P_0, m_0, T, r+\lambda, \delta+\lambda, \sigma\right) = E\int_0^T e^{-\lambda t} e^{-rt}\left(P_t - m_0\right)^+ dt \tag{23}$$

Clearly, parameters r and δ in the cap function C need to be adjusted for jumps i.e. replaced by $r + \lambda$ and $\delta + \lambda$. This means that to incorporate the risk of default we need to adjust the profit cap C to reflect the default intensity λ. Profit cap C cumulates a continuum of infinitesimal caplets on income P struck at m_0 over a continuum of maturities $t \in [0, T]$. Each caplet is weighted by the probability $e^{-\lambda t}$ of borrower's income source "surviving" at least until t. As we will see in what follows, because any income-contingent flow will cease to be paid as well, the lender will, similarly, need to adjust their expectations accordingly.

Handling the second term is, however, more delicate. First note that it can be decomposed as

$$E \int_0^T \left(1 - e^{-\lambda(t-\eta)}\right) e^{-rt} \left(\pi P_t - m_0\right)^+ dt \tag{24}$$

$$- E \int_0^{\eta \wedge T} \left(1 - e^{-\lambda(t-\eta)}\right) e^{-rt} \left(\pi P_t - m_0\right)^+ dt \tag{25}$$

$$= E \int_0^T e^{-rt} \left(\pi P_t - m_0\right)^+ dt \tag{26}$$

$$- e^{\lambda\eta} E \int_0^T e^{-\lambda t} e^{-rt} \left(\pi P_t - m_0\right)^+ dt \tag{27}$$

$$- E \int_0^{\eta \wedge T} e^{-rt} \left(\pi P_t - m_0\right)^+ dt \tag{28}$$

$$+ e^{\lambda\eta} E \int_0^{\eta \wedge T} e^{-\lambda t} e^{-rt} \left(\pi P_t - m_0\right)^+ dt \tag{29}$$

Therefore, the pledgable income becomes

$$I = C\left(P_0, m_0, T, r + \lambda, \delta + \lambda, \sigma\right) \tag{30}$$

$$+ C\left(\pi P_0, m_0, T, r, \delta, \sigma\right) \tag{31}$$

$$- C\left(\pi P_0, m_0, \eta \wedge T, r, \delta, \sigma\right) \tag{32}$$

$$+ e^{\lambda\eta} C\left(\pi P_0, m_0, \eta \wedge T, r + \lambda, \delta + \lambda, \sigma\right) \tag{33}$$

$$- e^{\lambda\eta} C\left(\pi P_0, m_0, T, r + \lambda, \delta + \lambda, \sigma\right) \tag{34}$$

Note that both η and π could, in general, be random and/or functions of time. However, this would require numerical integration and closed form solutions would not, in such case, be available. Also, it is important to monitor the minimum $\eta \wedge T$ in order to ensure convergence of the last four terms to zero whenever $T \leq \eta$.

3.5 Security Fund

A fraction of pledgable income equal to αI, where $\alpha < 1$, is contributed towards repayment of debt. The remaining portion, $(1-\alpha) I$, where $1-\alpha < 1$, can be used for consumption or buying required insurance. We assume that bank requires the household to deposit a fraction γ of the remaining portion to offset possible losses on arrears. Parameter γ is part of the innovative mortgage contract and must be evaluated and imposed beforehand, at time $t = 0$, when the loan is being granted.

If lending starts immediately, the lender will need to assess the possible lifetime loss over the time horizon $[0, T]$ in order to determine a fair fraction γ. The equilibrium condition which statistically covers such lifetime exposure, X, can in such case be expressed as

$$\gamma (1-\alpha) I = X \ , \tag{35}$$

where, under absence of defaults ($\lambda = 0$),

$$X = X(T) = E \int_0^T e^{-rt} (m_0 - P_t)^+ \, dt = F(P_0, m_0, T, r, \delta, \sigma) \tag{36}$$

and F is the floor function. Analogously, to the cap function C employed previously, the floor function F is the time integral of European put options expiring continuously. It measures the present value of arrears which are likely to accumulate over the lifetime $[0, T]$ and thus the exposure of the bank.

When there will be a job loss with intensity λ, the above condition needs to be adjusted.

The exposure increases to

$$X = X_1 + X_2 + X_3 = \tag{37}$$

$$E \int_0^T e^{-\lambda t} e^{-rt} (m_0 - P_t)^+ \, dt \tag{38}$$

$$+ E \int_{\eta \wedge T}^T \left(1 - e^{-\lambda(t-\eta)}\right) e^{-rt} (m_0 - \pi P_t)^+ \, dt \tag{39}$$

$$+ \int_0^T \left(1 - e^{-\lambda t}\right) e^{-rt} A(t) \, dt \tag{40}$$

where

$$A(t) = \int_t^{t+\eta \wedge (T-t)} e^{-r(s-t)} m_0 ds \tag{41}$$

$$= \frac{m_0}{r} \left(1 - e^{-r[\eta \wedge (T-t)]}\right) \tag{42}$$

and

$$\eta \wedge (T-t) = \min \{\eta, T-t\} \tag{43}$$

The first two terms, X_1 and X_2, resemble those encountered in the computation of income I. However, integrals here cumulate arrears in bad states of nature ($P_t < m_0$) i.e. infinitesimal floorlets $(P_t - m_0)^+ \, dt$. Therefore, they equal the value of the put option to default. In contrast, the third term, X_3, containing the arrears component $A(t)$, is new. It reflects the fact that upon occurrence of income disruption (i.e. $P_t = 0$ within the time interval $[t, t+\eta]$ or $[t, T]$ if $t + \eta > T$) the household will, for the length of time $\eta \wedge T - t$, be accumulating unpaid annuity at the rate m_0.

Clearly, the first integral in (37) can be evaluated using a floor formula adjusted for income loss

$$X_1 = F(P_0, m_0, T, r + \lambda, \delta + \lambda, \sigma) \tag{44}$$

The second integral in (37) can be decomposed and evaluated, too, with adjusted floor expressions

$$X_2 = E\int_0^T \left(1 - e^{-\lambda(t-\eta)}\right) e^{-rt} \left(m_0 - \pi P_t\right)^+ dt \tag{45}$$

$$- E\int_0^{\eta\wedge T} \left(1 - e^{-\lambda(t-\eta)}\right) e^{-rt} \left(m_0 - \pi P_t\right)^+ dt \tag{46}$$

$$= E\int_0^T e^{-rt} \left(m_0 - \pi P_t\right)^+ dt \tag{47}$$

$$- e^{\lambda\eta} E\int_0^T e^{-\lambda t} e^{-rt} \left(m_0 - \pi P_t\right)^+ dt \tag{48}$$

$$- E\int_0^{\eta\wedge T} e^{-rt} \left(m_0 - \pi P_t\right)^+ dt \tag{49}$$

$$+ e^{\lambda\eta} E\int_0^{\eta\wedge T} e^{-\lambda t} e^{-rt} \left(m_0 - \pi P_t\right)^+ dt \tag{50}$$

$$= F\left(\pi P_0, m_0, T, r, \delta, \sigma\right) \tag{51}$$

$$- F\left(\pi P_0, m_0, \eta\wedge T, r, \delta, \sigma\right) \tag{52}$$

$$+ e^{\lambda\eta} F\left(\pi P_0, m_0, \eta\wedge T, r+\lambda, \delta+\lambda, \sigma\right) \tag{53}$$

$$- e^{\lambda\eta} F\left(\pi P_0, m_0, T, r+\lambda, \delta+\lambda, \sigma\right) \tag{54}$$

The third integral can be split along $\tau = (T-\eta)^+$ into

$$X_3 = \int_0^\tau \left(1 - e^{-\lambda t}\right) e^{-rt} A\left(t\right) dt \tag{55}$$

$$+ \int_\tau^T \left(1 - e^{-\lambda t}\right) e^{-rt} A\left(t\right) dt \tag{56}$$

We obtain

$$X_3 = \frac{m_0}{r}\left(\frac{e^{-rT-\lambda\tau}}{\lambda} + \frac{e^{-r\eta}\left(e^{-r\tau} - 1\right) - e^{-rT} + 1}{r}\right. \tag{57}$$

$$+ \frac{e^{-r\eta}\left(1 - e^{-(r+\lambda)\tau}\right) - 1}{\lambda + r} \tag{58}$$

$$\left. - \frac{re^{-(r+\lambda)T}}{\lambda(\lambda+r)} + e^{-rT}(\tau - T)\right) \tag{59}$$

3.6 Statistical Cover and Time profile of Exposure

We use the equilibrium condition (35) to compute the required pledge and saving fractions α and γ based on expected net income under risk of default. When $\lambda > 0$ (defaults), proportion γ of the total capacity to save (left hand side of 35) is equal to the present value of expected lifetime arrears (right hand side of 35). Proportion γ of accumulated savings in excess of αI covers (on average) the bank against individual income disruptions. It provides average "statistical" guarantee for the set duration T of the product.

To refine our analysis, we perform a second pass approach using the pre-computed equilibrium "pledge" and "saving" fractions α and γ (for a specified lifetime T). This is done by "stopping" the time integrals at t, where t lies in the interval $[0, T]$. The $LHS\,(t) = \gamma\,(1-\alpha)\,I\,(t)$ then shows the expected cumulated guarantee up to time t. The $RHS\,(t) = X\,(t)$ shows the expected loss cumulated up to time t. A negative difference

$$\Delta\,(t) = LHS\,(t) - RHS\,(t) \tag{60}$$

gives a signal that within a particular time range (where $\Delta\,(t) < 0$) the product is more vulnerable to cumulative losses (mortgage in arrears) which are likely to exceed the protection fund cumulated up to date t.

4 Data and Further Assumptions

In this section we present the assumptions used in our numerical exercise and the key objective function used to obtain the required fraction α of income to be pledged in order to obtain a further increase in the initial LTV up to 100%

4.1 Base case and numerical values

The main numerical assumptions used in our simulation are reported in Table 1

[Insert Table 1 Here]

The base case of our simulation assumes an initial house price equal to 400,000 (ranging from 300,000 to 700,000) growing at 4%, which also represents the level of interest rates and growth rate of the annual income (25,000), which shows a volatility of 25% (ranging from 10% to 50% for other simulations) . The rent is equal to 15,000, while the mortgage has 30 year maturity and an LTV equal to 90%. Finally, in all our numerical simulations, the borrower does not need to have any accumulated savings to become homeowner ($\theta = 0$).

For our simulations we also allow for changes in the maturity of the mortgage (from 10 to 40 years), loan to value ratio (from 70% to 100%) and house price growth rate and interest rate (both from 2% to 8%). Finally, the annual net expendable income can vary between 15,000 to 30,000. As we do not change the annual rent paid before the house purchase, the net figure between the two will represent the ability for the borrower to save before becoming a homeowner. Clearly a higher figure will determine a higher capacity to save and hence a shorter time to homeownership.

4.2 Evaluating α when $\theta = 0$

For different values of our parameters, we obtain the percentage of expendable income to be be pledged (α satisfying (11). As we assume $\theta = 0$, we can rewrite equation (11) as follows:

$$H_0 = \underbrace{\frac{m_0}{r}\left(1 - e^{-rT}\right)}_{\text{modified annuity with } r, T, m_0} + \alpha \underbrace{E\left[\int_0^T e^{-rt}\left(P_t - m_0\right)^+ dt\right]}_{\text{income cap } C} \tag{61}$$

With m_0 computed using equation (13), we can further simplify as follows:

$$(1-L)\,H_0 = \alpha \lim_{\delta^+ \to 0} \underbrace{E\left[\int_0^T e^{-rt}\,(P_t - m_0)^+\,dt\right]}_{C(P_0,m_0,T,r,\delta,\sigma)} \tag{62}$$

The left hand side is the required deposit at time zero. As an alternative to save cash for this initial deposit, we introduce the possibility that this amount is covered by a fraction α of expected pledgeable income (right hand side). Given the uncertain nature of income for the borrower, this variable component of the mortgage-related payment represents an option for the bank, which can be computed in closed form using the income cap function C which is described in Appendix A.

4.3 Evaluating γ

If the mortgage contract is set up correctly, the payment of the pledged fraction α of net income $P_t - m_0$ substitutes the initial required deposit $(1-L)\,H_0$. However, even after this pledged amount, the borrower still has a residual income to be used for either consumption or saving:

$$(1-\alpha)\,(P_t - m_0)^+ \tag{63}$$

We hereby focus on a two-tier mortgage where the lender reclaims the contribution of a second tranche (γ) towards the accumulation of a guarantee against the risk of payments in arrears. As the salary P_t is stochastic, we may find periods when the borrower is unlikely to afford mortgage payments. Hence, as the probability to have $P_t < m_0$ is greater than zero, our contract requires the borrower to pledge a fraction γ of the residual income on top of the pledged fraction α of the disposable income. In the long run, the value of this accumulated amount money (from now security fund) can cover the temporary issue of payments in arrears $(m_0 - P_t)^+$.

In order to solve for γ we therefore start from the following condition:

$$\gamma\left(1-\alpha\right)E\left[\int_0^T e^{-rt}\left(P_t-m_0\right)^+dt\right]=E\int_0^T e^{-rt}\left(m_0-P_t\right)^+dt \tag{64}$$

The left hand side represents the present value of the expected accumulated security fund, which must be equal to the present value of the expected accumulated payments in arrears (right hand side). Solving for γ we obtain the following closed form solution:

$$\gamma=\frac{E\int_0^T e^{-rt}\left(m_0-P_t\right)^+dt}{E\int_0^T e^{-rt}\left(P_t-m_0\right)^+dt-\left(1-L\right)H_0}=\frac{F\left(P_0,m_0,T,r,\delta,\sigma\right)}{C\left(P_0,m_0,T,r,\delta,\sigma\right)-\left(1-L\right)H_0} \tag{65}$$

where C and F represent respectively the cap and floor on the continuous flow P. These functions can be computed in closed form according to **?**.

We focus on the magnitude of the guarantee required to study the variations of the value and properties of γ. For the time being, in this study we leave the rules of operation of the security fund unspecified, but we can imagine a setup where funds are cleared at maturity. This fund would then represent a further pension contribution made by the borrower and it may be assumed to be invested in assets/funds with low risk/return profile (e.g. government bonds). Any excess would be paid back to the borrower and any shortfall would be due to the lender. Alternatively, a more sophisticated contract would be likely to operate similarly to a margin account with margin calls, which could involve more frequent clearing, caps on the maximum accumulated security deposit or a possibility of early excess withdrawal.

5 Main Results

Using the assumptions of the base case scenario, we start to compute the borrower's waiting time to become homeowner and plot it against the level of rent before purchase as a percentage of expendable income. Figure 3 shows that our model is consistent with theoretical predictions where increasing LTV and rental payments (as percentage of expendable income) leads to a rise in waiting time before becoming homeowner. Mortgages with 100% LTV allow households to buy immediately, providing that the future income is sufficient to pay a fixed annual mortgage payment and the pledged component. However, as we reduce the LTV to 90%, we obtain a minimum waiting time of almost 2 years (when there is no payment of rent, e.g. assuming that the borrower lives with the natural family), which increases to almost 5 years if the rent represents 60% of the expendable income. Moreover, if borrowers decide to spend more than 80% of available income in rental payments, they forfeit the option to become homeowners during their lifetime. Finally, as the LTV is reduced further to 80% (70%), the minimum waiting time rises to 3.5 (6) years and the maximum threshold of annual rent payment as a percentage of expendable income to keep the accessibility to the property ladder open becomes 70% (60%).

[Insert Figure 3 Here]

As we have several combinations for our numerical assumptions, we present the main results of our simulations in Table 2. Particularly we focus on two outcomes: the fixed amount of mortgage payments for the initial LTV value (m_0) and the resulting percentage of future expendable income the borrower has to pledge to increase the LTV ratio to 100% (α). For our base case scenario, we find that the initial mortgage payment is 20,607 and the required α to be pledged is around 9% of the remaining income (after paying the mortgage). The sensitivity of this result to other parameters in our model will be discussed in the following figures. An important aspect to highlight is the existence of situations where this product cannot be offered because either α is greater than 1 (where borrowers are not actually able to pledge more than 100% of their net disposable income), or the mortgage payment exceeds the pledgeable income at origination (when borrowers do not have enough money to pay the first instalment and hence would be in arrears from the very beginning). Examples are given by high house prices (H_0 equal to 500,000 or700,000), a 10 year mortgage product ($T = 10$) and a market with 6% and 8% interest rate.

[Insert Table 2 Here]

In our simulation, we have only one stochastic variable (income) while all others are deterministic. In the last part of this section we present the sensitivity of our model - and particularly of the percentage to be pledged α - to several parameters.

5.1 Pricing Sensitivity of α with no Income Loss ($\lambda = 0$)

Panel A of Figure 4 shows α as a function of the volatility of disposable income. Clearly a mainstream mortgage with 100% LTV does not require any pledged income to reach the combined 100% LTV. With no volatility, the pledged income necessary to offset the remaining 10% (20% or 30%) of LTV to reach 100% for an original mortgage of with 90% (80% or 70%) LTV is equal to c.ca 10% (19% or 25%). The impact of volatility is also not hugely significant since a big increase in volatility (from 0% and 50%) leads to only 2% to 4% increase in the percentage of pledge income, with the biggest effect found for LTV = 70%. Finally, one may expect that volatility and α should have a positive relationship. However, in this case a higher income volatility increases the option the bank underwrites on future pledged income because the disposable income may increase quite significantly while it cannot fall below zero. Hence the second part of equation (62), which represents the price of this option becomes more valuable and the bank is then willing to ask for a lower percentage disposable income to be be pledged in the contract.

[Insert Figure 4 Here]

Panel B shows that α increases as the house price increases. Particularly adding a 10% LTV to reach 100% only requires the pledging of 5% of disposable income if the house price is equal to 200,000, while the percentage (and hence cost for borrowers) increases to 20% of their income if the house price is equal to 700,000. As the added amount of LTV increases to 20% or 30%, we obtain a much steeper curve with a requirement of respectively 8% or 10% for the lower house price and more than 40% or almost 60% for the higher price.

Panel C shows the impact of the amount of LTV for the fixed component of the payment (m_0) for mortgages with different maturities. As the maturity becomes longer, the required pledged income is smaller. Moreover, with maturities between 20 and 40 years is possible to reduce the fixed component of the annual payment to 0 at the expense of pledging between 40% and 80% of the disposable income. Particularly, the area of our interest is the bottom right end corner of the graph where $L \geq 0.7$.

The 10 year mortgage requires a minimum 95% LTV with fixed payments and the income to be pledged would be in this case near 100% or the available income. In fact we see that the transparent blue line for $L \leq 0.95$ is above the $\alpha = 1$ line, which suggest the non existence of such products because the pledged disposable income should be greater than 100% (technically impossible). Similar patterns are shown by 30- an 40-year mortgages, while the 20-year product reveals a much greater sensitivity, with required α is around 50% for $L = 0.7$.

As the initial disposable income increases, Panel D in Figure 4 shows that the percentage of future income to be pledged is reduced significantly. Particularly, when the income volatility goes from 10% to 50%, α only increases from 6% to 8% with initial disposable income of 30,000. However, the percentage of pledged income decreases much more (from 31% to 14%) for lower income households. This suggests that our mortgage product could become particularly useful for lower income household in periods of higher uncertainty.

The sensitivity of the percentage of pledged income to changes in interest rates is reported in Panel E. Firstly, this function is not monotonic for any of the income volatilities. This result is due to a competing effect of interest rates on our pricing. On one hand an increase in interest rates makes the fixed component of our payment (m_0) higher. However, the opposite effect may be seen as the price of the call option the bank holds on future income pledged by the borrower increases (and hence the bank is willing to receive a lower α). Initially (for low interest rate levels) the first component wins, while the second dominates after interest rates pass a threshold which is higher than the house price growth.

At last we determine the impact of an increase in the maturity of the mortgage product, particularly focusing on maturities of at least 20 years (reflecting market practice). As expected, Panel F shows that increasing the maturity decreases the required percentage of pledged income. For 20-year products, when an extra 30% of LTV needs to be added (L =0.7), a high percentage of income (around 50%) is required. However, an extra 10% of LTV only requires a pledging of 20% of future income. Finally for longer products with 40 year maturity the pledged income varies between 6% and 18% to obtain respectively an extra 10% and 30% of LTV.

5.2 Pricing Sensitivity of γ with no Income Loss ($\lambda = 0$)

Panel A in Figure 5 confirms our intuition that the higher the income volatility σ, the higher the risk of arrears (salary can go "more up" but also "more down" for higher σ). Therefore, the contribution to the "security tranche" should be higher, as evidenced by higher γ. Similarly, for higher loan-to-value ratios L, the mortgage payment m_0 is higher, increasing the risk of arrears. Therefore, a higher guarantee contribution is required, as measured by increased γ. On the other hand, for secure streams of revenues, when σ approaches zero, the contract can be tailored so that to select a correct α. As streams are almost predictable in this case, the need to build up a security fund is low and thus γ converges to zero for all loan-to-value ratios L (near the y axis).

[Insert Figure 5 Here]

Clearly, a higher house price H_0 will trigger a higher mortgage payment m_0, thus increasing the risk of arrears and resulting in a higher fraction γ of residual income to be pledged. This is illustrated in Panel B, which also shows a direct relationship with the loan-to-value ratio L.

For the extreme case of loan-to-value L approaching zero, the mortgage becomes a "pure" shared income contract. The annual mortgage payment m_0 becomes nil and the contract looses any fixed income character. Of course, an annual payment is still present, but it is 100% random and it is determined as a proportion α of disposable income P_t. Since the guarantee γ is meant to cover the fixed income portion, it is no longer needed when the contract becomes an equity contract fully contingent on the income stream P_t. Thus $\gamma \to 0$ as $L \to 0$ - see Panel C.

Issues are also found with very short loans. As T becomes smaller, the required payment m_0 becomes higher because the mortgage has to be repaid in a shorter time frame and this increases the risk of arrears. As a consequence, the security fraction γ increases significantly passing from 40 to 20 year maturity. In fact, we cannot even illustrate the numerical simulation for a 10 year maturity product because the payment and thus the risk of arrears is so high that the required γ would reach areas outside our boundary conditions. For example, a loan-to-value ratio around $L = 0.95$ would lead to a γ - computed from (65) - equal to about 8, which is clearly impossible as the value should be included in the range 0 - 1. The effect of maturity T on the further income pledge to the security fund γ can also be seen in Panel F, where the different LTV does not seem to show a significant impact.

For $L = 0.9$ (base case) and $T = 10$, the income flow P is insufficient to accumulate an initial deposit $(1-L)\,H_0$. According to Table 1 we would need α to be greater than one, but this is not possible because the borrower can only save up to the amount of disposable income. Inspecting the denominator in equation (65), we observe that such low income results in a very low value of the cap C compared to the deposit $(1-L)\,H_0$. This gives a negative denominator and a spurious result for γ, which would be negative too. Panel D shows that households with higher income will need to pay a smaller percentage of their future income after the initial pledge α.

Finally, interest rates seem to have an initial negative and subsequent positive effect on γ - see Panel E. We believe that this convex shape as interest rates increase could be the result of a trade off between the positive effect it has on the returns and hence cumulated value of the security fund and an increase of discount rate reducing the call option value embedded in the mortgage product.

5.3 Pricing Sensitivity of α introducing Income Loss ($\lambda \geq 0$)

When we introduce the possibility of income loss, we capture the price increase determined by the increased risk in the mortgage product Panel A in Figure 6 reports two lines of the same colour for each loan to value ratio used for the traditional part of our mortgage product. The upper lower line represents the results without income loss, while the upper line shows the higher percentage of income required due to the introduction of a probability of income loss. The difference between the no-risk to the risky scenario ranges between 3% and 5% for low level of income volatility and between 1% and 3% for higher volatilities. On average borrowers need to pledge a percentage similar to the extra percentage of LTV they want to access using this feature of the product.

[Insert Figure 6 Here]

In line with our expectation, we find that higher levels of λ (i.e. risk of income loss) require a higher percentage of disposable income, with an upper bound at 0.44 and 0.32 respectively for low and high volatilities - see Panel B. Our base case assumes an income loss frequency λ of 1/30. The upper bound pricing shows a 30% pledge is needed for very low income volatilities and this percentage decreases to a minimum of 25% for high volatilities. As the income loss frequency increases, the pledged income also rises to an upper bound of less than 45% as $\lambda \to \infty$. With lambda increasing, we are increasing the frequency of income loss and anticipating it in time (with the event still being one in 30 years time for the mortgage loan).

Hence to increase the severity of income loss we also present our simulations increasing the time η when the borrower is without income. Panel C presents the variation of pledged income considering a jobless time from 1 (base case) to 30 years. Remarkably the extra income to pledge for η increasing from 1 to 5 years is minimal (order of 2%) and it is not significantly affected by income volatility. The upper bound of our pricing is also shown by the line with with $\eta = 30$, where we assume that the borrower never receives an income, loosing his/her job the day after he/she signs the mortgage contract. Clearly this is still a profitable business for a bank considering the frequency of occurence, which makes this individual loss covered by the pledged income of other borrowers within a portfolio context (frequency $\lambda = 1/30$ is our base case)

Finally, Panel D presents a stress test on the income recovery rate, which ranges from full to no recovery. Assuming on average expectations may actually be for full recovery, our base case seems to be conservative, assuming a 3/4 recovery on average. This recovery rate may well be expected for some categories of borrowers with higher (and riskier) salaries. however, the successive income growth may be logarithmic with initial growth immediately after recovery that is in excess of the general income growth g we assume constant in our model. Hence, we our assumptions are in fact more pessimistic than one might expect. On full recovery the percentage of pledged income is reduced by 0.05, ranging between 0.20 and

0.25 depending upon the level of income volatility.

In Figure 7 we report the same simulations for a less stronger (and possibly more plausible) case, where the loan to value ratio is increased from 0.70 to 0.85 (hence α only covers 0.15 instead of 0.30 of extra funding to reach 100% LTV in our mortgage contract) and the recovery rate from 0.75 to 0.80 Panel A shows that the introduction of a job loss (upper line of the same color) increases the pledged income less than in our previous base case by only 1%. However, the upper bound of our pricing drops to less than 0.25 from almost 0.45. In particular, this is reached for low levels of income volatility with income loss frequency $\lambda = \infty$ in Panel B and jobless time $\eta = 30$ in Panel C. The overall pledged income is reduced to 0.17 and a further reduction to 0.15 is achievable if a full income recovery can be assumed - see Panel D.

[Insert Figure 7 Here]

5.4 Pricing Sensitivity of γ introducing Income Loss ($\lambda \geq 0$)

The final part of our analysis presents the main results for the percentage $\gamma$of pledged income accruing in the security fund to cover for income losses. Differently from the previous discussion, Figure 8 shows that lines for L=0.7 are below the ones for L=1.0. Even if this finding may seem counterintuitive, we believe a plausible explanation holds: bankers care for the fixed rate portion m_0 and not the "equity" portion (computed as alpha multiplied by the integral of call options). As a consequence, the "debt" portion is not priced correctly because it assumes zero default risk in our setup. This is why banks would require a further protection γ (accruing into a security fund) in place of hedging themselves. In particular, arrears are measured with regards to the m_0 level: puts are in the money if $P_0 < m_0$, i.e. if salary falls below m_0). A lower L also means a lower $m0$ and thus these put options happen to be less in the money, i.e. there are less arrears, should the income P_0 falls. Thus a lower loan to value ratio means there is proportionally less of fixed rate component (and higher equity component substituted by pledged income) in our mortgage product and thus less risk for banks.

In fact all our mortgages are 100% LTV, thus the interpretation of L is more similar to a debt/equity ratio. For example, L=0.70 means 70% of the mortgage contract corresponds to a debt-type component (classic plain vanilla fixed income product) and 30% is contingent on available excess to a percentage of the borrower's disposable income after the payment of m0, i.e. $\alpha * (P_0 - m_0)^+$. Should income never materialize in the region above m_0, this 30% portion might never be repaid, but the formula for cap (and the bank) obtained the pricing correctly. Banks took the risk at t=0 and they should delta hedge it if macro markets existed, or at least pool it (not all borrowers will lose their job on the same day).

[Insert Figure 8 Here]

All results in the different Panel A, B, C and D are then reflecting numerical solutions as expected with γ increasing with income volatility σ, income loss frequency λ and time without a job η and decreasing with the improvement of income recovery π. We also find inaccessible regions (lighter/transparent lines), where $\gamma \geq 1$). For example: a low π towards zero means almost no recovery after job loss, and hence the bank rejects those prospects. In fact, the threshold $\gamma = 1$ plays a role of revenue constraint and highlights 'unaffordable areas', akin to 3.5 annual salary multiple (now more than 10 for London and some US cities such as New York and San Francisco).

In the final Figure 9 we report the variation of γ to our other assumptions in the presence of income loss (upper lines for each color) compared with the previous case where income loss was assumed away (lower lines). Because of added Poisson risk, these "bundles" gamma must be higher to reassure the banker. Some variables reduce gamma like as for example maturity T: more time gives greater access to the borrower's lifetime revenue, and hence more security to the bank. The same is reflected in income P_0: the higher the level of revenue, the more security is available for the bank, and therefore the lower the gamma requirement to cover for risk of arrears.

[Insert Figure 9 Here]

For typical values of σ, λ, π and η the behaviour of $\Delta(t)$ is illustrated in Figure 10. By construction $\Delta(0) = 0$ because the accumulation process always starts with an empty protection account at time $t = 0$. Also by construction, $\Delta(T) = 0$ because the protective fund is assumed to balance out any losses at maturity T (even if a lack of income loss would result in a positive cumulation of funds at maturity). In particular, we observe that for higher income volatilities (50% rather than 10%) and higher frequencies of income disruption (i.e. as λ increases from 1 in 30 years to 1 in 15 years), the fund becomes more exposed, especially around year $t \approx 20$. Lowering the recovery rate π from 3/4 to 1/4 commands increased initial cumulation of security money into the fund with a peak around year $t \approx 7$ to decrease subsequent exposure. An effect of similar character and magnitude is also observed while increasing the disruption time window η from 1/2 to 2 years.

[Insert Figure 10 Here]

5.5 Further results in a less stress testing environment

So far our product was tested for mortgages with a very high loan to income (LTI) ratio (i.e. 10 in our base case scenario), as to allow borrowers to access bigger and more expensive houses. US and UK banks are currently offering standard mortgages with an LTI up to 5. If our product allowed access to higher levels of debt, a reduction in the loan amount should therefore lead to a much smaller income sharing component (along with a lower "m" component). Table 3 presents the values of α, γ and m_0 for different loan amounts corresponding to LTI of 10.0, 7.5 and 5.0. The base case is reported in the first line. Panel A represents the case where m_0 is computed on a 90% LTV and α and γ are obtained respectively to cover the remaining 10% (which is not due as a deposit as our product has 100% LTV) and to cover for temporary arrears in m_0 payments. When in a realistic case scenario, the LTI value decreases to 5.0 (i.e. reaching the current limit), α drops from just below 11% to 4% while γ significantly decreases from 48% to just below 13%. Finally, if the borrower prefers to reduce the fixed component to an 80% LTV and increase the income sharing component to cover the remaining 20% (i.e. Panel B), m_0 falls from 10,303.3 to 9,158.5, but the lender requires a higher income sharing component, with α going from around 4% to just below 8%, while γ decreases by almost two percentage points to 11% because the potential arrears for the "m" component are smaller. Overall, our mortgage product allows immediate homeownership raising the LTV to 100% and requires a reasonable level of income sharing at the current LTI levels to compensate for the lack of initial downpayment.

[Insert Table 3 Here]

6 Conclusion

Accessibility to homeownership has become a concern in several western countries, with reduced household saving rates, house prices and rents increasing more than salaries and therefore an increasing inability to save for the initial downpayment of a plain vanilla mortgage product. In this paper we present a FRM product that allows an immediate access to the property ladder through an increase of the initial LTV to 100% in exchange of a percentage of future disposable income pledged by the borrower until maturity. The components of the payment are made by a constant amount "m" computed on the original LTV and a variable payment linked to the disposable income of the borrower in the future (which correspond to the value of a call option for the bank). This variable amount is partly paid as a compensation to the lender for the extra LTV ("α" component) and partly cumulated in a saving/retirement account ("γ" component)

The pricing of this product shows that a reasonable percentage of income (19% in our base case) can be pledged to allow immediate homeownership. This product is particularly sensitive to the affordability of the purchased property and the initial income (maybe functioning as a proxy for future income growth). We also find that this product may be useful for lower income households in periods of higher uncertainty and that it may become less expensive in a high interest rate environment.

Clearly this product also embeds an incentive to increase the propensity to save, keeping a protection for consumption related to basic needs (i.e. non-discretionary expenses). In fact, the willingness to become homeowners requires first time buyers to save on the rent before the purchase and on consumption of the "surplus income" after it. In consumption-based societies in the western world (e.g. US and UK), we believe that this effect may also be beneficial for the overall systemic risk of the banking system making households more aware of their lifetime expenditure.

A Appendix: Cap and call formulae

Caps C on flow s with strike flow level k for finite horizon T can be computed using the following closed-form formula (see **?**):

$$C\left(s_0,k,T,r,\delta,\sigma\right) = -As_0^a\left(\mathbf{1}_{s_0>k} - N\left(d_a\right)\right) + \frac{s_0}{\delta}\left(\mathbf{1}_{s_0>k} - e^{-\delta T}N\left(d_1\right)\right) \\ - \frac{k}{r}\left(\mathbf{1}_{s_0>k} - e^{-rT}N\left(d_0\right)\right) + Bs_0^b\left(\mathbf{1}_{s_0>k} - N\left(d_b\right)\right). \tag{66}$$

where

$$\mathbf{1}_z = \begin{cases} 1 & \text{if} \quad z \text{ is true} \\ 0 & \text{if} \quad z \text{ is false} \end{cases} \tag{67}$$

and

$$A = \frac{k^{1-a}}{a-b}\left(\frac{b}{r} - \frac{b-1}{\delta}\right), \\ B = \frac{k^{1-b}}{a-b}\left(\frac{a}{r} - \frac{a-1}{\delta}\right), \tag{68}$$

and

$$a,b = \frac{1}{2} - \frac{r-\delta}{\sigma^2} \pm \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}, \tag{69}$$

whereas the cumulative normal integrals $N\left(\cdot\right)$ are labelled with parameters d_β

$$d_\beta = \frac{\ln s_0 - \ln k + \left(r - \delta + \left(\beta - \frac{1}{2}\right)\sigma^2\right)T}{\sigma\sqrt{T}} \tag{70}$$

(different to the standard textbook notation) for elasticity β which takes one of four values $\beta \in \{a,b,0,1\}$.

Standard Black-Scholes **?** call on S with strike value of K can be computed using

$$c\left(S_0,K,r,\delta,\sigma,T\right) = S_0e^{-\delta T}N\left(d_1\right) - Ke^{-rT}N\left(d_0\right) \tag{71}$$

where d_0 and d_1 can be computed using formula (70) in which values S_0 and K can (formally) be used in place of flows s_0 and k.

Both floor (66) and put (71) formulae assume that the underlying flow s or asset S follows the stochastic differential equation

$$\frac{ds_t}{s_t} = \frac{dS_t}{S_t} = (r - \delta)\, dt + \sigma dZ_t \tag{72}$$

with initial values s_0 and S_0, respectively. Clearly, (72) describes a geometric Brownian motion under risk-neutral measure where Z_t is the standard Brownian motion, σ is the volatility, r is the riskless rate and δ is the payout flow rate.

B Appendix: Tables

Table 1: Numerical Assumptions

Var	Description	Unit	Values			
			low	*base case*	*high*	*extreme*
H_0	Initial house price	£	$300,000$	$400,000$	$500,000$	$700,000$
g	Growth house price	p.a.	0.02	0.04	0.06	0.08
r	Interest rate level	p.a.	0.02	0.04	0.06	0.08
R_0	Annual rent	£ p.a.		$15,000$		
P_0	Annual disposable income	£ p.a.	$15,000$	$20,000$	$25,000$	$30,000$
T	Mortgage term	years	10	20	30	40
L	Loan-to-value	-	0.7	0.8	0.9	1
θ	Initial margin	years		0		
σ	Earnings volatility	p.a.	0.1	0.25	0.3	0.5

Table 2: Simulation Values of Pledged Disposable Income (α), Pledged Residual Income (γ) and Mortgage Payment (m_0)

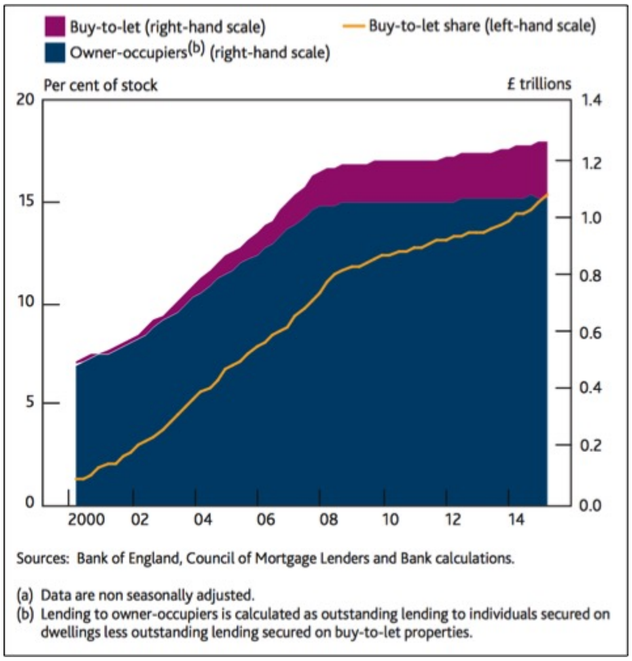
	α	γ	m_0
base case	0.0919242	0.114241	20606.6
σ			
0.1	0.102238	0.00353653	20606.6
0.3	0.0874658	0.161319	20606.6
0.5	0.0734717	0.306144	20606.6
L			
0.7	0.243479	0.0613015	16027.3
0.8	0.172833	0.085863	18317.
1	0.	0.145895	22896.2
H_0			
300000	0.0599177	0.0439504	15454.9
500000	0.131024	0.246099	$25758.2^{(1)}$
700000	0.228381	0.788589	$36061.5^{(1)}$
T			
10	$1.47376^{(2)}$	-10.6654	$43678.7^{(3)}$
20	0.197382	0.385192	$26149.9^{(3)}$
40	0.0588763	0.0616067	18042.8
P_0			
15000	0.210399	0.666923	20606.6
20000	0.131024	0.246099	20606.6
30000	0.0697944	0.0621108	20606.6
r			
0.02	0.0909298	0.124781	15957.9
0.06	0.0917953	0.115608	$25877.5^{(4)}$
0.08	0.0903694	0.13071	$31673.3^{(4)}$
g			
0.02	0.0919242	0.114241	20606.6
0.06	0.0919242	0.114241	20606.6
0.08	0.0919242	0.114241	20606.6

Notes: Pledgable fraction α of net earnings *after* mortgage or rent payment and non-basic spending. The base case is: volatility of net earnings *before* mortgage or rent payment and non-basic spending $\sigma = 0.25$, loan-to-value ratio $L = 0.9$, initial house price $H_0 = 400,000$, term of the mortgage $T = 30$ years, initial net earnings *before* mortgage or rent payment and non-basic spending $P_0 = 25000$, interest rate level $r = 0.04$, house price growth rate $g = 0.04$. Mortgages in cases (1), (3) and (4) should not be granted on the basis of insufficient annual income ($P_0 < m_0$). Cases such as (2) where $\alpha > 1$ which is impossible should be rejected on the basis of insufficient pledgable income net of annual mortgage payment.

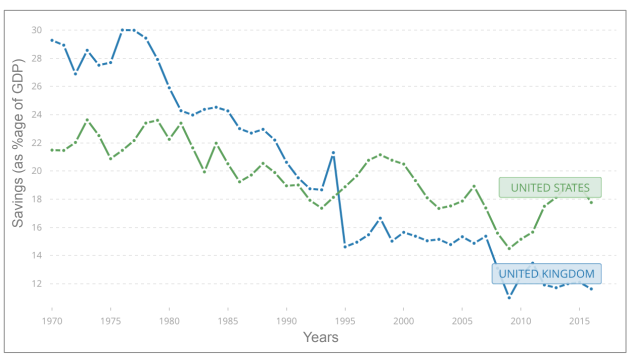
Table 3: Realistic Cases for Lower Levels of Loan to Income (LTI) Ratios

	LTI	α	γ	m_0
base case	10.0	0.108562	0.478244	$20,606.6$
Panel A: H_0 with *L=0.9*				
$200,000$	5.0	0.0398785	0.127084	$10,303.3$
$300,000$	7.5	0.0698282	0.259679	$15,454.9$
$400,000$	10.0	0.108562	0.478244	$20,606.6$
Panel B: H_0 with *L=0.8*				
$200,000$	5.0	0.0771002	0.110768	$9,158.5$
$300,000$	7.5	0.132606	0.225386	$13,737.7$
$400,000$	10.0	0.202924	0.420016	$18,317.0$

C Appendix: Figures

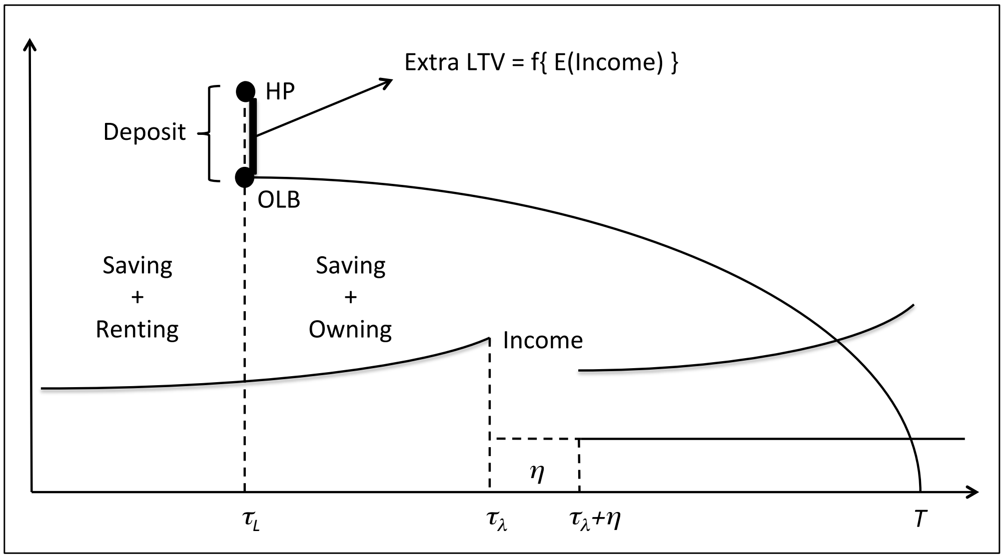


Panel A: UK Buy-to-Let market share

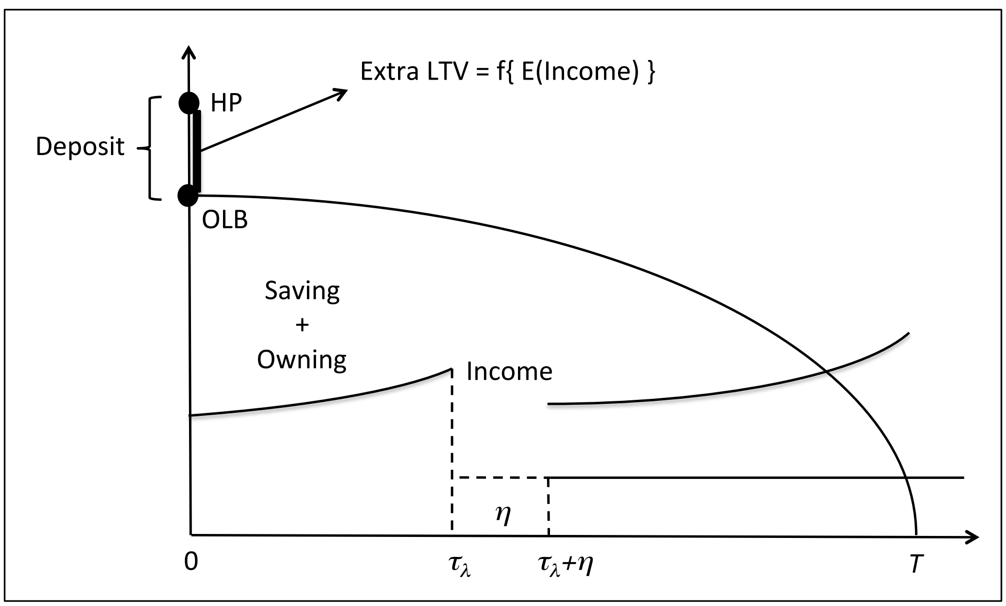


Panel B: UK and US Saving rates as a percentage of GDP

Figure 1: Buy-to-Let and saving rates



Panel A: Mortgage product with income sharing and saving components over the life cycle



Panel B: Modelling of mortgage product with income sharing and saving components

Figure 2: Mortgage risk transfer mechanism over the life cycle

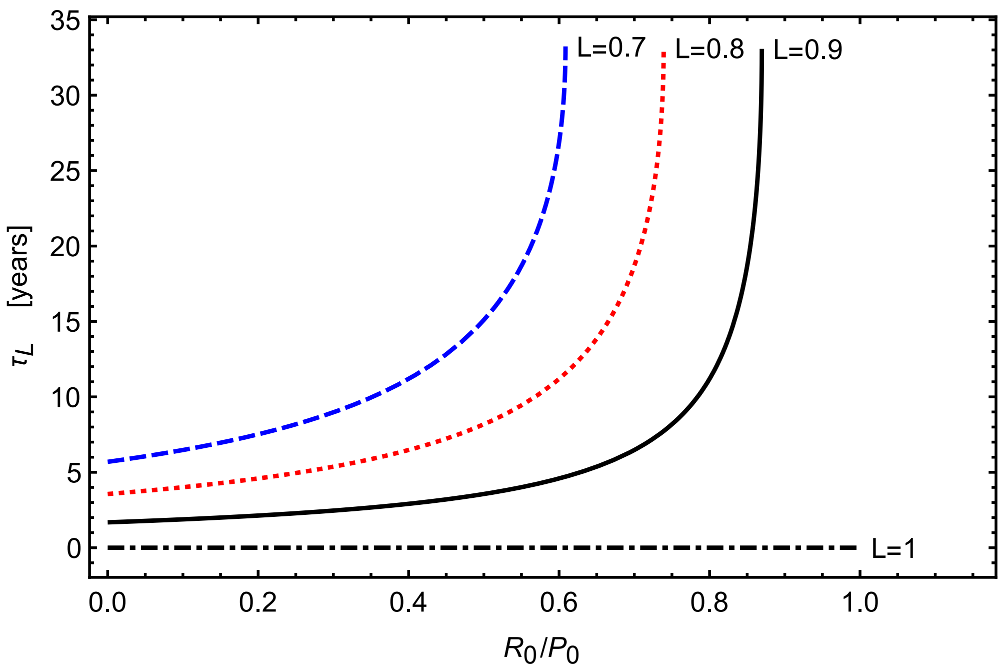
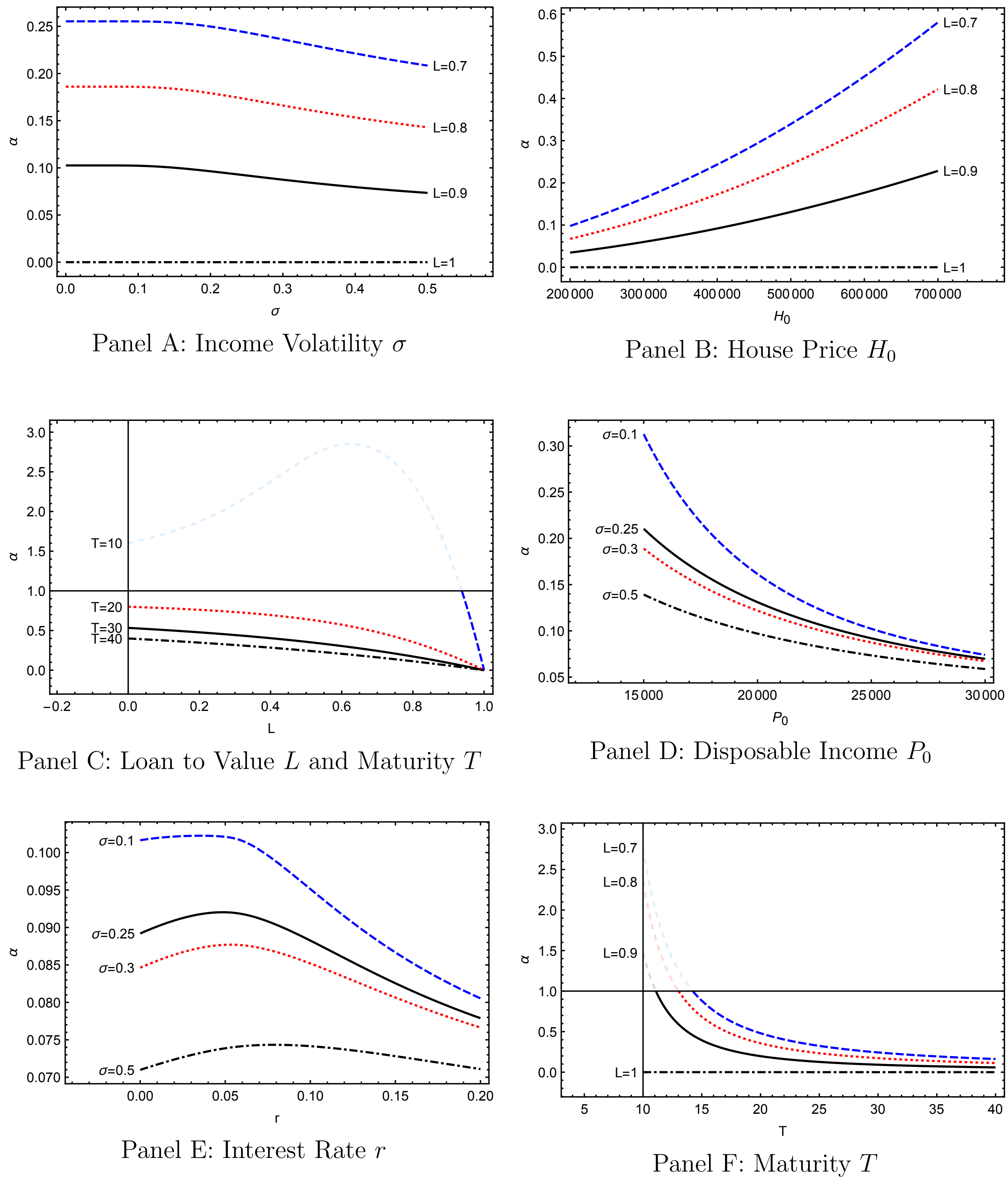


Figure 3: Time to save τ_L as a function of rent to available income $\frac{R_0}{P_0}$ for different values of the loan-to-value ratio L.



Panel A: Income Volatility σ

Panel B: House Price H_0

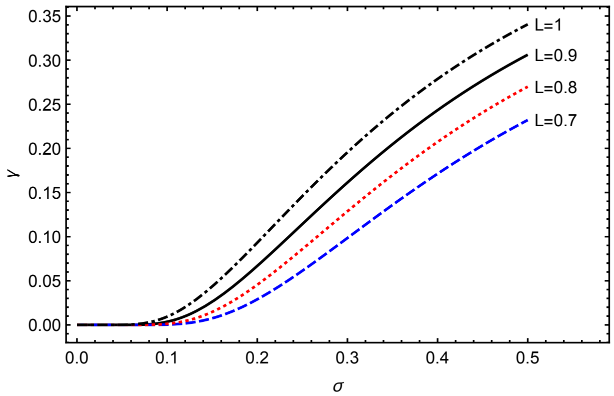
Panel C: Loan to Value L and Maturity T

Panel D: Disposable Income P_0

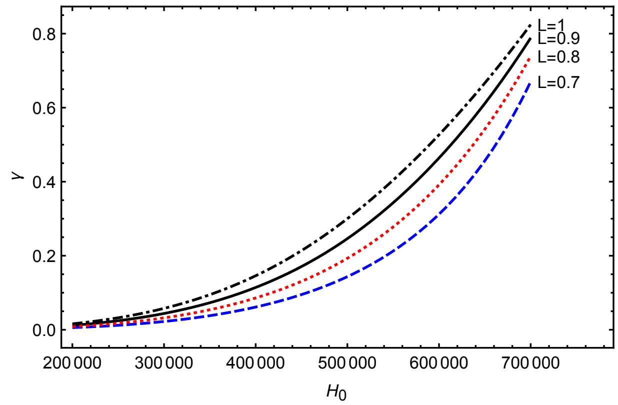
Panel E: Interest Rate r

Panel F: Maturity T

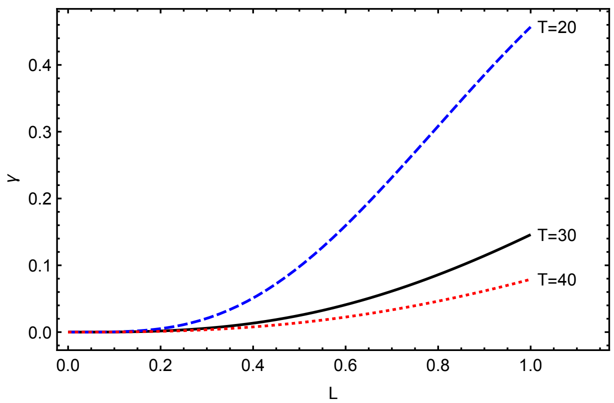
Figure 4: Pledgeable fraction α of disposable income *after* mortgage payment and basic spending.



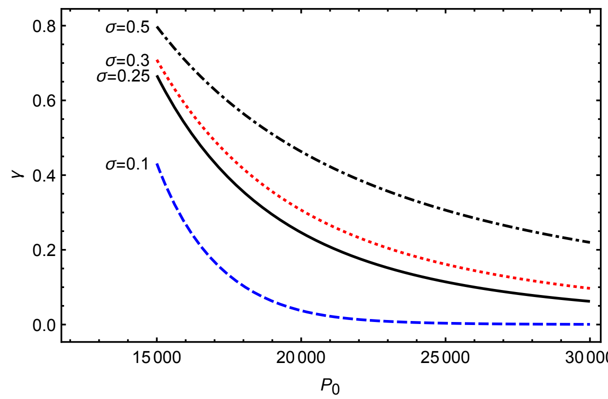
Panel A: Income Volatility σ



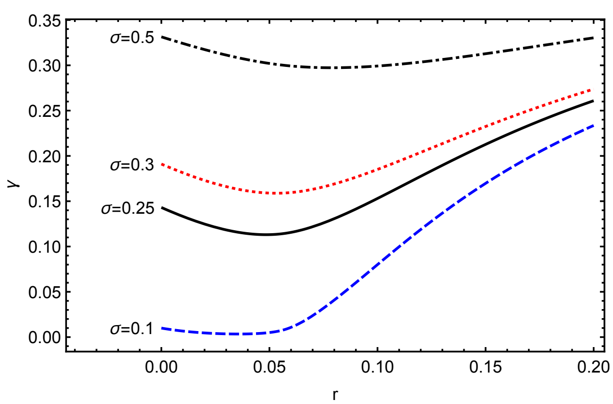
Panel B: House Price H_0



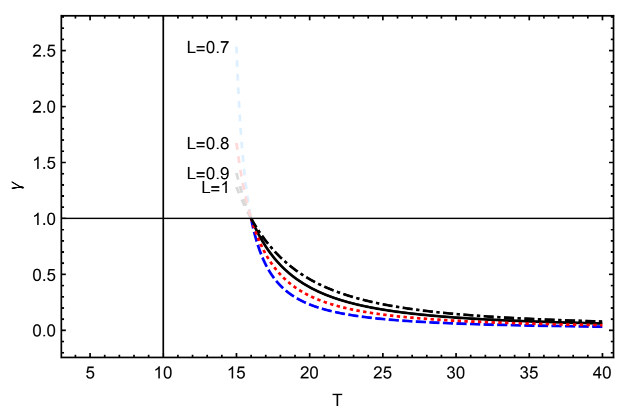
Panel C: Loan to Value L and Maturity T



Panel D: Disposable Income P_0

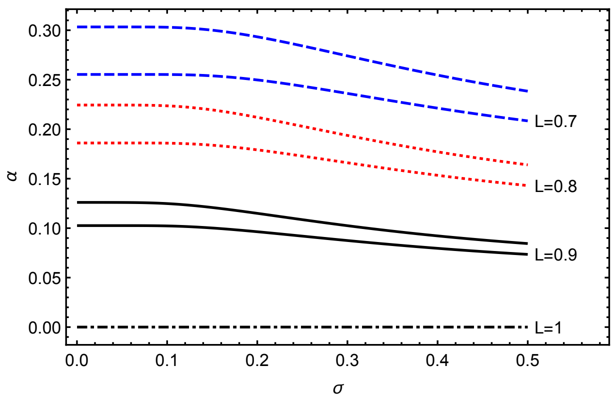


Panel E: Interest Rate r

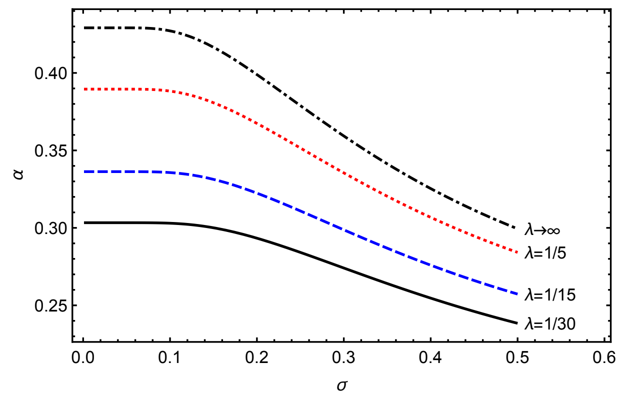


Panel F: Maturity T

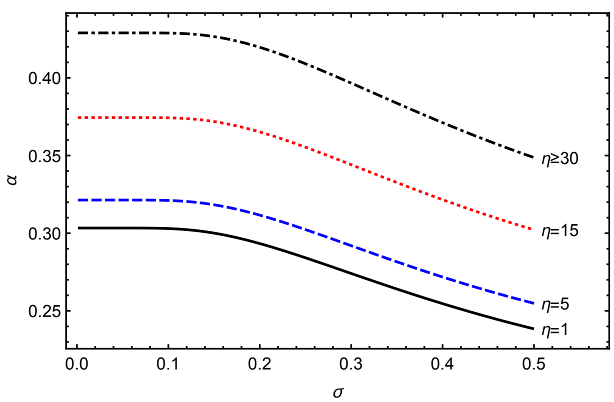
Figure 5: Security fraction γ *after* mortgage and covenant payment.



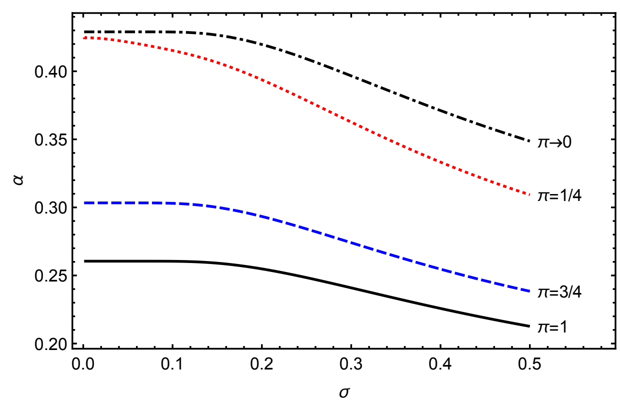
Panel A: Income Volatility σ



Panel B: Income Loss Frequency λ

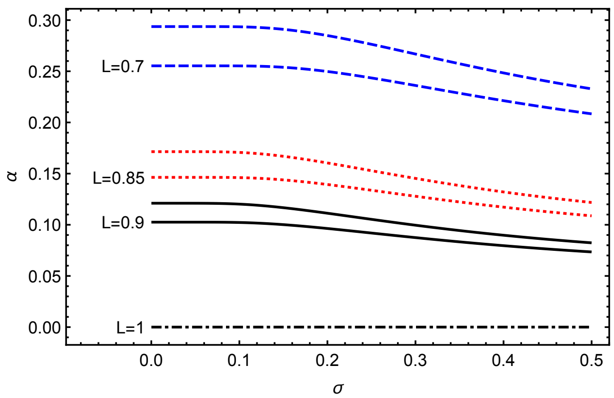


Panel C: Jobless Time η

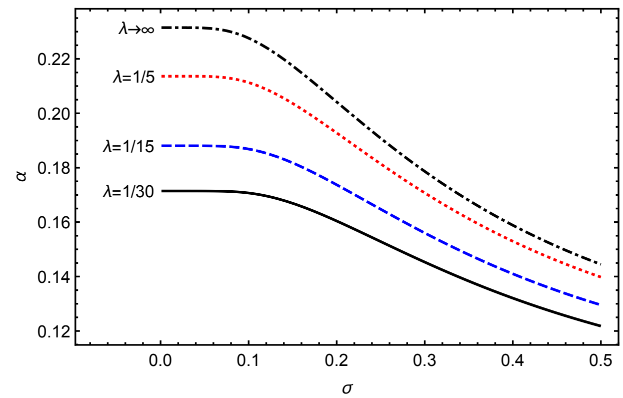


Panel D: Income Recovery π

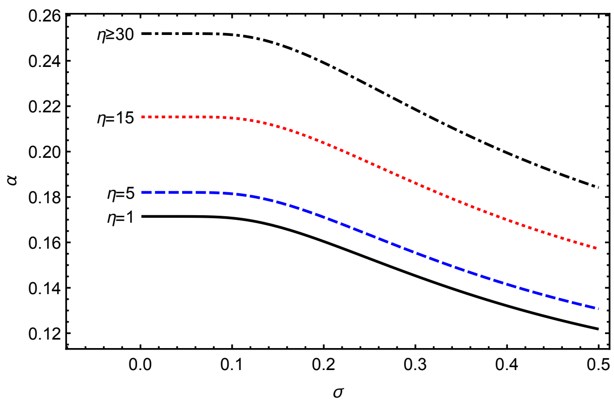
Figure 6: Pledged Income α. Base Case: L=0.7; λ=1/30; η=1; π=0.75



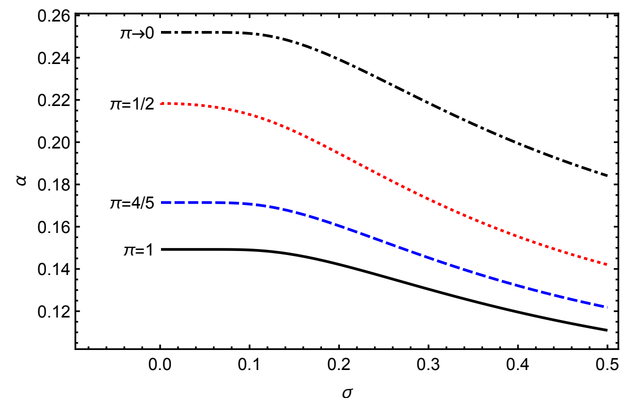
Panel A: Income Volatility σ



Panel B: Income Loss Frequency λ

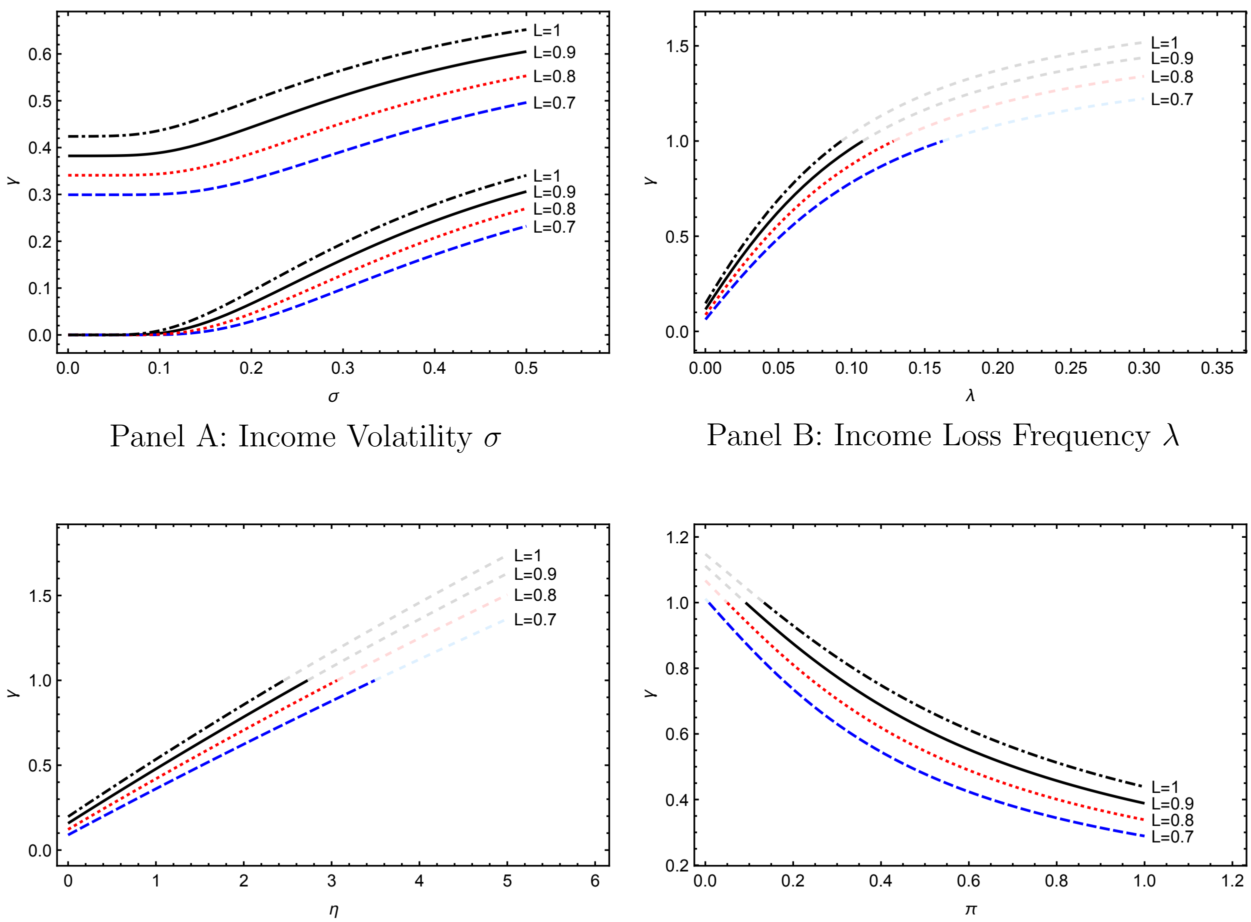


Panel C: Jobless Time η



Panel D: Income Recovery π

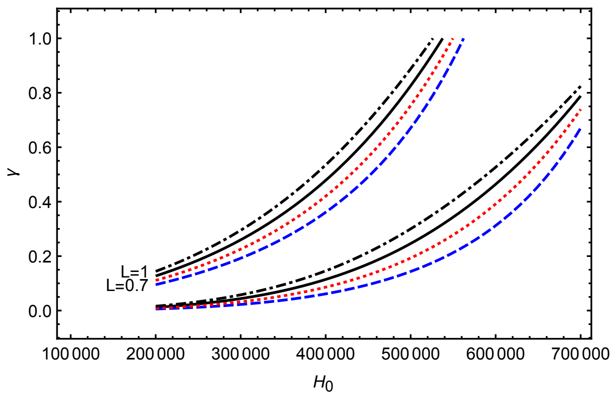
Figure 7: Pledged Income α. Base Case: L=0.85; λ=1/30; η=1; π=0.80.



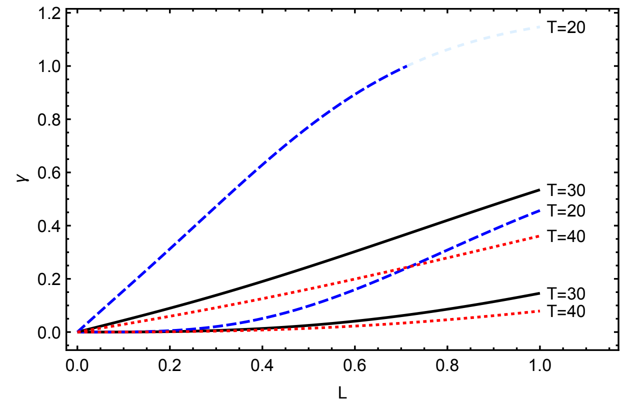
Panel A: Income Volatility σ Panel B: Income Loss Frequency λ

Panel C: Jobless Time η Panel D: Income Recovery π

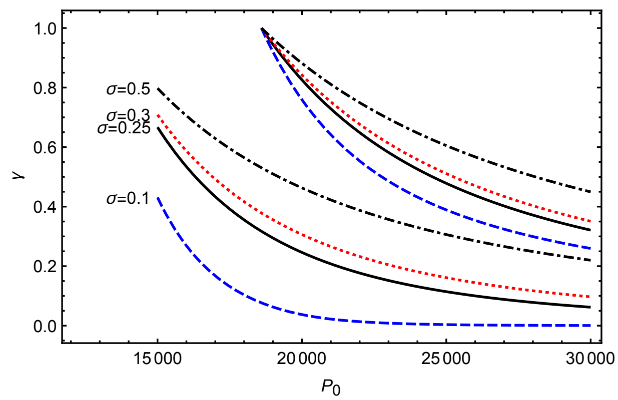
Figure 8: Pledged Security Fund γ. Base Case: L=0.7; λ=1/30; η=1; π=0.75.



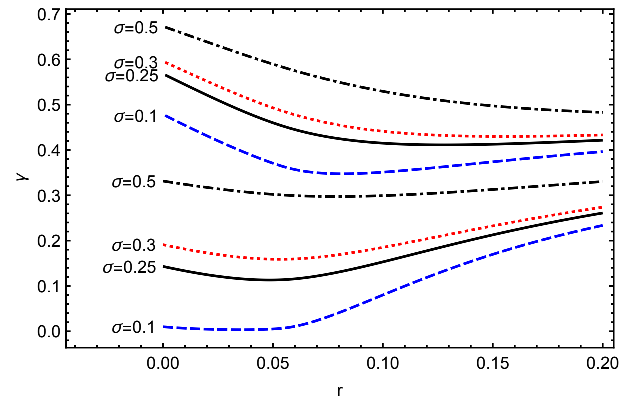
Panel A: House Price H_0



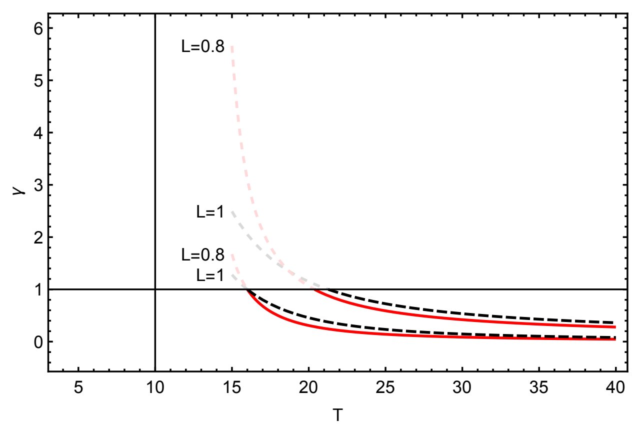
Panel B: Loan to Value L and Maturity T



Panel C: Disposable Income P_0

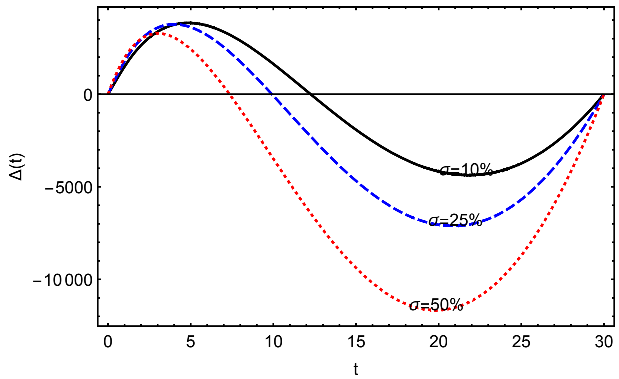


Panel D: Interest Rates r

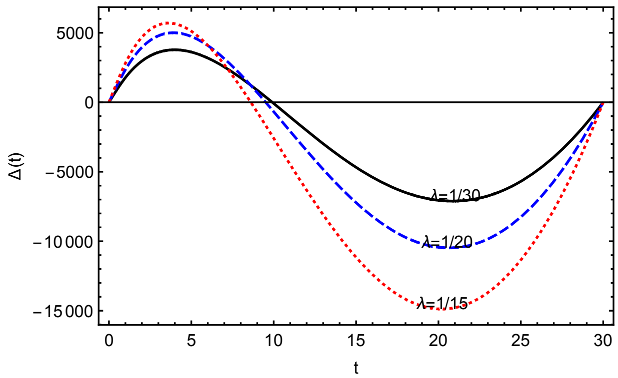


Panel E: Maturity T

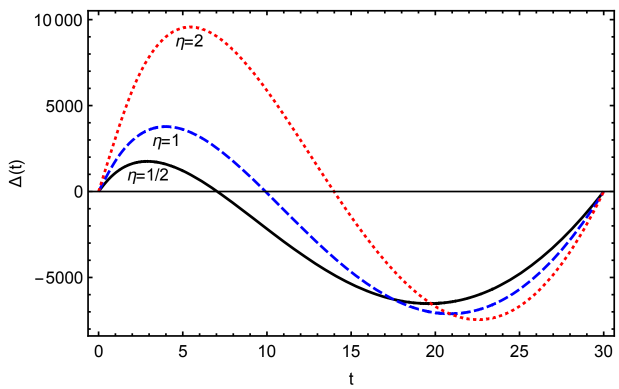
Figure 9: Pledged Security Fund γ. Base Case: L=0.7; λ=1/30; η=1; π=0.75.



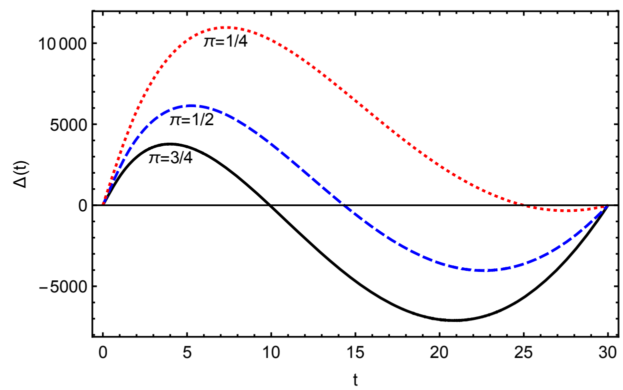
Panel A: Income Volatility σ



Panel B: Income Loss Frequency λ



Panel C: Jobless Time η



Panel D: Income Recovery π

Figure 10: Time profile exposure of the statistical cover $\Delta(t)$. Base Case: L=0.7; λ=1/30; η=1; π=0.75.