A Spatial Model for Market Concentration Measure

Kerem Yavuz Arslanlı
Istanbul Technical University

Christopher Hannum
Istanbul Technical University

Wendy Usrey
Colorado State University

Laurie Dufloth
Home Again Properties

Abstract:
Anecdotal evidence suggests that firms and individuals participating in property-related markets, such as residential brokers, consider that competition among them intensifies during property market downturns. This is at odds with conventional microeconomic theory, which suggests that in a downturn competition will decrease as some firms faced with losses leave the market. We argue that this disconnect may be caused by an unnoticed spatial dimension to competition in these markets, where firms choose the area in which they will operate. We first demonstrate using a spatial agent-based model (ABM) that the desire by a broker to restrict market area to appeal to clients within that local area is consistent with expanded market areas by brokers and hence intensified competition during downturns. We then calculate market areas and shares within those market areas for individual brokers using MLS data and regress the size of market area and concentration within that market area on market condition variables. We find that during downturns, brokers' market areas expand and competition within their market areas increases.

Introduction

Research has shown that a primary role for Realtors© has been to increase the sales price of homes, by increasing demand from home buyers in a similar way to salesmen or advertising agencies (Jud & Frew, 1986; Jud, 1983). Increased market power would allow real estate brokers to further raise sales prices and justify higher commissions. As it is typical in real estate brokerage for firms to compete for clients on quality of service and other considerations rather than price alone (Micelli 1992, Turnbull 1996), there has been some concern in the past that this might signal a form of collusion through which real estate brokers could artificially inflate home prices and/or commissions. This has led to court rulings forbidding the explicit setting of commissions by MLS agencies or the recommendation of commissions by MLS agencies. However, it has since been demonstrated that nationally and in all but a handful of metropolitan area markets Herfindahl-Hirschman Indices of market concentration in real estate brokerage fall well below levels considered problematic for competition (Beck, Scott & Yelowitz, 2012).

Relatively little research has been done on the impact on housing markets of variations in HHI between markets and submarkets and over time. Among submarkets in the Lexington, Kentucky metropolitan area an increase in HHI (indicating less competition between brokers) is shown to increase time on market but to have no significant impact on sales prices (Beck, Scott & Yelowitz, 2012). It has been demonstrated by Turnbull & Dombrow (2007) that greater geographical concentration of listings and sales do increase prices. Taken together, this suggests that an HHI measure taken for an entire city or county may not accurately capture the geographic concentration of brokerage firms' activities and that an alternative market concentration measure which takes into account brokers individual ranges.

Economies of scale and economies of scope give moderate cost advantages to larger brokerage firms (Zumpano, Elder & Crellin, 1993; Zumpano & Elder, 1994). Brokerage firms are more successful when locating offices near areas in which they want to sell and when selling in areas in which they have sold before (Richins, Black & Sirmans, 1987) consistent with the idea of local knowledge or of reputation. Based on survey results, desirable characteristics of a particular agent rather than of a brokerage firm were found to strongly influence selection of a real estate firm, particularly local knowledge and personal connections with the seller or buyer of the home (Johnson, Nourse & Day, 1988). Taken together these findings are broadly supportive of a model for real estate brokerage similar to that of Miller & Shedd (1979), Yinger (1981) and Crockett (1982) who propose theoretical models for the industry akin to monopolistic competition rather than an oligopoly or cartel. In such a framework, individual brokerage firms provide a similar but not identical service which allows higher prices or profit margins for customers who prefer or are loyal to them. In such a market structure easy entry into the market serves to keep brokerage firms inefficiently small (Hsieh & Moretti, 2002; Anderson et al. 1998). It is these findings that form the basis of our theoretical model of spatial competition in the real estate brokerage industry.

This study will consist of three components: a theoretical and simulation model of spatial activity in real estate brokerage, the creation of a measure of market concentration which factors in spatial concentration within a market and analysis of the determinants of variation in this measure over time, and from market to market.

We first present a theoretical model of spatial dynamics in the real estate brokerage industry with simulation results derived using the agent-based modeling software NetLogo. Many brokerage firms compete over listings across space at a fixed price. Individual home sellers have a preference for a nearby brokerage firm, a brokerage firm with spatially concentrated listings and a stochastic preference for given brokerage firms based on personal factors or other service characteristics. The spatial interaction of sellers and agents generates a restricted but non-exclusive operating range for each brokerage firm, similar to what is seen in the industry. The simulation model is used to explore the possible effects of changes in market conditions on the size and shape of brokerage firms' operating ranges as well as market concentrations within them.

Anecdotal evidence suggests that when local housing market conditions worsen, brokers and agents increase their operating range in search of new clients. This would increase the number of brokers competing over a given listing or sale, increasing costs or decreasing commission rates. For the purposes of testing this hypothesis, we propose the creation of a better measure of spatial competition between brokers and statistical analysis of its determinants.

Hotelling-Type Model

The first simulation is a model of real estate brokerage based upon Hotelling's seminal work in location theory (Hotelling, 1929). It draws heavily upon Otto & Wilensky's (1999) Hotelling's Law of retail location choice in NetLogo, with extensions for stochastic preferences for brokers among homeowners and a housing market cycle. In Hotelling's formulation, and most modern game-theoretical interpretations and extensions (-), consumers have preferences regarding distance to a seller as well as the price charged for the homogeneous good. Buyers choose one seller taking both into consideration. Sellers participate in what reduced to a two-stage, repeated game; first simultaneously choosing location and then simultaneously choosing price in order to maximize expected profits (or revenues assuming constant marginal cost). Whereas Hotelling's original formulation imagined a clear-cut spatial equilibrium, more recent research (d'Aspremont, Gabszewicz & Thisse, 1979) has demonstrated this to be false. More complex variations, particularly those involving many agents, tend towards multiple or no spatial equilibria (Osborne & Pitchik, 1987) with agents constantly in motion and market areas in flux.

The Hotelling-type brokerage model imagines a 41 x 41 grid of homeowners with identical homes which will be put on the market with probability P (set by the slider "conditions" in the model) in each period. Each homeowner who puts their home on the market will choose one of ten brokers i with the following formulation: $min_i\left[P_i + Distance_{i,h} + pref_{i,h}\right]$ where $pref_{i,h}$ represents a stochastic preference for broker i by homeowner h. $Pref_{i,h}$ is normally distributed with mean 2 and standard deviation s. Broker location is initially determined randomly. Each round, brokers evaluate each potential move of distance 1 simultaneously and choose the location with the highest expected revenue. They then simultaneously choose the revenue maximizing price. Simulation based market area research is not a novelty, be they individual agent models (Pasek & Mehrjoo, 2010) or multi-agent systems (Kuscsik & Horvath, 2007) such as this.

Each broker's market area is determined by first calculating the distance to their farthest client and then the area within the radius defined by that distance. A broker's market share within her own market area is defined as the broker's number of sales or listings as a proportion of the total within their respective market area during that round. For example, a broker with 45 transactions and a market area which encompasses 300 homes which transacted during that period will be considered to have a market share of 15%. Simulations are run using the Hotelling-type brokerage model for "conditions" from 1 to 10 and for values for "s" ranging from 0 to 9. A model run with conditions = 10 and s = 0 reproduces the Otto & Wilensky (1999) Hotelling's Law model results. S=0 mimics many models of market area (Mills & Lav, 1964; Erlenkotter, 1989) in which a geometric market area can be clearly defined in which there is no competition. Simulations with S>0 are similar to a variety of market areas with probabilistic demand (O'Kelly & Miller, 1989; Webster & Gupta, 1995) although here probability contours are never explicitly mapped. Figure 1.1 (below) shows a single-period equilibrium for conditions = 1 and s = 2, such that each homeowner has only a 10% probability of putting their home on the market. Figure 1.2 shows a single-period equilibrium for conditions = 9 and s = 2, such that each homeowner has a 90% probability of putting their home on the market. As such models use abstract definitions of time and space it is difficult to say which conditions are most representative of a typical housing market.

Figure 1.1

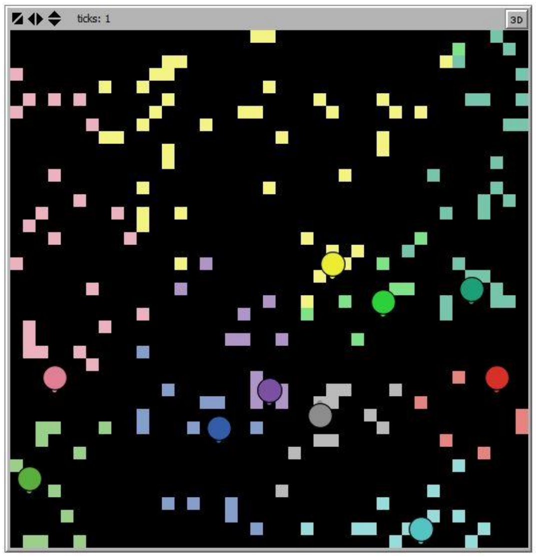
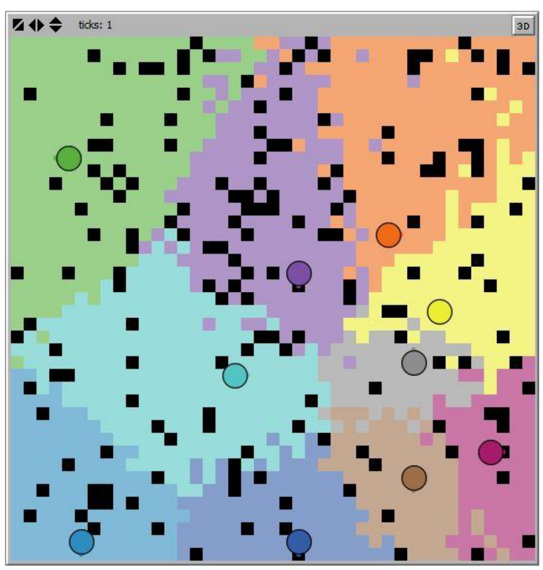


Figure 1.2



The key output of these simulation models for the purposes of this study is the average market share of brokers and the relationship between the average market share of brokers and economic conditions. Figure 1.3 (below) shows average market shares by "conditions" ranging from 1 to 10 and "s" ranging from 0 to 9. As "S" represents the standard deviation of the stochastic preference term, it is unsurprising that simulations run with low values for the “S” parameter result in lower average market shares. When S is large, the likelihood increases that faraway homeowners will prefer a given broker in spite of their general preference for proximity. When S is small, preference for proximity is more likely to dominate.

Figure 1.3

		S									
		0	1	2	3	4	5	6	7	8	9
Conditions	1	48%	50%	37%	36%	29%	26%	23%	17%	18%	17%
	2	47%	40%	37%	32%	30%	21%	19%	15%	16%	16%
	3	41%	38%	31%	27%	25%	22%	18%	16%	14%	13%
	4	39%	40%	34%	30%	27%	21%	19%	15%	14%	12%
	5	40%	44%	30%	25%	24%	22%	16%	16%	12%	12%
	6	36%	35%	32%	28%	24%	20%	18%	15%	14%	11%
	7	38%	32%	31%	25%	22%	17%	15%	13%	14%	12%
	8	41%	37%	34%	25%	25%	19%	14%	14%	13%	12%
	9	44%	38%	31%	26%	22%	18%	15%	14%	12%	12%
	10	38%	33%	31%	29%	24%	16%	17%	13%	12%	12%

Figure 1.4 (below) shows the results of simple OLS regression, with robust standard errors

Figure 1.4

	Dependent variable:
	Average Broker Market Share
Time	0.0003^{***}
	(0.0001)
Conditions	-0.009^{***}
	(0.0004)
S	-0.036^{***}
	(0.0005)
Conditions:S	0.0004^{***}
	(0.0001)
Constant	0.437^{***}
	(0.003)
Observations	3,575
R^2	0.871
Adjusted R^2	0.871
Residual Std. Error	0.038 (df = 3570)
F Statistic	$6{,}021.336^{***}$ (df = 4; 3570)
Note:	$^{*}p^{**}p^{***}p<0.01$

In Hotelling-type model experiments, average market shares increased over time, albeit slowly, as brokers move from their initial, random locations to seek higher revenues. Average market shares predictably decrease sharply with higher values for S. As housing market conditions worsen, average market shares (within the broker's own radius) increase. The change resulting from a decrease from maxium (10) to minimum (1) for conditions is slightly less than an eight percentage point increase in average market share. The interaction term between S and conditions has a positive and significant impact on average market share. However, for relevant values of S the overall effect of conditions remains negative ranging between -0.008687 and -0.005084.

A Hotelling-type model is not consistent with the anecdotally observed phenomenon of decreasing market shares and increasing competition due to poor economic conditions. When market area is driven purely by homeowners' desire for proximity and stochastic preference, the effect of poor economic conditions is to reduce the likelihood of a faraway homeowner choosing a given broker at any point in time. This is particularly true for conditions values at the low end of the spectrum. In order to explain such a phenomenon as increasing competition due to economic conditions it would be necessary to make significant modifications to the assumptions of the model, particularly the homeowner's preference for distance or the broker's own willingness to take on a listing.

Service Radius Model

The second model simulated differs from the standard stochastic Hotelling model in important ways. Homeowners are assumed to be have no preference for proximity to an agent, which is the driving force in the Hotelling model. Rather homeowners are assumed to be limited in their selection of agents by the agent's service areas, which are defined by a radius around the location of the agent. Homeowners are assumed to prefer agents with a small radius, assumed to correspond to local concentration and local knowledge. Brokers sequentially choose location, a service radius and price in order to maximize expected revenues. As in the Hotelling-type model, homeowner preferences are assumed to include a stochastic component. Simulations are run for a variety of values of S and conditions.

Figure 1.5

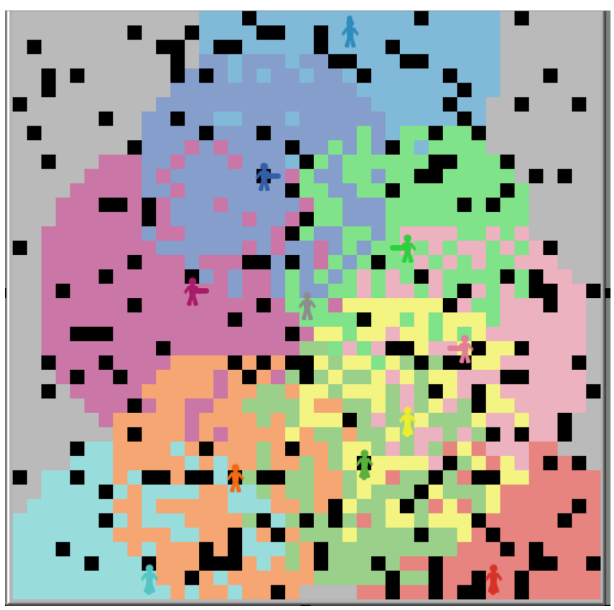


Figure 1.6

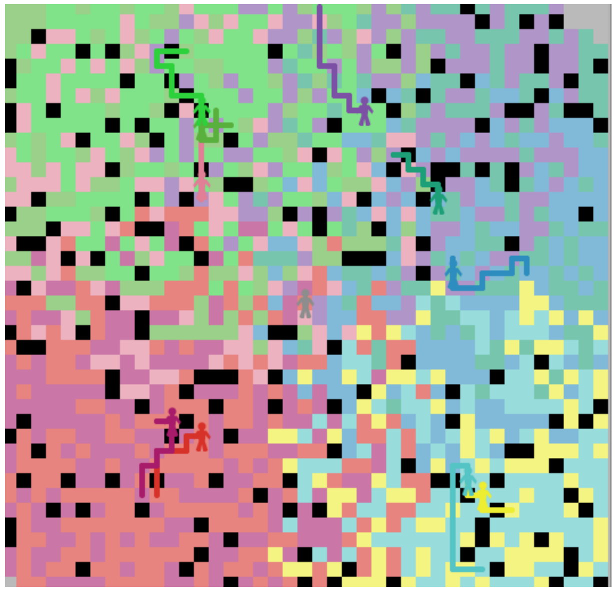


Figure 1.7

		S									
		0	1	2	3	4	5	6	7	8	9
Conditions	1	60%	54%	45%	39%	40%	36%	32%	29%	29%	24%
	2	50%	51%	47%	42%	39%	35%	33%	27%	29%	26%
	3	52%	44%	47%	41%	39%	36%	32%	31%	27%	23%
	4	53%	52%	43%	42%	37%	37%	32%	29%	26%	25%
	5	56%	51%	39%	45%	40%	34%	31%	30%	23%	24%
	6	50%	46%	49%	41%	38%	35%	30%	26%	26%	24%
	7	51%	48%	49%	44%	38%	35%	34%	29%	26%	24%
	8	47%	50%	46%	44%	40%	34%	31%	30%	27%	23%
	9	57%	48%	50%	44%	39%	36%	33%	29%	26%	26%
	10	61%	51%	49%	41%	39%	35%	34%	28%	25%	25%

As shown in figure 1.8 (below) in the radius model Average Market Share is higher with good economic conditions than with bad economic conditions unlike in the Hotelling model. However, with a negative interaction term between conditions and S not only is the positive effect of economic conditions stronger for low values of S there is a tipping point at a value of S of 6.39 where the overall effect of conditions on AMS becomes negative. As in Hotelling the preference standard deviation (S) causes AMS to fall.

Figure 1.8

	Dependent variable:
	Average Broker Market Radius
Time	-0.00001
	(0.00002)
Conditions	0.002***
	(0.001)
S	-0.031***
	(0.0005)
Conditions:S	-0.0003***
	(0.0001)
Constant	0.550***
	(0.003)
Observations	10,000
R^2	0.853
Adjusted R^2	0.853
Residual Std. Error	0.040 (df = 9995)
F Statistic	14,531.640*** (df = 4; 9995)
Note:	*p**p***p<0.01

Radius results:

Average Market Share is higher with good economic conditions than with bad economic conditions unlike in the Hotelling model. The coefficient on sellers/conditions is significant, though not by a large margin (considering 10000 obs.). As in Hotelling the preference standard deviation causes AMS to fall. In radius model, the average radius (rar) falls as economic conditions improve and rises as economic conditions worsen. Price - an alternative if imperfect measure of competition in these models - rises when economic conditions worsen. Price also rises when PSD falls. The highest prices would be observed when individuals have the smallest radii and are indifferent between brokers. The proportion of homes sold "by owner" (not by any broker), which is zero in the Hotelling version of the model, is positively related to conditions and negatively related to PSD. This means that when economic conditions are good, a larger proportion of total sales go to for-sale-by-owner (FSBO) and when economic conditions are bad, a smaller proportion of total sales go to FSBO.

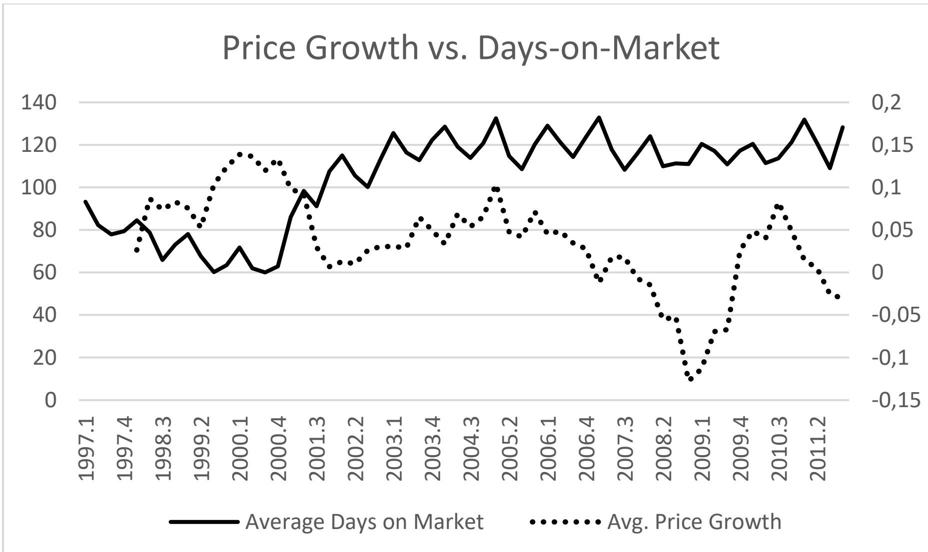
Empirical Model:

Methodology:

In order to empirically assess the degree of competition between residential brokers in their individually selected market areas as well as how this competition evolves over time, we began with a sample of MLS data from IRES, LLC covering housing sales in Northern Colorado for the years 1997 to 2011. This dataset encompasses more than 200,000 individual transactions as well as several thousand brokers. As

Northern Colorado was affected more strongly than the United States at large by the dotcom boom and bust, these 15 years include two pronounced cycles of the housing market.

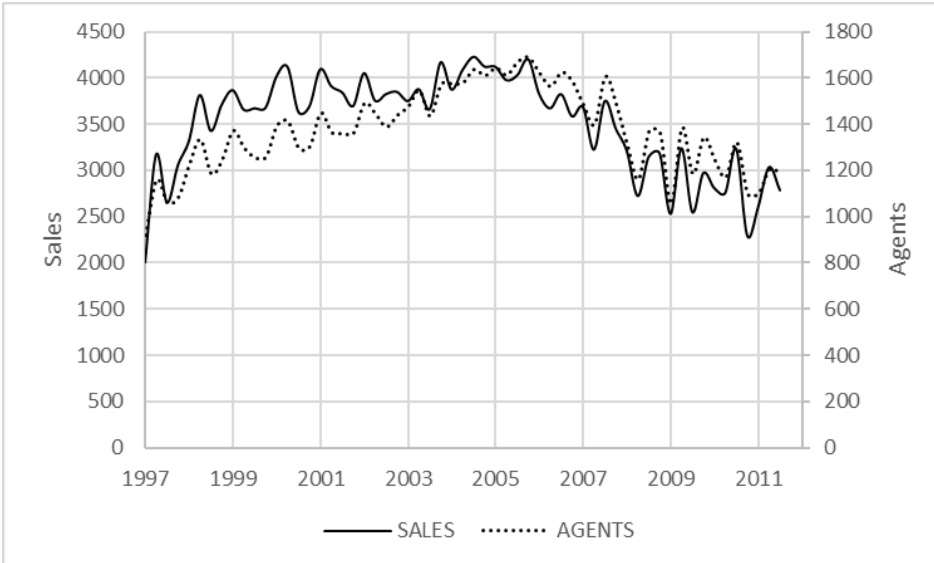
Figure 2.1



Source: IRES, LLC

We use as our basic agent definition the individual broker responsible for the listing, rather than the office to which the listing agent belongs or the office responsible for the sale (which may or may not be the same office). Earlier has demonstrated that preferences for specific listing agents are most important in customer choice and that listing agent market concentration may be more likely to have relevant market impacts than selling agent concentration. The number of agents with at least one listing, as seen in figure 1.8 below, has broadly moved in tandem with total sales with a (seasonally adjusted) quarterly correlation of (+) 0.823. This is exactly the pro-cyclical rather than counter-cyclical competition predicted by conventional economic theory.

Figure 2.2



Source: IRES, LLC

To provide as close a match in measurement of broker market areas as possible to that employed in the simulation-based analysis above, a decision was made to calculate circular market areas. Using data for transactions by each broker, in each quarter, a circular area was defined for each broker in each quarter provided that he or she was involved in at least three transactions during that quarter. Based on the listed latitude and longitude for each property sold, the midpoint of the brokers circular market area during that quarter is taken to be the average of the latitudes and the average of the longitudes of each home sold by that broker during that quarter. The radius of the circle is calculated as the maximum linear distance between the estimated midpoint and any one of the homes sold by that broker during that quarter. This radius is calculated only for a subsample of total brokers, those with 3 or more listings during that particular quarter. Over the 15 years of the sample, this averaged 28.6% of total brokers with listing activity during the quarter but 63.5% of total listings. If the radii are calculated annually, we are able to define market areas for 54.5% of agents responsible for 89.2% of total listings.

Broker market shares within the entire Northern Colorado region are calculated as a ratio of sales by that broker during that quarter to total home sales in the entire Northern Colorado region. However, many brokers will not operate in the whole of the Northern Colorado region but rather in a self-selected smaller market area that we have endeavored to estimate using a circular market area. All homes sold by that particular broker in that quarter will fall within that market area. We then determine which market area(s) each home sold during that quarter falls within in order to calculate a total number of homes sold within each broker-specific circular market area. Broker market shares within their own circular market area are then calculated as the ratio of sales by that broker during that quarter to the total number of homes sold during that quarter which were located within their circular market area. We consider that this provides a better estimate of true market share, market concentration and the degree of competition than do market shares calculated for arbitrarily defined regions such as cities or counties. We compare the two market share measures by calculating the ratio of market share within the brokers radius to market share in the overall region. We find that the median ratio in a given quarter is, on average, 6.6.

Figure 2.3

Descriptive Statistics

Statistic	N	Mean	St. Dev.	Min	Max
Median Market Radius	60	0.107	0.019	0.065	0.167
Median # of Sales In Radius	60	549.6	157.3	160	846.5
Median Regional Market Share	60	0.001	0.0003	0.001	0.003
Median Market Share Within Radius	60	0.009	0.003	0.006	0.025
Median Market Share Ratio	60	6.603	1.121	4.463	9.766
Quarterly Average Home Price	60	250,300	39,277	170,678	302,090
Year-on-Year Average Price Growth	56	1.036	0.058	0.872	1.139
Average Days-on-Market	60	104.3	21.67	60.04	132.9
Quarterly Total Sales	60	3,554.4	884.5	1,562	5,263
Quarterly Total Attached Sales	60	688.2	218.2	258	1,105
% of Attached Sales	60	0.191	0.017	0.155	0.240
Average Transactions Per Broker*	60	5.658	0.313	4.326	6.391

Results

The results of our regressions of broker market radius on market descriptives and broker market share ratio (in-radius to regional) on market descriptives are displayed below. Radius and ratio are modelled as simple linear functions of the independent variables using OLS. It is assumed that a larger proportion of attached homes in the overall transaction mix during that quarter will decrease the size of market areas and increase market shares within them as attached homes cluster. It is expected that a larger average number of transactions per broker (among brokers with at least 3 transactions) will correspond to a larger market radius but have an ambiguous impact on market share ratio.

Two market cycle variables are included, the rate of growth in the average sales price of a home (calculated as the ratio of the average during that quarter and the average during that quarter in the preceding year) and the average number of days that a home sold during that quarter remained on the market prior to sale. A robust housing market is indicated by low Days-on-Market combined with high growth in house prices. Three quarter dummies are included in order to control for seasonal effects. Three sets of regressions are run, the first including only price growth as a cycle variable, the second including only days-on-market and the third including both price growth and days-on-market.

Figure 2.4

	Dependent variable:		
	Median Broker Market Radius		
	(1)	(2)	(3)
		(OLS)	
1st Quarter	0.005	0.003	0.003
	(0.005)	(0.004)	(0.004)
2nd Quarter	0.009	0.007^{**}	0.008^{**}
	(0.006)	(0.003)	(0.004)
3rd Quarter	0.012^{**}	0.015^{***}	0.014^{***}
	(0.005)	(0.003)	(0.003)
Average Transactions per Broker	0.019^{**}	0.029^{***}	0.023^{***}
	(0.010)	(0.004)	(0.006)
% Attached	-0.412^{***}	-0.344^{***}	-0.353^{***}
	(0.135)	(0.065)	(0.073)
Year-on-Year Average Price Growth	-0.138^{***}		-0.064^{**}
	(0.018)		(0.030)
Average Days-on-Market		0.0005^{***}	0.0004^{***}
		(0.0001)	(0.0001)
Constant	0.213^{***}	-0.049^{*}	0.069
	(0.068)	(0.025)	(0.062)
Observations	56	60	56
R^2	0.654	0.784	0.783
Adjusted R^2	0.611	0.759	0.751
Residual Std. Error	0.011 (df = 49)	0.009 (df = 53)	0.009 (df = 48)
F Statistic	15.414^{***} (df = 6; 49)	31.997^{***} (df = 6; 53)	24.716^{***} (df = 7; 48)
Note:	*p**p***p<0.01		

The results of the radius regressions above indicate that a long period on the market for the average home leads to a larger median broker radius and that falling housing prices lead to a larger broker radius. Both are statistically significant at the 1% level. As expected, a larger average number of transactions among brokers in the sample (restricted to those with at least three transactions) leads to a larger average radius as does a smaller proportion of attached properties in the quarterly transaction mix.

Figure 2.5

	Dependent variable:		
	Market Share Ratio		
	(1)	(2)	(3)
1st Quarter	-0.355	-0.125	-0.220
	(0.279)	(0.203)	(0.243)
2nd Quarter	-1.094^{***}	-0.942^{***}	-1.058^{***}
	(0.304)	(0.206)	(0.232)
3rd Quarter	-1.255^{***}	-1.380^{***}	-1.333^{***}
	(0.249)	(0.113)	(0.155)
Average Transactions per Broker	-0.653	-1.596^{***}	-0.821^{*}
	(0.593)	(0.260)	(0.443)
% Attached	21.071^{***}	16.659^{***}	17.991^{***}
	(7.854)	(3.786)	(4.781)
Year-on-Year Average Price Growth	9.060^{***}		5.230^{**}
	(1.087)		(2.052)
Average Days-on-Market		-0.026^{***}	-0.019^{***}
		(0.004)	(0.005)
Constant	-2.529	15.829^{***}	4.924
	(4.549)	(1.800)	(4.888)
Observations	56	60	56
R^2	0.656	0.756	0.754
Adjusted R^2	0.614	0.728	0.718
Residual Std. Error	0.634 (df = 49)	0.584 (df = 53)	0.541 (df = 48)
F Statistic	15.552^{***} (df = 6; 49)	27.377^{***} (df = 6; 53)	20.981^{***} (df = 7; 48)
Note:			*p**p***p<0.01

Results from the median market share ratio regressions indicate that as broker radii grow when housing market conditions are weak, the median ratio of in-radius market share to overall regional market share decreases. High average price growth corresponds to a high ratio, indicating a larger difference between market concentration in the region as a whole and within the self-determined market areas of individual brokers. The ratio decreases as days-on-market rises. Both cycle variables are statistically significant at the 1% level in each regression. A high proportion of attached properties in the transaction mix is found to lead to a high ratio, relatively more concentrated broker market areas.

Conclusions:

Firstly, our findings should lay to final rest any argument that the market for residential brokerage is unfairly concentrated and non-competitive – at least at the level of individual agents. We focus only on the market shares of individual agents (with at least 3 sales in the quarter) within their circular market areas. We find that while by this market definition average or median market shares are substantially higher, but in no quarter do they approach a level which would be deemed inappropriate by regulatory bodies. Competition in the market for residential brokerage services appears generally robust.

As concerns the behavior of competition in residential brokerage over time and across the housing market cycle, we find at the aggregate level that our findings lend support for the argument that competition between brokers may intensify during housing market downturns. While the overall number of brokers participating in the market may be constant or fall when demand is weak, as predicted by general microeconomic theory, this is not the sole determinant of the extent of competition. During downturns, brokers remaining in the market appear to expand their market areas, seeking or accepting commissions farther afield. This increase in radius, due to the housing market cycle, leads to a decreased ratio of in-radius market share to general market share. While each broker may, in fact, compete with fewer other brokers in the region as a whole during downturns they may be competing with more other brokers within their own self-determined market area as they expand their own areas and as other brokers likewise expand into theirs.

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