Commercial leases, terms and options in the light of game theory*

Charles-Olivier Amédée-Manesme[†]**, François des Rosiers, Philippe Grégoire**

Université Laval, Department of Finance, Insurance and Real Estate
Pavillon Palasis Prince, G1V 0A6 Québéc ,QC, Canada

Early draft version: do not quote

June 23, 2017

Abstract

In this paper, we analyse the drivers of the office lease market with a particular emphasis on components such as lease duration and leased area as rent determinant. Using a *leasing game*, we built-up a conceptual framework to determine the terms of the leases –regarding duration, size of premises, conditions– negotiated between landlords and tenants. This model allows us to derive conclusions on the leasing market structure and in particular on bargaining power. In particular, we show why longer leases can command a premium under some market conditions. The conditions leading to opposite results are also discussed in the paper. Using data from Costar for New York City and Chicago, we discuss lease rates in light of our theoretical model. In particular, we find that, for both markets, the rent-size relationship is not monotonic but rather U-shaped, with unit rent, hence the premium to the landlord, rising beyond a given surface threshold.

Keywords: Leases, Bargaining power, Lease length, Tenants.

JEL codes: C60, G32, R30, R33.

*The authors acknoledge generous support from the Social Sciences and Humanities Research Council of Canada Insight Program and from the Chaire d'assurance et de services financiers L'Industrielle-Alliance in Laval University.

†Contact author: charles-olivier.amedee-manesme@fsa.ulaval.ca

1 Introduction

A commercial lease[1] is a legally binding contract made between a landlord and a business tenant for for occupying space within a commercial real estate asset. The lease gives a tenant the right to to use a specific property for a given business or commercial activity and for a given period of time. The leases outline the rights and responsibilities of both the landlord and tenant during the lease term. Leases contracts are the basis of the value of commercial real estate property as they reflect the capacity of a property to generate cash-flows. The rent paid by a tenant is the principal cash flow generated by the property and thus corresponds to the main building block of commercial real estate valuation, investment and analysis. Basically, and by analogy with bond valuation, the value of a given piece of property is obtained by dividing its yearly stabilized net operating income (the net rent) by the relevant market yield (or, inversely, by multiplying it by the corresponding multiple). Being at the basis of rental flows, the lease contract is therefore of primary importance in commercial real estate.

The rent paid by a tenant is the primary cash flow in commercial real estate and is thus the most influential building block in the valuation of office properties. Office leases vary with property types and countries: length, expenses and services are non-standard and can be subject to local practices, market specificities and bargaining power. There exist an infinity of lease contracts and almost each contract is a special case.[2] However, the most important points can often be reduced to the rental value, lease duration and flexibility and the tenant's signing incentives. The cost of rent is negotiable and depends on both the bargaining power of each party and the offers available on the market.

A tenant usually cannot terminate a commercial lease before the term is over without facing some penalties, unless the lease allows for early termination. While this assertion is generally accepted, we argue in this paper that it is not always the case and that the office rent-setting process is dependent upon the local market state and structure, the bargaining power of parties as well as on the size and type of tenants. First, longer lease contracts bring more security to the landlord who is therefore more prone to providing the tenant with a rent discount. Second, a tenant is likely to negotiate a discount for waiving the termination option embedded in the lease. We show in this work how this is not always the case given market structure, bargaining power, size of premises and market rent.

[1] A commercial lease contract typically includes many details, among which a description of the premises being let, the term of the tenancy and whether it is fixed or subject to periodic renewals, the lease rate (rent per sq. foot or sq. meter), the type of business that may be conducted on the premises, subletting and termination clauses as well as any signing incentive (rent-free months and/or improvements to the premises).

[2] While residential lease contracts involving individuals or households are generally standardized and rent payments heavily regulated, commercial lease contracts widely vary in duration, leased space as well as payment timing and arrangements, and include many features such as the right to cancel, the option to renew or to expand, and so on.

In this paper, we analyze the value of commercial lease contracts and derive equilibrium values for lease payments, taking into account different features inherent to commercial real estate such as size of the premises or of the companies, bargaining power and current leasing market conditions. We do so using a standard equilibrium model coupled with a leasing game and derive an analytical solution to value lease contracts. Our theoretical model allows us to explain the discount (premium) accruing to the tenant (landlord) according to expected cash flows and negotiation power. In particular, we investigate the rent discount/premium issue on the New-York and Chicago office markets and find a counterintuitive, and substantial, premium for longer lease contracts in the former case. We also find that, for both markets, the rent-size relationship is not monotonic but rather U-shaped, with unit rent, hence the premium to the landlord, rising beyond a given surface threshold. All findings can be explained by local market structures that drive the bargaining power.

Focusing on the rental market structure, the contribution of this paper is twofold. First it proposes a theoretical model to value the lease contract taking into account - among other things - the size of the premises, the bargaining power of the parties and the lease duration. Secondly, it provides an empirical analysis of the New York City and Chicago office markets. In particular, a counterintuitive result is obtained for NYC where longer leases command a premium, thereby suggesting a specific market structure issue.

This paper is organized as follows. Section 2 presents the relevant literature on buildings, rental market structure and leases. Section 3 introduces the theroetical model in three steps: first it introduces the modelling of leases' cash-flows, then we show how the bargaining power of the tenant is taken into account and finally, using a "leasing game", we derive theoretical conditions under which small or large firms sign short-term or long-term leases with a premium/discount. In Section 4, regression results are reported and analyzed in detail. Finally, Section 5 provides a discussion of the findings in light of the literature on market conditions and structures. A conclusion on the database limitations ends the paper.

2 Literature Review

The academic literature on office rent determinants covered in this paper is confined to quantitative studies performed in various urban and national contexts. This literature has developed since the early 1980s and has evolved around a series of issues that can be grouped into five main categories: (i) location factors; (ii) building intrinsic attributes; (iii) market context and landlord-tenant relationships; and (iv) lease-related factors and tenant quality issues and (v) leases' structure (terms and conditions).

2.1 Location factors

Location factors affecting office rents encompass a wide range of spatial issues ranging from the influence of building surroundings' quality to the impact that transportation modes and proximity to the CBD, to the labour market or to railway stations have on rent levels. The importance of face-to-face contacts and of the synergy between economic agents generated by central locations and capitalized into office rents are also widely addressed by authors, among which Clapp (1980), Bollinger et al. (1998), Gat (1998), Archer & Smith (2003), Dunse et al. (2002) and Nitsch (2006).

2.2 Building attributes

The influence of building intrinsic attributes has been addressed by numerous econometric studies on office rent determination. Issues addressed include building quality and/or prestige Glascock et al. (1990); Mills (1992); Wheaton & Torto (1995); Dunse & Jones (1998); Gat (1998); Archer & Smith (2003); Desyllas (2000); Slade (2000); Dunse et al. (2002); Gunnelin & Söderberg (2003); Bond et al. (2008); Nitsch (2006), internal services and amenities (Mills, 1992; Slade, 2000; Nitsch, 2006) as well as layout and functionality of the rented premises (Dunse et al., 2002).

2.3 State of the market

Fewer studies deal with the role played by the state of the market at the time of lease negotiation, hence by economic cycles, on the rent setting process through the landlord's and tenant's respective bargaining power. Webb & Fisher (1996), D'Arcy et al. (1997), Robinson (1999) and Slade (2000) are among the few authors that have addressed that issue.

2.4 Lease factors

Finally, lease factors – which are most relevant in the context of this paper - have triggered quite a few publications over the past two-and-a-half decades. Lease factors include size of rental area, lease duration, rent review provisions as well as options and incentives. Type and quality of tenants are also investigated as office rent determinants.

Benjamin et al. (1992) were among the first to investigate the influence of the size of rental area on office rents. His study on Greensboro, NC, yields a significant and negative relationship. A similar conclusion is arrived at by Bond et al. (2008) for the London office market while Desyllas (2000) finds a negative, but statistically insignificant coefficient for Berlin. In their study on U.S. metropolitan areas, Wheaton & Torto (1995) obtain significant, both positive and negative, relationships between size of rental area and office rent. Finally, while Dunse & Jones (1998) and Gunnelin & Söderberg (2003) find highly statistically significant coefficients for Glasgow and Stockholm, respectively, their influence emerges as being negligible.

As can be seen from the literature, the relationship between size of rental area and office rent is far from obvious and deserves being further investigated. According to Benjamin et al. (1992) though, landlords will generally prefer renting large spaces to fewer tenants due to the economies of scale realized on building management operations. Consequently, one should expect that the larger the size of the rental area, the lower the rent.

Turning to lease length, Benjamin et al. (1992), in their study on the Greensboro office market, find that lease length is statistically significant and negatively correlated with office rent. For Berlin though, Desyllas (2000) obtains a positive, although statistically insignificant relationship. While their research on three US metropolitan areas is performed on various property types and includes only 130 office leases, Stanton & Wallace (2009) find a positive and statistically significant relationship between lease length and rent. A similar conclusion is arrived at by Englund et al. (2004) in their investigation of three Swedish office markets involving more than 4,000 lease contracts negotiated between 1998 and 2002. Gunnelin & Söderberg (2003) focus on the Stockholm CBD office market, with their study, performed over 15 years, being based on 1,300 leases. They find that the relationship between rent and lease term is upward-sloping in a bullish market and downward-sloping in a bearish market. Such findings, which are at odds with the rationale put forward by Grenadier (1995), point towards a direct link between market cycles and the office rent setting process. In line with McCann & Ward (2004), findings from Crosby et al. (2006) on the office market of England and Wales suggest that lease duration is dependent upon the size of the tenant organization, with small office tenants negotiating shorter lease terms for flexibility reasons while large and medium-size occupiers tend to prefer longer terms as they provide greater stability. Finally, in their investigation of the London office market over the 1994-2004 period, Bond et al. (2008) find a positive, although counterintuitive, relationship between lease length and office rents.

Considering that leasing for longer periods reduces search costs, refurbishment costs, negotiation costs as well as occupancy risk for the landlord (Kempf, 2015, p.95) on top of enhancing capital values (Crosby et al., 2003), the tenant should be placed at an advantage over the landlord in the rent negotiation process, particularly in a recessionary context (McAllister, 2001; Englund et al., 2004). This argument is all the more sensible considering that, for the tenant, longer leases tend to reduce flexibility while raising capitalized liability (Robinson, 1999). Consequently, tenants would tend to favour shorter contracts over longer ones while the opposite applies to landlords, which should, by and large, support the idea of a negative relationship between office rent and lease term. As brought out by empirical findings though, this proves to be too simplistic an explanation for dealing with a highly complex issue.

In the lease negotiation process, landlord and tenants are ultimately interested by the level of effective, rather than contract, rent, with the former being affected by rent review provisions. These may vary substantially among markets and between countries. In their hedonic rent index model for the Chicago office market, Webb & Fisher (1996) introduce the CPI as a rent determinant together with an interactive term linking the CPI adjustment with the term of the lease, assuming that the latter would be statistically significant and positive for longer lease term. Regression findings though were not conclusive due to a low inflation period of analysis. Englund et al. (2004) also tested for the impact of escalation clauses in their study on the Swedish office market. The dummy variables used emerged as statistically significant, which stems from the fact that the vast majority of Swedish leases are inflation-indexed.

Options and incentives embedded in the lease contract include cancellation and renewal options as well as subletting, downsizing or expansion clauses. These are most important aspects for the would-be tenant to consider if he wants to avoid or minimize search and contract negotiation costs, as well as business disruption and loss-of-goodwill costs that a move involves (Kempf, 2015, p.101). A study by Mooradian & Yang (2000) on 311 US leases suggests that tenants are ready to pay a substantial premium for a downsizing provision, while subletting or cancellation options have no significant effect on rents. Bond et al. (2008) test for the influence that the length of the lease break option has on rents and, contrary to expectations, find that the relationship is statistically significant, but negative. The explanation they provide is that the farther the option is from expiration, the lower its value. Finally, in their US study on commercial leases, Stanton & Wallace (2009) test for the impact of tenant improvements on rents and find they are significantly and negatively correlated with rent. The low representativeness of their sample with regard to office leases may be responsible for this counterintuitive result (Kempf, 2015, p.102).

The type and quality of tenants have been addressed by a number of authors. Benjamin et al. (1992) use several dummy variables to test for creditworthiness as well as a continuous variable for the amount of security deposit payable in advance. In line with theoretical expectations, they find that tenants with good credit ratings and paying a large deposit can negotiate at a substantial rent discount. Stanton & Wallace (2009) arrive at an opposite result, but the coefficient of their credit rating dummy is not statistically significant. In their study on Baton Rouge, La, Glascock et al. (1993) find a statistically significant and positive relationship between rent and the degree of owner-occupation of the building. Finally, findings from Webb & Fisher (1996) suggest that new tenants pay less rent than in-place tenants who are negotiating a renewal. Such findings though are at odds with both Englund et al. (2004) and Gunnelin & Söderberg (2003) who find the opposite, with in-place tenants benefitting from a 4% to 10% rent discount over new ones.

2.5 Lease structures

Despite their importance, options embedded in office lease contracts have received little attention in the literature.[3] Grenadier (1995), Buetow & Albert (1998) and Stanton & Wallace (2009) develop partial differential equations models to value lease-embedded options but they rely on numerical methods in order to characterize the equilibrium. Clapham (2003), on the other hand, obtain an analytical solution using partial differential equations by modifying the approach of Buetow & Albert (1998). However, Clapham (2003) provides a model to value an option in a risk-free, Black-Scholes-Merton, setup.

In exchange for more flexibility, a tenant may accept to pay a higher rent (see Miceli & Sirmans, 1999; Amédée-Manesme et al., 2013), and the value of an option can then be expressed as the difference between the rent of a plain-vanilla lease and that of an option-embedded lease. The value of the option represents the price paid in each period by the tenant in order to get the flexibility to leave or to renew on an agreed date, and thus corresponds to an option-adjusted lease spread. With this option, the tenant may decide - at a specified date - to continue the current lease or to enter a new contract if market rents drop.

3 The model

We developp a theoretical model to analyse the leasing market and in particular the lease structure. Our development is based on the following steps: first, we introduce the way we model leases with a particular emphasys on short-term (renewable) and long-term (fixed) leases; second we introduce the bargaining power of the tenant in a setting that holds whatever the current state of the leasing market and finally, using a "leasing game", we derive theoretical conditions under which small or large firms sign short-term or long-term leases with a premium/discount.

Time is discrete and represented by $t = 1, 2, \ldots$, measured in years, and $\Delta t = 1/12$ represents one

[3] This is not the case though in other fields of research such as automobile lease contracts (see Gamba & Rigon, 2008 and Giaccotto et al., 2007) or options to expand businesses (see Agliardi, 2006) where the literature proves to be quite abundant.

month. The one-month market rent prevailing at the beginning of period t is denoted R_t and evolves according to a binomial model with time steps Δt. Over a period Δt, the market rent may go up by a factor $u = e^{\sigma\sqrt{\Delta t}}$ or down by a factor $d = e^{-\sigma\sqrt{\Delta t}}$, σ being the volatility of the market rent. Let p_u denote the actual probability of an up movement and let μ denote the expected growth rate of market rents. Then $E_t[R_{t+\Delta t}] = p_u R_t u + (1-p_u) R_t d = R_t e^{\mu \Delta t}$, with $p_u = \frac{e^{\mu\Delta t}-d}{u-d}$. Note that to have $p_u \leq 1$, we need $\mu \leq \sigma/\sqrt{\Delta t}$.

3.1 Modelling leases

In what follows, it is assumed that the tenant has the choice between a long-term fixed-lease contract ($2T$) or a short-term lease contract (T). By assumption (that can be relaxed), the shorter term contract is half the longer term contract and all short term lease contract are renewable for another T years.[4] It is also assumed that rent conditions are fixed throughout the full lease contract and known from start, excluding, more often than not, rent indexation and negotiation dimensions.

Consider a W-year lease is signed at the beginning of year t with a first Δt-payment of L_t^W, indexed every Δt at the rate $i < \mu$. he present value of all rent payments is given by

$$V_t^W = L_t^W + e^{(i-k)\Delta t} L_t^W + e^{(i-k)(2\Delta t)} L_t^W + \ldots + e^{(i-k)(W-\Delta t)} L_t^W = \frac{\left(1-e^{(i-k)W}\right) L_t^W}{1-e^{(i-k)\Delta t}},$$

where k is the annual discount rate. When a contract is signed over W years, the value of the fixed-lease contract has to be equal to the expected present value of the sum of all market rents (R_t)[5] over the same period, i.e.

$$\begin{aligned} V_t^W &= E_t\left[R_t + e^{-k\Delta t} R_{t+\Delta t} + e^{-k(2\Delta t)} R_{t+2\Delta t} + \ldots + e^{-k(W-\Delta t)} R_{t+W-\Delta t}\right] \\ &= R_t + e^{(\mu-k)\Delta t} R_t + e^{(\mu-k)(2\Delta t)} R_t + \ldots + e^{(\mu-k)(W-\Delta t)} R_t \\ &= \frac{\left(1-e^{(\mu-k)W}\right) R_t}{1-e^{(\mu-k)\Delta t}}, \end{aligned}$$

and thus

$$L_t^W = \frac{\left(1-e^{(\mu-k)W}\right)\left(1-e^{(i-k)\Delta t}\right)}{\left(1-e^{(\mu-k)\Delta t}\right)\left(1-e^{(i-k)W}\right)} R_t. \tag{1}$$

With $i < \mu < k$, the lease payment under an infinite contract would be

$$\lim_{W\to\infty} L_t^W = L_t^\infty = \frac{1-e^{(i-k)\Delta t}}{1-e^{(\mu-k)\Delta t}} R_t,$$

[4] In the current setting, the short-term renewable contract is equivalent to a long-term cancelable contract, signed for $2T$ years and cancelable at time T.

[5] Contractual rents (L_t) charged to tenants rarely follow market rents (R_t). Rents are usually contracted at a value close to the market rents at the initiation of the lease. Later on, rents have usually been indexed and do not necessarily represent the current market value, which may collapse in bear markets or raise in bull markets. In uncertain economic times, many previously determined contractual rents may end up being above market rents. Market rents can thus be defined as the most likely lease rate a property would command in an open market.

and the present value of all future expected market rents as of time t is given by

$$V_t^{\infty} \;=\; L_t^{\infty} \;+\; e^{-k\Delta t} L_t^{\infty} \;+\; e^{-2k\Delta t} L_t^{\infty} \;+\; e^{-3k\Delta t} L_t^{\infty} \;+\; \ldots \;=\; \frac{L_t^{\infty}}{1 - e^{(i-k)\Delta t}} \;=\; \frac{R_t}{1 - e^{(\mu-k)\Delta t}}. \quad (2)$$

The solution to our model assumes that a tenant considers the value of the rent paid into infinity when considering different types of contract terms (the tenant will never own its real estate). The equilibrium also assumes that an infinite sequence of W-year contracts has to be equal to the present value of expected market rents into infinity, i.e.

$$V_t^W \;+\; e^{-kW} E_t\left[V_{t+W}^W\right] \;+\; e^{-2kW} E_t\left[V_{t+2W}^W\right] \;+\; \ldots \;=\; V_t^{\infty}, \quad (3)$$

and thus the present value of an infinite sequence of W-year contracts can be written as

$$V_t^W \;+\; e^{-kW} E_t\left[V_{t+W}^{\infty}\right] \;=\; V_t^W \;+\; e^{(\mu-k)W} V_t^{\infty}. \quad (4)$$

3.2 Bargaining/negotiation power

Our basic assumption stands as follow: the lower the negociation power of a company, the more likely it will sign a cancelable lease, i.e. if a company has a dominant bargaining power, it will sign a long-term lease since this ensures stability over time and minimizes search and moving costs. We denote α the negotiation power of a company ($\alpha = 0$ means no negotiation/bargaining power). We demonstrate how our setting holds whatever the leasing market. Therefore, our model is independant of the leasing market (i.e. the market can be or not at the advantage of the landlord). In the next section (3.3), the state of the market is taken into account with a segmentation between availabilities of spaces (large or small) and rental values.

Consider now a short term T-year renewable contract with a initial Δt-payment $\tilde{L}_t^T$, indexed at the rate i every Δt. After T years, the tenant can renew the lease for another short term T years or enter into another T-year renewable contract at a lower rate. Two scenarios may happen:[6]

- If, at time $t+T$, the available renewable contract requires that $\tilde{L}_{t+T}^T > e^{iT}\tilde{L}_t^T$, then the tenant will prefer to renew the contract signed at time t. When renewing, negotiation takes place and the tenant obtains a new renewable contract with an initial lease $\alpha e^{iT}\tilde{L}_t^T + (1-\alpha)\tilde{L}_{t+T}^t$, where $\alpha \in [0,1]$ denotes the tenant's negociation power. When $\alpha \to 1$, the tenant renews into a lease consisting in the continuity of the previous lease. When $\alpha \to 0$, the tenant has no negociation power and the new lease is the market lease.

- If, on the other hand, $\tilde{L}_{t+T}^T \leq e^{iT}\tilde{L}_t^T$, then the tenant will exit the current contract and sign a new renewable contract.

[6] In this letter, transaction costs (broker fees, searching costs, moving costs...) have not been considered. However, one can add the transaction costs δ in the moving decision by comparing $\tilde{L}_{t+T}^T + \delta$ with $\tilde{L}_t^T$.

Note that equation (3) must hold with renewable leases as well, and thus the present value of an infinite sequence of T-year renewable leases at time $t+j$ must equal V_{t+j}^{∞} for all j. The present value of an infinite sequence of T-year renewable contracts is then equal to:

$$\begin{aligned}
&\frac{\left(1-e^{(i-k)T}\right)\tilde{L}_t^T}{1-e^{(i-k)\Delta t}} \\
&+ \Pr\left(\tilde{L}_{t+T}^T > e^{iT}\tilde{L}_t^T\right)e^{-kT}\left(\frac{1-e^{(i-k)T}}{1-e^{(i-k)\Delta t}}\left(\alpha e^{iT}\tilde{L}_t^T + (1-\alpha)E\left[\tilde{L}_{t+T}^T\middle|\tilde{L}_{t+T}^T > e^{iT}\tilde{L}_t^T\right]\right) + e^{-kT}E_t\left[V_{t+2T}^{\infty}\middle|\tilde{L}_{t+T}^T > \tilde{L}_t^T\right]\right) \\
&+ \Pr\left(\tilde{L}_{t+T}^T \le e^{iT}\tilde{L}_t^T\right)e^{-kT}E_t\left[V_{t+T}^{\infty}\middle|\tilde{L}_{t+T}^T \le \tilde{L}_t^T\right].
\end{aligned}$$

Since $\tilde{L}_{t+T}^T \le e^{iT}\tilde{L}_t^T$ if and only if $R_{t+T} \le e^{iT}R_t$, the last expression can be rewritten as:

$$\begin{aligned}
&\frac{\left(1-e^{(i-k)T}\right)\tilde{L}_t^T}{1-e^{(i-k)\Delta t}} \\
&+ \Pr\left(R_{t+T} > e^{iT}R_t\right)e^{-kT}\left(\frac{1-e^{(i-k)T}}{1-e^{(i-k)\Delta t}}\left(\alpha e^{iT}\tilde{L}_t^T + (1-\alpha)E\left[\tilde{L}_{t+T}^T\middle|R_{t+T} > e^{iT}R_t\right]\right) + e^{-kT}E_t\left[V_{t+2T}^{\infty}\middle|R_{t+T} > e^{iT}R_t\right]\right) \\
&+ \Pr\left(R_{t+T} \le R_t\right)e^{-kT}E_t\left[V_{t+T}^{\infty}\middle|R_{t+T} \le R_t\right].
\end{aligned}$$

Note that

$$E_t\left[V_{t+2T}^{\infty}\middle|R_{t+T} > e^{iT}R_t\right] = E_t\left[\frac{R_{t+2T}}{1-e^{(\mu-k)\Delta t}}\middle|R_{t+T} > e^{iT}R_t\right] = \frac{e^{\mu T}}{1-e^{(\mu-k)\Delta t}}E_t\left[R_{t+T}\middle|R_{t+T} > e^{iT}R_t\right] \quad (5)$$

and

$$E_t\left[V_{t+T}^{\infty}\middle|R_{t+T} \le e^{iT}R_t\right] = \frac{1}{1-e^{(\mu-k)\Delta t}}E_t\left[R_{t+T}\middle|R_{t+T} \le e^{iT}R_t\right]. \quad (6)$$

As shown in Appendix A, $E_{t+j}\left[R_{t+j+T}\middle|R_{t+j+T} \le e^{iT}R_{t+j}\right] = \Omega_d^i R_{t+j}$ for any $j \ge 0$, where Ω_d^i does not depend on j, and $E_{t+j}\left[R_{t+j+T}\middle|R_{t+j+T} > e^{iT}R_{t+j}\right] = \Omega_u^i R_{t+j}$ for any $j \ge 0$, where Ω_u^i does not depend on j. This gives

$$E_t\left[V_{t+2T}^{\infty}\middle|R_{t+T} > e^{iT}R_t\right] = e^{\mu T}\Omega_u^i V_t^{\infty} \quad \text{and} \quad E_t\left[V_{t+T}^{\infty}\middle|R_{t+T} \le e^{iT}R_t\right] = \Omega_d^i V_t^{\infty}.$$

Let $\theta_i = \Pr\left(R_{t+T} \le e^{iT}R_t\right)$. The present value of an infinite sequence of T-year once-renewable contracts is then equal to

$$\begin{aligned}
&\frac{\left(1-e^{(i-k)T}\right)\tilde{L}_t^T}{1-e^{(i-k)\Delta t}} + (1-\theta_i)e^{-kT}\left(\frac{1-e^{(i-k)T}}{1-e^{(i-k)\Delta t}}\left(\alpha e^{iT}\tilde{L}_t^T + (1-\alpha)\Omega_u^i\tilde{L}_t^T\right) + e^{(\mu-k)T}\Omega_u^i V_t^{\infty}\right) + \theta_i e^{-kT}\Omega_d^i V_t^{\infty} \\
&\frac{\left(1-e^{(i-k)T}\right)\tilde{L}_t^T}{1-e^{(i-k)\Delta t}} + (1-\theta_i)e^{-kT}\left(\frac{1-e^{(i-k)T}}{1-e^{(i-k)\Delta t}}\left(e^{iT} + (1-\alpha)\left(\Omega_u^i - e^{iT}\right)\right)\tilde{L}_t^T + e^{(\mu-k)T}\Omega_u^i V_t^{\infty}\right) + \theta_i e^{-kT}\Omega_d^i V_t^{\infty} \\
&\frac{\left(1-e^{2(i-k)T}\right)\tilde{L}_t^T}{1-e^{(i-k)\Delta t}} - \theta_i e^{(i-k)T}\frac{1-e^{(i-k)T}}{1-e^{(i-k)\Delta t}}\tilde{L}_t^T + (1-\theta_i)e^{-kT}\frac{1-e^{(i-k)T}}{1-e^{(i-k)\Delta t}}(1-\alpha)\left(\Omega_u^i - e^{iT}\right)\tilde{L}_t^T \\
&\qquad + (1-\theta_i)e^{(\mu-2k)T}\Omega_u^i V_t^{\infty} + \theta_i e^{-kT}\Omega_d^i V_t^{\infty}
\end{aligned}$$

which can be written as

$$\begin{aligned}
&\frac{\left(1-e^{2(i-k)T}\right)\tilde{L}_t^T}{1-e^{(i-k)\Delta t}} - \theta_i e^{(i-k)T}\frac{1-e^{(i-k)T}}{1-e^{(i-k)\Delta t}}\tilde{L}_t^T + (1-\theta_i)e^{-kT}\frac{1-e^{(i-k)T}}{1-e^{(i-k)\Delta t}}(1-\alpha)\left(\Omega_u^i - e^{iT}\right)\tilde{L}_t^T \\
&\qquad + (1-\theta_i)e^{(\mu-2k)T}\Omega_u^i V_t^{\infty} + \theta_i e^{-kT}\Omega_d^i V_t^{\infty}
\end{aligned}$$

Using $(1-\theta_i)\Omega_u^i = e^{\mu T} - \theta_i \Omega_d^i$, we can write

$$\frac{\left(1-e^{2(i-k)T}\right)\tilde{L}_t^T}{1-e^{(i-k)\Delta t}} \; + \; \left((1-\alpha)(1-\theta_i)\left(\Omega_u^i - e^{iT}\right) - \theta_i e^{iT}\right)\frac{1-e^{(i-k)T}}{1-e^{(i-k)\Delta t}} e^{-kT}\tilde{L}_t^T$$
$$+ \; e^{2(\mu-k)T}V_t^\infty \; + \; \theta_i\Omega_d^i e^{-kT}\left(1-e^{(\mu-k)T}\right)V_t^\infty.$$

$$\frac{\left(1-e^{2(i-k)T}\right)\tilde{L}_t^T}{1-e^{(i-k)\Delta t}} \; + \; \left((1-\alpha)\left(e^{\mu T} - \theta_i\Omega_d^i\right) - (1-\alpha+\alpha\theta_i)e^{iT}\right)\frac{1-e^{(i-k)T}}{1-e^{(i-k)\Delta t}} e^{-kT}\tilde{L}_t^T$$
$$+ \; e^{2(\mu-k)T}V_t^\infty \; + \; \theta_i\Omega_d^i e^{-kT}\left(1-e^{(\mu-k)T}\right)V_t^\infty. \tag{7}$$

for the value of an infinite sequence of T-year renewable contracts.

Proposition 1. *Consider a world where tenants have two options regarding lease contracts: A long-term $2T$-year contract, or a short-term (renewable) T-year contract. Let $\tilde{L}_t^T$ and L_t^{2T} represent the short-term T-year rent and the long-term $2T$-year rent, respectively.*

(i) Then the short-term rent can be expressed as

$$\tilde{L}_t^T \; = \; \left(1 + \frac{(1-\alpha)(1-\theta_i)(\Omega_u^i - e^{iT})}{e^{kT}+e^{iT}}\right)^{-1}\left(L_t^{2T} + P_t^i\right),$$

where $P_t^i > 0$ represents the value of the option to continue with the same lease agreement at time T given the indexation rate i.

(ii) There exists a $\alpha^* \in (0,1)$ such that

$$\tilde{L}_t^T \; \begin{cases} > \; L_t^{2T} & \text{for all } \alpha \in (\alpha^*, 1], \\ = \; L_t^{2T} & \text{when } \alpha = \alpha^*, \\ < \; L_t^{2T} & \text{for all } \alpha \in [0, \alpha^*). \end{cases}$$

Proof. See proof in Appendix B □

3.3 Leasing games

In this part, we developp a leasing game (in the sense of game theory) in order to characterise leasing market and more broadly rental market according to the size of tenants (small or large), to bargaining power (α in $\tilde{L}_t^T$) and to cash flows (A_T and B_T with π later on).

There are two types of firm, large (denoted a) and small (denoted b). There are N available buildings, K_a large firms and K_b small firms, with $K_b > N > K_a$. The square footage of a building is given by S (all buildings have the same size), S_a denotes the space needed by an a-firm and S_b denotes the space needed by a b-firm. There are fewer a-firms than buildings, i.e. $K_a < N$, and a building can host at most one a-firm. We also impose:

$$K_a S_a \; + \; K_b S_b \; < \; NS,$$

i.e. all buildings cannot be completely filled, i.e. the vacancy rate is always positive.

Let A_0 and A_T denote the present value of the cash flow of an a-firm, excluding its rent, from time 0 to $T-1$ and from time T to $2T-1$, respectively. Let B_0 and B_T play the same role for a b-firm.

Each building is owned by a landlord willing to rent the office spaces for long-term leases ($2T$ years). After T years (a short term lease), an a-firm may be tempted to move to another building if the latter offers better facilities and is such that it increases the firm's profit compared to staying in the previous building. For example, a partially filled building may spend some resources, or the area surrounding it may have developped, in which case an a-firm would expect a cash flow of $(1+\lambda)A_T$ instead of A_T by moving into this building at time T instead of staying in the current one. The moving cost for an a-firm is given by C_a. The net benefit of moving for an a-firm is then equal to $\lambda A_T - C_a$. *Ex-ante*, the perceived probability that a partially filled building will help generate an extra payoff λA_T to an a-firm is given by δ_a, and the parameters λ and δ_a are specific to an a-firm so that not all a-firms move at time T. For simplicity, we assume that the empty space in a partially filled building is exactly equal to S_a.

While an a-firm is not expected to cease its activities within the next $2T$ years, a b-firm may cease to operate after T years with probability ω_b. When a b-firm disappears, it is immediately replaced by another b-firm, so the number of firms remains constant. When a firm, a or b, signs a $2T$-year lease contract and leaves at time T, then it must pay a cancellation penalty D (or sublet).

The timing of events is as follows:

1. All a-firms decide where to locate, given the lease contracts proposed to them by landlords;
2. Once the buildings with an a-firm are known, landlords offer the menu of lease contracts to b-firms;
3. The b-firms then choose the building where they want to operate;
4. After T years, some a-firms may decided to change building;
5. After $2T$ years, the game ends.

3.3.1 Payoffs

Throughout the paper, all rents must satisfy (1) and thus all rents are set according to the market rent, which implies that

$$V_0^{2T} \;=\; V_0^T + E_0\left[V_T^T\right] \;=\; \tilde{V}_0^T \;=\; M_0^{2T}.$$

We also only consider the renewable contract in (7) with $\alpha = 1$. All lease rate are expressed in square foot.

Expected payoff of a a-firm

Three possibles leases structure must be distinguished:

- If an a-firm signs a $2T$-year lease with initial rent L_0^{2T} indexed at the rate i, then its expected payoff is (we assume the penalty cost D to be sufficiently high to prevent an a-firm with a long-term contract to switch building at time T)

$$U_a^{2T} \;=\; A_0 \;-\; \frac{\left(1-e^{(i-k)T}\right) L_0^{2T} S_a}{1-e^{(i-k)\Delta t}} \;+\; e^{-kT}\left(A_T - \frac{\left(1-e^{(i-k)T}\right) e^{iT} L_0^{2T} S_a}{1-e^{(i-k)\Delta t}}\right).$$

- If an a-firm signs two short-term leases with initial rent L_0^T, then its expected payoff is

$$\begin{aligned} U_a^T \;=\; & A_0 \;-\; \frac{\left(1-e^{(i-k)T}\right) L_0^T S_a}{1-e^{(i-k)\Delta t}} \;+\; e^{-kT}(1-\delta_a)\left(A_T - \frac{\left(1-e^{(i-k)T}\right) E_0\left[L_T^T\right] S_a}{1-e^{(i-k)\Delta t}}\right) \\ & +\; e^{-kT}\delta_a\left((1+\lambda)A_T - C_a - \frac{\left(1-e^{(i-k)T}\right) E_0\left[L_T^T\right] S_a}{1-e^{(i-k)\Delta t}}\right) \end{aligned}$$

  where $E_0\left[L_T^T\right]$ denotes the expected short-term rent that will prevail at time T.

- An a-firm could also sign a T-year renewable contract with $\alpha = 1$. To alleviate notation, let

$$E_0\left[\tilde{L}_T^T\right] \;=\; \Pr\left(L_T^T > e^{iT}\tilde{L}_0^T\right) e^{iT}\tilde{L}_0^T \;+\; \Pr\left(L_T^T \le e^{iT}\tilde{L}_0^T\right) E_0\left[L_T^T \,\middle|\, L_T^T \le e^{iT}\tilde{L}_0^T\right].$$

  If an a-firm signs a T-year renewable contract at time 0, then it will move at time T if

$$\lambda A_T \;-\; C_a \;-\; \frac{\left(1-e^{(i-k)T}\right) L_T^T S_a}{1-e^{(i-k)\Delta t}} \;>\; -\,\frac{\left(1-e^{(i-k)T}\right) \tilde{L}_T^T S_a}{1-e^{(i-k)\Delta t}},$$

  where $\tilde{L}_T^T = \min\left\{e^{iT}\tilde{L}_0^T, L_T^T\right\}$. Let $\hat{L}_T^T$ be such that[7]

$$\lambda A_T \;-\; C_a \;-\; \frac{\left(1-e^{(i-k)T}\right) \hat{L}_T^T S_a}{1-e^{(i-k)\Delta t}} \;=\; -\,\frac{\left(1-e^{(i-k)T}\right) e^{iT}\tilde{L}_0^T S_a}{1-e^{(i-k)\Delta t}}.$$

[7]Note that

$$\hat{L}_T^T \;=\; e^{iT}\tilde{L}_0^T \;+\; \frac{\left(1-e^{(i-k)\Delta t}\right)(\lambda A_T - C_a)}{\left(1-e^{(i-k)T}\right) S_a} \;>\; e^{iT}\tilde{L}_0^T.$$

Then, under a renewable contract, the expected payoff to an a-firm is

$$\begin{aligned}\tilde{U}_a^T &= A_0 - \frac{\left(1-e^{(i-k)T}\right)\tilde{L}_0^T S_a}{1-e^{(i-k)\Delta t}} + e^{-kT}(1-\delta_a)\left(A_T - \frac{\left(1-e^{(i-k)T}\right)E_0\left[\tilde{L}_T^T\right]S_a}{1-e^{(i-k)\Delta t}}\right)\\ &+ e^{-kT}\delta_a \Pr\left(L_T^T \geq \hat{L}_T^T\right)\left(A_T - \frac{\left(1-e^{(i-k)T}\right)e^{iT}\tilde{L}_0^T S_a}{1-e^{(i-k)\Delta t}}\right)\\ &+ e^{-kT}\delta_a \Pr\left(L_T^T < \hat{L}_T^T\right)\left((1+\lambda)A_T - C_a - \frac{\left(1-e^{(i-k)T}\right)E_0\left[L_T^T\middle| L_T^T < \hat{L}_T^T\right]S_a}{1-e^{(i-k)\Delta t}}\right).\end{aligned}$$

Expected payoff of a b-firm

Because of the probability to cease its activies at time T and the penalty D when cancelling a long-term lease contract, a b-firm never signs a long-term ($2T$-year) lease. It either signs two short-term T-year leases or a T-year renewable contract (with $\alpha = 1$, i.e. no need to negotiate at the end of the contract). The b-firm expects to increase its revenues by π.

- If it signs two short-term lease contracts:

$$\begin{aligned}U_{ab}^T &= (1+\pi)B_0 - \frac{\left(1-e^{(i-k)T}\right)L_0^T S_b}{1-e^{(i-k)\Delta t}}\\ &+ (1-\omega_b)e^{-kT}(1-\delta_a)\left((1+\pi)B_T - \frac{\left(1-e^{(i-k)T}\right)E_0\left[L_T^T\right]S_b}{1-e^{(i-k)\Delta t}}\right)\\ &+ (1-\omega_b)e^{-kT}\delta_a \Pr\left(L_T^T \geq \hat{L}_T^T\right)\left((1+\pi)B_T - \frac{\left(1-e^{(i-k)T}\right)E_0\left[L_T^T\middle| L_T^T \geq \hat{L}_T^T\right]S_b}{1-e^{(i-k)\Delta t}}\right)\\ &+ (1-\omega_b)e^{-kT}\delta_a \Pr\left(L_T^T < \hat{L}_T^T\right)\left(B_T - \frac{\left(1-e^{(i-k)T}\right)E_0\left[L_T^T\middle| L_T^T < \hat{L}_T^T\right]S_b}{1-e^{(i-k)\Delta t}}\right)\end{aligned}$$

- If it signs a T-year renewable contract:

$$\begin{aligned}\tilde{U}_{ab}^T &= (1+\pi)B_0 - \frac{\left(1-e^{(i-k)T}\right)\tilde{L}_0^T S_b}{1-e^{(i-k)\Delta t}}\\ &+ (1-\omega_b)e^{-kT}(1-\delta_a)\left((1+\pi)B_T - \frac{\left(1-e^{(i-k)T}\right)E_0\left[\tilde{L}_T^T\right]S_b}{1-e^{(i-k)\Delta t}}\right)\\ &+ (1-\omega_b)e^{-kT}\delta_a \Pr\left(L_T^T \geq \hat{L}_T^T\right)\left((1+\pi)B_T - \frac{\left(1-e^{(i-k)T}\right)e^{iT}\tilde{L}_0^T S_b}{1-e^{(i-k)\Delta t}}\right)\\ &+ (1-\omega_b)e^{-kT}\delta_a \Pr\left(L_T^T < \hat{L}_T^T\right)\left(B_T - \frac{\left(1-e^{(i-k)T}\right)E_0\left[\tilde{L}_T^T\middle| L_T^T < \hat{L}_T^T\right]S_b}{1-e^{(i-k)\Delta t}}\right)\end{aligned}$$

3.3.2 Results to the leasing game

Proposition 2. *When $\delta_a = 0$ and ω_b is sufficiently large, then*

1. *If π is sufficiently large, then*

   (a) *All a-firms sign $2T$-year fixed contracts;*

   (b) *b-firms sign a T-year renewable contract;*

   (c) ***If the number of a-firms is sufficiently large, then the average short-term rent as of time 0, $\frac{K_a}{N}\tilde{L}_0^T + \left(1 - \frac{K_a}{N}\right) L_0^T$, is greater than the long-term rent L_0^{2T}.***

2. *If π is sufficiently small, then*

   (a) *All a-firms sign $2T$-year fixed contracts;*

   (b) *All b-firms sign a sequence to two T-year non-renewable contracts;*

   (c) ***The long-term lease rate at time 0, L_0^{2T}, is then greater than the short-term rent as of time 0, which is L_0^T.***

Proposition 3. *When $\delta_a > 0$ and $\lambda A_T - C_a > 0$, then*

1. ***All a-firms sign non-renewable short-term contracts;***

2. *b-firms sign renewable contracts if π is sufficiently large;*

To sum up the theoretical model, when the availability of large spaces is small, the negotiation power is in the hand of the landlord and this case, the large tenants all sign long-term lease contracts at a premium. On the other hand, when small-size premises are easily available, small companies sign short-term lease contracts at a discount. This theoretical model provides clues to a better understanding of the relationship between leasing market on the one hand, and, on the other hand, contract duration. The result of the relative availability of office space for various types of tenants with different needs and preferences.

4 Empirical analysis

In this section, we analyse two american cities regarding their leasing market according to a set of leases where we characterize the size/rental value relationship acording to leases duration.

4.1 Database and Descriptive Statistics

4.1.1 Database

The database used for testing our theoretical model was kindly provided by the CoStar Group. After filtering, the initial database provides detailed information on 5,125 office leases settled between 2007 and 2015 in three major SMSAs in the USA, namely New-York (2,133 leases), Los Angeles (1,709) and Chicago (1,283). The information includes the contract gross rent, the gross leasable area of the premises, lease term and duration, whether the contract is a new lease or a renewal, whether it is a direct lease or a subrental, the state of the premises (renovated or not), the number of free months, building quality class and location as well as year and quarter of lease negotiation. Following a thorough analysis of the descriptive statistics for each sub-sample though, it was decided to confine our analysis to the Chicago and New York office markets. Los Angeles was thus discarded considering the pronounced non-normality of the unit rent variable distribution. Major discrepancies were observed between mean and median values of key variables, with ensuing inconsistencies in the related regression coefficients leading to a potentially biased interpretation of results.

4.1.2 Descriptive statistics

Detailed descriptive statistics for Chicago and New York have been computed for both the dependent (contract unit rent) and main independent variables (leased area, lease term and number of free months) as well as by lease type (new or renewal), spatial cluster, building Class (A, B or C), occupancy status (direct or sublease) and by quarter (Q1 through Q4). For simplicity purposes, these are not reported in the core of the paper, where focus is laid on the unit rent distribution by SMSA. Detailed descriptive statistics for selected variables can be consulted in Annex 1.

Starting with the Chicago sub-sample (1,283 cases), Table 1 provides us with a quite consistent picture of the office market in this first SMSA. With an overall mean and median unit rent standing at \$17.32 and \$16.50, respectively, and a standard deviation of 37.3%, rent distribution in Chicago can be said to approach a normal distribution, in spite of extreme levels negotiated in 2009 (\$110) and 2012 (\$5.18). Furthermore, the relative stability of office rent in Chicago holds throughout the whole period although unit rents display a slight downward trend until 2014, followed by a sharp rise in 2015.

Detailed statistics (see Annex 1) provide further information on the Chicago office market rent structure. While unit rent does not substantially differ depending on whether the lease is a new one (mean at \$17.23) or a renewal (\$18.10), it does with regard to the building location. The Chicago sub-sample includes eleven spatial clusters.[8] Unit rents are highest for metro Chicago ($20.61) and lowest for Kenosha County ($10.21). Mean rent stands at $19.69, $17.01 and $16.06 for Class A, B and C buildings, respectively. Subleases negotiate at a premium ($19.66) relative to direct leases ($17.18) while the rent pattern displays a great stability among quarters.

Table 1: Office rent levels per square foot, Chicago, 2007-2015

Transaction Year	**Mean**	**Median**	**Sd**	**Min**	**Max**	**Count**
			Unit Gross Rent			
2007	$18.13	$17.64	$4.89	$7.00	$32.00	67
2008	$17.24	$17.00	$4.23	$8.00	$30.00	49
2009	$17.34	$16.19	$7.82	$7.15	$110.00	310
2010	$17.17	$15.75	$6.91	$6.74	$54.50	258
2011	$17.15	$16.50	$5.91	$5.25	$53.47	275
2012	$16.84	$15.75	$5.61	$5.18	$40.00	186
2013	$17.05	$16.75	$5.33	$7.00	$30.00	76
2014	$16.66	$16.13	$2.62	$13.50	$22.00	14
2015	$20.55	$20.63	$5.93	$8.30	$33.00	48
Total	**$17.32**	**$16.50**	**$6.46**	**$5.18**	**$110.00**	**1,283**

Overall, leased surface area averages 6,640 square feet but major discrepancies are reported for renewals (15,326 sq. ft.), Metro Chicago (10,415 sq. ft.) and Class A buildings (15,992 sq. ft). As for mean lease term, it stands at 53 months (roughly 4.5 years), with longer durations applying to Metro Chicago (66 months) and Class A buildings (74 months). Finally, Chicago office tenants are granted, on average, a 2.6 free month concession but again, this advantage is more generous for Metro Chicago locations (3.7) and for Class A buildings (4.7).

The New York office market (2,133 cases) emerges as being relatively homogeneous when compared with Chicago's, at least with regard to the rental structure (Table 2). With an overall mean unit gross rent standing at $43 per square feet, this market met its low in 2010 (with a minimum at $9.18) before rents surged again to reach $61 per square feet in 2015 (with a maximum at $200). According to detailed descriptive statistics (see Annex 1), tenants under a renewal pay slightly higher rents ($47) while the mean rent for Midtown locations stands at $47, as opposed to between $37 and $39 elsewhere. As expected, Class A buildings command substantially higher rents ($59) than Class B ($40) and Class C ($34) ones.This being said, it should be noted that major discrepancies exist

[8]These clusters consist of: East/West Corridor, South Suburban, Kenosha County, Metro Chicago, Near West, North, North Chicago, Northwest, O'Hare, South Chicago and Indiana.

with regard to leased surface in relation to lease type, location and building class: with an overall mean surface standing at 9,909 square feet, renewals are negotiated, on average, at 31,085 square feet and Downtown locations at 26,902 square feet. Once again, very large spaces (22,886 sq. ft.) characterize Class A buildings. Also, there seems to be a trend for larger surfaces (12,747 sq. ft.) to be rented during the fourth quarter.

Office leases in New York extend over much longer terms than in Chicago, with the mean duration standing at 238 months (nearly 20 years). Renewals and Downtown locations command slightly shorter terms (around 17 years) while lease duration tends to decrease with building quality, from 23 years for Class C buildings to below 17 years for Class A ones. Finally, rental incentives stand, overall, at 2.4 free months, although they are substantially larger for Class A building tenants (nearly 4 years), compared with Class B (2 years) and Class C (1.6 year) building clienteles.

Table 2: Office rent levels per square foot, New York, 2007-2015

Transaction Year	**Mean**	**Median**	**Sd**	**Min**	**Max**	**Count**
			Unit Gross Rent			
2007	$47.05	$42.00	$21.42	$13.63	$132.00	130
2008	$53.92	$47.00	$24.23	$19.00	$185.00	243
2009	$36.60	$33.00	$13.25	$19.00	$105.00	243
2010	$35.75	$33.00	$13.18	$9.18	$120.00	415
2011	$38.10	$35.00	$13.84	$15.00	$125.00	399
2012	$39.79	$37.00	$13.64	$15.77	$125.00	217
2013	$42.31	$39.02	$12.66	$20.10	$135.00	202
2014	$55.04	$49.00	$21.21	$26.18	$152.00	160
2015	$61.23	$54.50	$23.57	$23.49	$200.00	124
Total	**$43.01**	**$38.50**	**$18.67**	**$9.18**	**$200.00**	**2,133**

4.2 Modelling Procedure, Regression Results and Discussion

Several functional forms were tested in order to model the influence of rented area (surface) and lease term on office rents. Both total and unit gross rents were alternately used as the dependent variable, either in their original form or with a log transformation. The two independent variables of interest (leased surface and lease term) were also tested, with and without mathematical transformations. While empirical analyses are still preliminary, models obtained using a log-log functional form (Model A)[9] and then by adding a log-quadratic term on the independent variables of interest (Model B)[10] prove to yield the most interesting results, which are reported and discussed below. Since quarter dummy variables were found to be statistically non-significant in any model, they are not reported in the tables.

4.2.1 Model specification

The standard log-linear hedonic rent model takes the following form:

$$lnR_i = \alpha_i + \beta lnX_i + \phi Z_i + \epsilon_i \tag{8}$$

where R_i is the natural log of the average rent per square foot in a given building, X_i is a vector of the natural log of the leased surface and lease term, Z_i is a vector of several physical, lease, locational and time characteristics, β and ϕ are the respective vectors of parameters to be estimated and ϵ_i is a random error and stochastic disturbance term that is expected to take the form of a normal distribution with a mean of zero and a variance of σ^2.
Once developed, the hedonic equation can be expressed as follows:

$$\begin{aligned} lnR_i = \beta_0 + \beta_1 lnSurf_i + \beta_2 lnLTerm_i + \beta_3 Free_i + \beta_4 Class_i + \beta_5 New_i \\ + \beta_6 Direct_i + \beta_7 Notrenov_i + \beta_8 Loc_i + \beta_9 Year_i + \epsilon_i \end{aligned} \tag{9}$$

where $lnSurf_i$ is the natural log of the leased surface, $lnLTerm_i$ is the natural log of the lease term, $Free_i$ is the number of free months, $Class_i$, New_i, $Direct_i$ and $Notrenov_i$ are a series of dummy variables to account for building class, a new lease (vs. a renewal), a direct lease from the landlord (vs. a sublease from an in-place tenant) and renovated premises (vs. not renovated), Loc_i is a vector of location dummies, $Year_i$ is a vector of time dummies (i.e. year of lease negotiation) and ϵ_i is a random error and stochastic disturbance term.

4.2.2 Regression results – Model A

Regression results for Model A are reported in Table 3. They first suggest that the relationship between contract unit rent and leased area is, unsurprisingly, not linear and that the larger the size of the premises, the lower the rent. This holds for both Chicago and New York (although, in the latter case, the coefficient is not significant at the 0.05 level) and is in line with theoretical expectations and the decreasing marginal utility principle. Findings are also consistent with the

[9] Model A takes the following form : $log(UnitRent) = f[log(Surface), log(Leaseterm), ...]$.
[10] Model B takes the following form : $log(UnitRent) = f[log(Surface), log(Surface)^2, log(Leaseterm), log(Leaseterm)^2, \ldots]$.

economies-of-scale argument put forward by Benjamin et al. (1992). In this case, and for Chicago, doubling the surface will result in a 6.9% drop in unit gross rent.

While regression coefficients obtained for the lease term variable are statistically significant in both SMSAs, they have opposite signs, which makes their interpretation trickier. In Chicago, and contrary to expectations, unit rent is positively, and significantly, related to the duration of the contract. Thus, doubling the term would raise the unit rent by 6.3%. Such a finding may seem quite surprising in light of the arguments developed by Kempf (2015, p. 95), Crosby et al. (2003), McAllister (2001), Englund et al. (2004) and Robinson (1999), all of which suggest that a negative relationship should prevail. The coefficient derived from the New York submarket is more in line with theoretical expectations as it is negatively signed. It is also smaller in magnitude, the elasticity coefficient standing at -0.017; that is, doubling the contract duration will lower the unit rent by 1.7%.

Under Model A specification, free months incentives does not seem to have any significant impact on rent levels. On the contrary, and in line with Bond et al. (2008), building class, a proxy for the overall quality and functionality of the building, emerges as a most powerful office rent determinant in both submarkets. In Chicago, Class B and C buildings command a rent discount of 9.6% and 18.5%, respectively, compared with Class A ones. Discounts for New York are substantially larger, standing at 28.8% and 41.1%, respectively. The type of lease also exerts a significant effect on Chicago office rents while it does not in New York. In the former case, and in line with Webb & Fisher (1996), new contracts negotiate at a 10.5% discount relative to renewals. This suggests that, in order to counter a depressed market – which was the case over most of the 2007-2015 period -, landlords tend to attract new tenants by lowering unit rent.

Upon lease renewal though, tenants already in place will generally agree to pay the lessor's asking rent in order to avoid search and moving costs. As for the occupancy status, results suggest that, in Chicago at least (the coefficient is positive, although not significant in New York), leasing directly from the landlord, as opposed to subletting from a tenant, translates into an 11.5% rent premium. Such a finding is consistent with theoretical expectations since subleases often result from tenants' financial distress, from a downward adjustment of space needs or simply from a move to other premises. Quite surprisingly, the state of the premises (renovated or not) does not emerge as a significant rent determinant.

Turning to location dummies, regression coefficients prove to be highly significant for all but one spatial cluster in Chicago as well as for the Downtown area in New York. In Chicago, relative to the reference (Metro Chicago), office rental contracts settle at a discount which ranges from a minimum of 10% (South Chicago, n.s.) to a maximum of 46% (Kenosha County). As for New York, leases in the Downtown area negotiate at an 18.1% discount compared to the Uptown area. Finally, year dummies, a proxy for rent index, clearly show that the 2007-2015 period was, by and large, a depressed one for the office market, although the severity of the rent slump that followed the subprime mortgage crisis of 2008-2009 differs among SMSAs. With regard to the Chicago submarket, 2015 is the only year displaying a significant, and positively signed, coefficient (at the 0.05 level). This suggests that office rents have stalled at their 2007 level for most of the period before bouncing back in 2015 (14.7% above the reference). The market slump was obviously more pronounced in New York, where unit rents dropped substantially from 2009 through 2012. Following a low reached in 2010 (-18.5%), the New York office market quickly recovered thereafter; by 2015, rents stood at 27.5% above their 2007 level.

Model A specification yields an adjusted R-squared which stands at 0.262 for the Chicago submarket as opposed to 0.513 for the New York submarket.

Table 3: Regression results – Model A

Dep. Var: Log(Unit Gross Rent)	**Chicago**	**New York**
Log(Surface)	*-.0692*****	*-0.0043*
Log(Surface)2	-	-
Log(Leaseterm)	*.0625*****	*-.0167****
Log(Leaseterm)2	-	-
Freemonth	-0.0013	0.0039
Class A (ref.)	-	-
Class B	-.1009***	-.3400***
Class C	-.2047***	-.5300***
Newlease (ref. Renewal)	-.1110***	-0.0163
Direct (ref. Subrental)	.1087*	0.0891
Notrenov (ref. Renovated)	-0.0746	0.02
Metro Chicago (ref.)	-	
East/West Corridor	-.3211***	
Indiana	-.4327***	
Kenosha County	-.6180***	
Near West	-.2334***	
North	-.1944***	
North Chicago	-.1741***	
Northwest	-.2962***	
O'Hare	-.2690***	
South Chicago	-0.105	
South Suburban	-.3550***	
Uptown (ref.)		-
Downtown		-.2060***
Midtown		0.0195
Midtown South		0.0607
Year2007 (ref.)	-	-
Year2008	-0.0108	.1510***
Year2009	-0.0119	-.1540***
Year2010	-0.0435	-.2040***
Year2011	-0.0638	-.1320***
Year2012	-0.0308	-.0935**
Year2013	0.0013	-0.0007
Year2014	0.0362	.1470***
Year2015	.1373*	.2430***
Constant	3.5063***	4.0700***
Observations	1,283	2,133
Adjusted R2	**0.262**	**0.513**

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

4.2.3 Regression results – Model B

Regression results from Model B specification are reported in Table 4. Overall model performance is substantially improved for Chicago (adjusted R-squared raises from 0.262 to 0.294) but remains stable for New York (0.515 vs. 0.512 previously). By and large, introducing quadratic terms in the model has a very limited impact on the sign, magnitude and statistical significance of most of the control variables' regression coefficients. Among changes that are worth noting, the negative contribution of rental incentives (Freemonth) for the Chicago submarket turns significant at the 0.05 level, with each additional free month reducing unit gross rent by some 0.6%. Building quality (Class A, B and C) and type of lease (new lease vs. renewal) descriptors experience a slight drop in their marginal contribution while the occupancy status (direct lease vs. subrental) is no more statistically significant in Chicago. Safe for the North Chicago and South Chicago areas whose location discounts are raised, location dummy coefficients are unaffected under Model B specification. Finally, time dummy coefficients are virtually identical in magnitude to those obtained with Model A, safe for Chicago where the enhanced coefficient for the year 2015 (20% above 2007 rent values) suggests that the recovery there was actually even stronger than initially expected.

Findings regarding key variables under investigation (size of rental area and lease term) are quite interesting and partly conclusive. Starting with the surface descriptor, it can be seen that the coefficients of the log-quadratic terms emerge as being statistically significant for both submarkets, although they are more robust for Chicago. In both cases, the signs of the coefficients (negative for the first term and positive for the second) imply that the rent function is not monotonic but rather follows a parabolic U-shape curve, with unit rent first decreasing as the leased surface increases, reaching a minimum at some surface threshold and then rising again beyond this point.

Regression findings also suggest that the slope of the log-quadratic function, given by the first parameter, is much steeper for the Chicago submarket than it is for New York's (Figures 1 and 2). In the former case, the critical distance (S*) is reached at 13,200 square feet, with the corresponding minimal rent (LCR) standing at roughly $16 per square feet while, in the latter case, the minimal rent stands at $37 per square feet for a critical distance of slightly below 6,000 square feet. The relatively low threshold found for New York stems from the fact that relatively few very large surfaces have been leased over the period under analysis, most leases referring to medium and small-sized premises. Moreover, detailed descriptive statistics (Annex 1) clearly show that, in both submarkets, very large surfaces refer to renewals in central locations (Metro Chicago and Downtown New York) and in Class A buildings.

Turning to the lease term dimension, findings from Model B suggest that the relationship between unit rent and contract duration does not follow a log-quadratic form as is the case with surface.

Regression coefficients for Chicago and New York yield opposite signs while none emerges as being statistically significant at the 0.05 level. Consequently, Model A specification still holds for this descriptor. According to the latter, longer lease terms drive office unit rents up in Chicago, and down in New York.

Table 4: Regression results – Model B

Dep. Var: Log(Unit Gross Rent)	**Chicago**	**New York**
Log(Surface)	*-.4801****	*-.1160***
Log(Surface)2	*.0253****	*.0067**
Log(Leaseterm)	*-0.0396*	*0.0415*
Log(Leaseterm)2	*0.0162*	-0.0068
Freemonth	-.0062*	0.0021
Class A (ref.)	-	-
Class B	-.0854***	-.3390***
Class C	-.1885***	-.5290***
Newlease (ref. Renewal)	-.0989***	-0.0133
Direct (ref. Subrental)	0.0972	0.084
Notrenov (ref. Renovated)	-0.0718	0.0212
Metro Chicago (ref.)	-	
East/West Corridor	-.3220***	
Indiana	-.4319***	
Kenosha County	-.5989***	
Near West	-.2348***	
North	-.1954***	
North Chicago	-.2036***	
Northwest	-.3007***	
O'Hare	-.2634***	
South Chicago	-.1243*	
South Suburban	-.3633***	
Uptown (ref.)		-
Downtown		-.2080***
Midtown		0.0207
Midtown South		0.0616
Year2007 (ref.)	-	-
Year2008	0.0242	.1500***
Year2009	0.0112	-.1560***
Year2010	-0.0075	-.1980***
Year2011	-0.0218	-.1280***
Year2012	0.0101	-.0885**
Year2013	0.0437	0.001
Year2014	0.0692	.1500***
Year2015	.1800***	.2460***
Constant	5.2495***	4.4300***
Observations	1,283	2,133
Adjusted R2	**0.294**	**0.515**

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

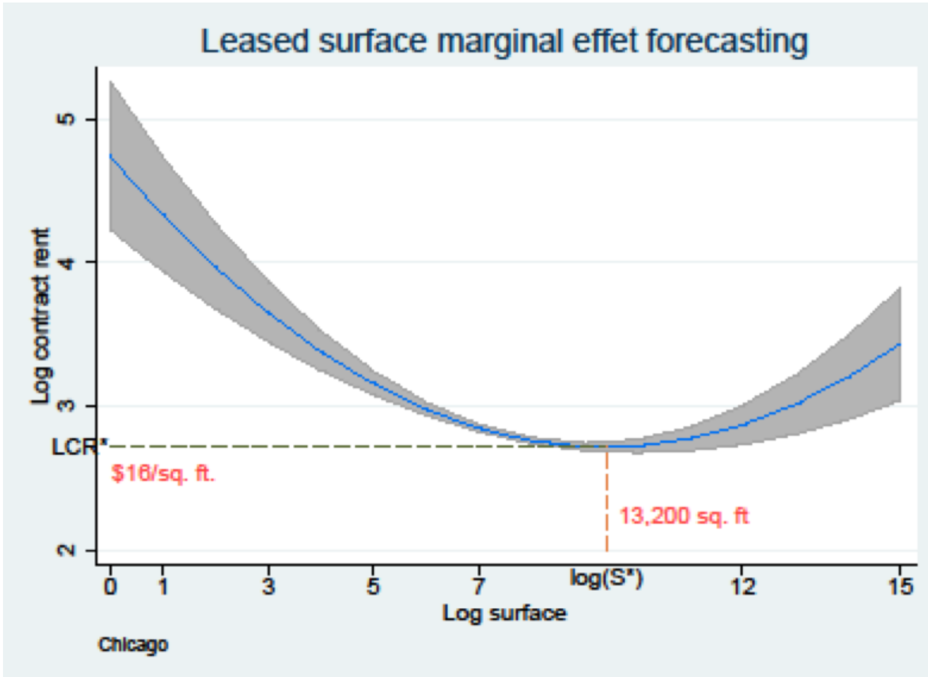

Figure 1: Log-Quadratic function on Surface – Chicago submarket

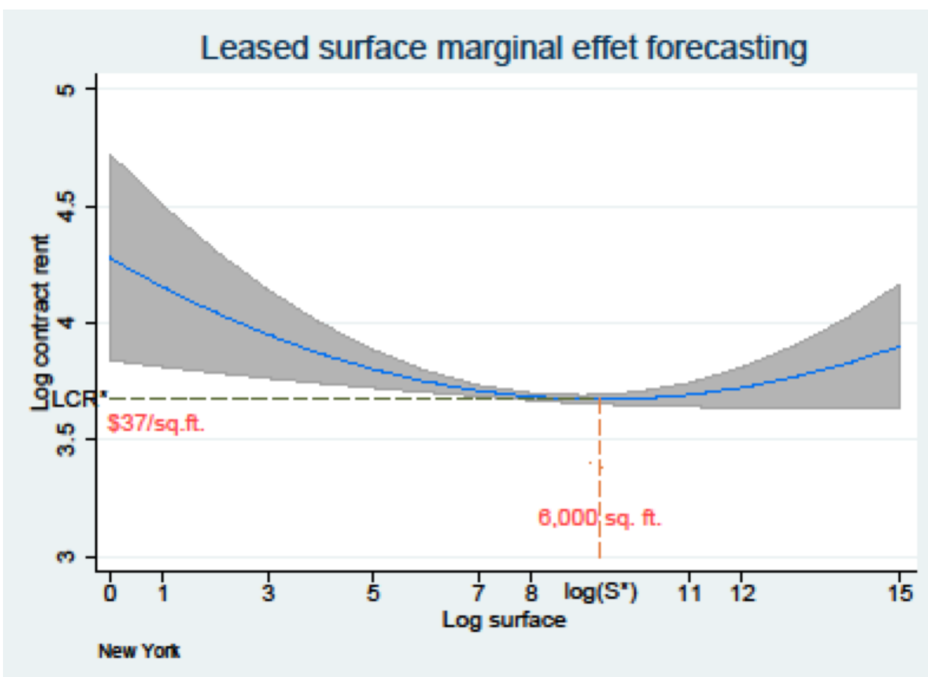

Figure 2: Log-Quadratic function on Surface – New York submarket

5 Discussion and conclusion

While still preliminary, findings accruing from this research provide clues to a better understanding of the relationship between office unit rent on the one hand, and, on the other hand, leased surface and contract duration. With regard to leased surface, this paper addresses the issue of the inconclusive, and apparently contradictory, results found in the literature as to whether the size of rentable area influences rents upwards or downwards Kempf (2015, p.99), depending on the market analyzed. Our findings suggest that the sign of the surface variable coefficient ultimately depends upon the structure of the local market and on whether the supply of, and the demand for, office space at a given location, for a given economic context and for specific clienteles is, or not, in equilibrium. In this regard, (Kempf, 2015, p.99) provides a reasonable explanation to our findings. To summarize, in a bearish market, landlords will cope with increasing vacancy rates by providing large tenants with rent discounts. On the other hand, the relatively scarce supply of office buildings offering large, continuous space will drive large firms to pay a premium for such premises. In other words, the influence of rentable area on rents is dependent upon the cycle stage of the local office market (Wheaton & Torto, 1995).

As for lease term, its positive – and highly significant - influence on rents in Chicago, while at odds with the dominant view, corroborates studies by Gunnelin & Söderberg (2003) and Englund et al. (2004) on the Swedish office market as well as that by Bond et al. (2008) on the London market, all of which point towards a positive relationship. In their study on the office market of England and Wales, Crosby et al. (2006), following McCann & Ward (2004), argue that the influence of lease term on rent is also affected by the size, hence the type, of the tenant organization: while large corporate or institutional tenants tend to look for long-term stability that minimizes moving cost, smaller companies are more concerned by flexibility in order to allow for future expansion, thereby avoiding lock-in effect of long leases (Grenadier, 1995).

In this paper, it is assumed that the above argument provides an adequate explanation to the regression results obtained. Quite clearly, different profiles of tenants will command different needs and preferences with respect to both leased surface and lease length. While several determinants are at stake in the office rent setting process, further research is needed to investigate the role the latter exert on rents. Our study is among the few that explicitly includes contract duration as an independent variable in the modelling process, while controlling for most of the dimensions accounted for in the literature (rental incentives, building and premises quality, lease type, location, and cyclical dimension). Yet, at this point, neither the identity, or type of tenant, nor the sector of economic activity of office occupiers – yet another potentially significant rent determinant - is available through the CoStar database, which is clearly a limitation of this paper. Further research should address this issue.

References

Agliardi, R. (2006). Options to expand: some remarks. *Finance Research Letters*, *3*(1), 65–72.

Amédée-Manesme, C.-O., Barthélémy, F., Baroni, M., & Dupuy, E. (2013). Combining Monte Carlo simulations and options to manage the risk of real estate portfolios. *Journal of Property Investment & Finance*, *31*(4), 360–389.

Archer, W. R., & Smith, M. T. (2003). Explaining location patterns of suburban offices. *Real Estate Economics*, *31*(2), 139–164.

Benjamin, J. D., Boyle, G. W., & Sirmans, C. (1992). Price discrimination in shopping center leases. *Journal of Urban Economics*, *32*(3), 299–317.

Bollinger, C. R., Ihlanfeldt, K. R., & Bowes, D. R. (1998). Spatial variation in office rents within the atlanta region. *Urban Studies*, *35*(7), 1097–1118.

Bond, S. A., Loizou, P., & McAllister, P. (2008). Lease maturity and initial rent: is there a term structure for uk commercial property leases? *The Journal of Real Estate Finance and Economics*, *36*(4), 451–469.

Buetow, G. W., & Albert, J. D. (1998). The pricing of embedded options in real estate lease contracts. *Journal of Real Estate Research*, *15*(3), 253–266.

Clapham, E. (2003). A note on embedded lease options. *Journal of Real Estate Research*, *25*(3), 347–360.

Clapp, J. M. (1980). The intrametropolitan location of office activities. *Journal of Regional Science*, *20*(3), 387–399.

Crosby, N., Gibson, V., & Murdoch, S. (2003). Uk commercial property lease structures: Landlord and tenant mismatch. *Urban Studies*, *40*(8), 1487–1516.

Crosby, N., Hughes, C., & Murdoch, S. (2006). Flexible property leasing and the small business tenant. *Journal of Property Research*, *23*(2), 163–188.

D'Arcy, E., McGough, T., & Tsolacos, S. (1997). National economic trends, market size and city growth effects on european office rents. *Journal of Property Research*, *14*(4), 297–308.

Desyllas, J. (2000). *The relationship between urban street configuration and office rent patterns in berlin* (Unpublished doctoral dissertation). University of London.

Dunse, N., & Jones, C. (1998). A hedonic price model of office rents. *Journal of property valuation and investment*, *16*(3), 297–312.

Dunse, N., Leishman, C., & Watkins, C. (2002). Testing for the existence of office sub-markets: A comparison of evidence from two cities. *Urban studies*, *39*(3), 483–506.

Englund, P., Gunnelin, Å., Hoesli, M., & Söderberg, B. (2004). Implicit forward rents as predictors of future rents. *Real Estate Economics*, *32*(2), 183–215.

Gamba, A., & Rigon, R. (2008). The value of embedded real options: Evidence from consumer automobile lease contracts - a note. *Finance Research Letters*, *5*(4), 213–220.

Gat, D. (1998). Urban focal points and design quality influence rents: The case of the tel aviv office market. *Journal of Real Estate Research*, *16*(2), 229–247.

Giaccotto, C., Goldberg, G. M., & Hegde, S. P. (2007). The value of embedded real options: Evidence from consumer automobile lease contracts. *The Journal of Finance*, *62*(1), 411–445.

Glascock, J. L., Jahanian, S., & Sirmans, C. F. (1990). An analysis of office market rents: some empirical evidence. *Real Estate Economics*, *18*(1), 105–119.

Glascock, J. L., Sirmans, C., & Turnbull, G. K. (1993). Owner tenancy as credible commitment under uncertainty. *Real Estate Economics*, *21*(1), 69–82.

Grenadier, S. R. (1995). Valuing lease contracts a real-options approach. *Journal of Financial Economics*, *38*(3), 297–331.

Gunnelin, Å., & Söderberg, B. (2003). Term structures in the office rental market in stockholm. *The Journal of Real Estate Finance and Economics*, *26*(2), 241–265.

Kempf, S. (2015). *Development of hedonic office rent indices: examples for german metropolitan areas.* Springer.

McAllister, P. (2001). Pricing short leases and break clauses using simulation methodology. *Journal of Property Investment & Finance*, *19*(4), 361–374.

McCann, P., & Ward, C. (2004). Real estate rental payments: application of stock-inventory modeling. *The Journal of Real Estate Finance and Economics*, *28*(2), 273–292.

Miceli, T. J., & Sirmans, C. (1999). Tenant turnover, rental contracts, and self-selection. *Journal of Housing Economics*, *8*(4), 301–311.

Mills, E. S. (1992). Office rent determinants in the chicago area. *Real Estate Economics*, *20*(2), 273–287.

Mooradian, R. M., & Yang, S. X. (2000). Cancellation strategies in commercial real estate leasing. *Real Estate Economics*, *28*(1), 65–88.

Nitsch, H. (2006). Pricing location: a case study of the munich office market. *Journal of Property Research*, *23*(2), 93–107.

Robinson, J. (1999). Commercial lease terms and the property cycle. *Facilities*, *17*(5/6), 177–182.

Slade, B. (2000). Office rent determinants during market decline and recovery. *Journal of Real Estate Research*, *20*(3), 357–380.

Stanton, R., & Wallace, N. (2009). An empirical test of a contingent claims lease valuation model. *Journal of Real Estate Research*, *31*(1), 1–26.

Webb, R. B., & Fisher, J. D. (1996). Development of an effective rent (lease) index for the chicago cbd. *Journal of urban Economics*, *39*(1), 1–19.

Wheaton, W. C., & Torto, R. G. (1995). *Retail sales and retail real estate*. Massachusetts Institute of Technology, Center for Real Estate.

A Derivation of Ω^i

Suppose that the market rent at time $t+j$ is equal to R_{t+j}, let $N = T/\Delta t$ denote the number of time steps during a time period T, and let N_i be such that $u^{N_i} d^{N-N_i} R_{t+j} \leq e^{iT} R_{t+j}$ and $u^{N_i+1} d^{N-N_i-1} R_{t+j} > e^{iT} R_{t+j}$. Then ($p_d = 1 - p_u$)

$$
\begin{aligned}
& E_{t+j}\left[R_{t+j+T} \middle| R_{t+j+T} \leq e^{iT} R_{t+j} \right] \\
&= \frac{\binom{N}{N_i} p_u^{N_i} p_d^{N-N_i} u^{N_i} d^{N-N_i} R_{t+j} + \binom{N}{N_i-1} p_u^{N_i-1} p_d^{N-N_i+1} u^{N_i-1} d^{N-N_i+1} R_{t+j} + \ldots + \binom{N}{0} p_d^N d^N R_{t+j}}{\Pr\left(R_{t+j+T} \leq R_{t+j}\right)} \\
&= \frac{\binom{N}{N_i} p_u^{N_i} p_d^{N-N_i} u^{N_i} d^{N-N_i} + \binom{N}{N_i-1} p_u^{N_i-1} p_d^{N-N_i+1} u^{N_i-1} d^{N-N_i+1} + \ldots + \binom{N}{0} p_d^N d^N}{\binom{N}{N_i} p_u^{N_i} p_d^{N_i-1} + \binom{N}{N_i-1} p_u^{N_i-1} p_d^{N-N_i+1} + \ldots + \binom{N}{0} p_d^N} R_{t+j} \\
&= \Omega_d^i R_{t+j}.
\end{aligned}
$$

Since $d = 1/u < 1$, $\Omega_d^i < e^{iT}$. Note that Ω_d^i does not depend on j. Similarly, we have

$$
E_{t+j}\left[R_{t+j+T} \middle| R_{t+j+T} > e^{iT} R_{t+j} \right] = \Omega_u^i R_{t+j},
$$

where $\Omega_u^i > e^{iT}$. Note further that

$$
E\left[R_{t+j+T}\right] = \Pr\left(R_{t+j+T} \leq e^{iT} R_{t+j}\right) \Omega_d^i R_{t+j} + \Pr\left(R_{t+j+T} > e^{iT} R_{t+j}\right) \Omega_u^i R_{t+j} = e^{\mu T} R_{t+j}
$$

and thus

$$
\Pr\left(R_{t+j+T} \leq e^{iT} R_{t+j}\right) \Omega_d^i + \Pr\left(R_{t+j+T} > e^{iT} R_{t+j}\right) \Omega_u^i = e^{\mu T}.
$$

Note also that since L_t^T is a linear function of R_t, $\Omega_d^i L_{t+j}^T = E\left[L_{t+j+T}^T \middle| R_{t+j+T} \leq e^{iT} R_{t+j} \right]$.

B Proof of Proposition 1

Proof. This section provides the proof of Proposition 1. In equilibrium, (3) and (7) must be equal and thus, writing (4) with $W = 2T$, replacing V_t^{2T} by $\frac{\left(1-e^{2(i-k)T}\right)L_t^{2T}}{1-e^{(i-k)\Delta t}}$, and replacing V_t^{∞} by $\frac{R_t}{1-e^{(\mu-k)\Delta t}}$, we have

$$
\begin{aligned}
\frac{\left(1-e^{2(i-k)T}\right)L_t^{2T}}{1-e^{(i-k)\Delta t}} &= \frac{\left(1-e^{2(i-k)T}\right)\tilde{L}_t^{T}}{1-e^{(i-k)\Delta t}} \\
&+ \left((1-\alpha)(1-\theta_i)\left(\Omega_u^i - e^{iT}\right) - \theta_i e^{iT}\right)\frac{1-e^{(i-k)T}}{1-e^{(i-k)\Delta t}}e^{-kT}\tilde{L}_t^{T} \\
&+ \theta_i \Omega_d^i e^{-kT}\frac{\left(1-e^{(\mu-k)T}\right)R_t}{1-e^{(\mu-k)\Delta t}} \\
&= \frac{\left(1-e^{2(i-k)T}\right)\tilde{L}_t^{T}}{1-e^{(i-k)\Delta t}} \\
&+ \left((1-\alpha)(1-\theta)\left(\Omega_u^i - e^{iT}\right) - \theta_i e^{iT}\right)\frac{1-e^{(i-k)T}}{1-e^{(i-k)\Delta t}}e^{-kT}\tilde{L}_t^{T} \\
&+ \theta_i \Omega_d^i e^{-kT}\frac{\left(1-e^{(i-k)T}\right)L_t^{T}}{1-e^{(i-k)\Delta t}}.
\end{aligned}
$$

Rearranging the last equation and developping $1-e^{2(i-k)T} = \left(1-e^{(i-k)T}\right)\left(1+e^{(i-k)T}\right)$, we obtain

$$
\begin{aligned}
\tilde{L}_t^{T} &= \left(1+\frac{(1-\alpha)(1-\theta_i)(\Omega_u^i - e^{iT})e^{-kT}}{1+e^{(i-k)T}}\right)^{-1}\left(L_t^{2T} + \frac{\theta_i e^{-kT}}{1+e^{(i-k)T}}\left(e^{iT}\tilde{L}_t^{T} - \Omega_d^i L_t^{T}\right)\right) \\
&= \left(1+\frac{(1-\alpha)(1-\theta_i)(\Omega_u^i - e^{iT})}{e^{kT}+e^{iT}}\right)^{-1}\left(L_t^{2T} + \frac{\theta_i}{e^{kT}+e^{iT}}\left(e^{iT}\tilde{L}_t^{T} - \Omega_d^i L_t^{T}\right)\right) \\
&= \left(1+\frac{(1-\alpha)(1-\theta_i)(\Omega_u^i - e^{iT})}{e^{kT}+e^{iT}}\right)^{-1}\left(L_t^{2T} + \frac{\Pr(R_{t+T}\le e^{iT}R_t)}{e^{kT}+e^{iT}}\left(e^{iT}\tilde{L}_t^{T} - E_t\left[L_{t+T}^{T}\middle| R_{t+T}\le e^{iT}R_t\right]\right)\right)
\end{aligned}
\tag{10}
$$

Define

$$
P_t^i = \frac{\Pr(R_{t+T}\le e^{iT}R_t)}{e^{kT}+e^{iT}}\left(e^{iT}\tilde{L}_t^{T} - E_t\left[L_{t+T}^{T}\middle| R_{t+T}\le e^{iT}R_t\right]\right). \tag{11}
$$

When $\alpha = 1$, i.e. when the tenant has the maximum negociation power when renewing the lease, then $P_t^i > 0$ represents the premium paid each period under the T-year renewable contract over the fixed $2T$-year rent. P_t^i corresponds to the option-adjusted lease spread between the T-year renewable contract and the fixed $2T$-year contract. In the above equations, L_t^T corresponds to the monthly rent paid, indexed at the rate i, on a T-year lease.

When $\alpha = 1$, $\tilde{L}_t^T > L_t^{2T}$, i.e. the short-term renewable lease is strictly greater than the $2T$-year lease. When $\alpha = 0$, then

$$
\begin{aligned}
\tilde{L}_t^T &= \left(1 + \frac{(1-\theta_i)(\Omega_u^i - e^{iT})}{e^{kT} + e^{iT}}\right)^{-1} \left(L_t^{2T} + \frac{\theta_i}{e^{kT} + e^{iT}} \left(e^{iT} \tilde{L}_t^T - \Omega_d^i L_t^T\right)\right) \\
&= \left(\frac{e^{kT} + e^{iT} + (1-\theta_i)\Omega_u^i - e^{iT} + \theta_i e^{iT}}{e^{kT} + e^{iT}}\right)^{-1} \left(L_t^{2T} + \frac{\theta_i}{e^{kT} + e^{iT}} \left(e^{iT} \tilde{L}_t^T - \Omega_d^i L_t^T\right)\right) \\
&= \left(\frac{e^{kT} + e^{\mu T} - \theta_i \Omega_d^i + \theta_i e^{iT}}{e^{kT} + e^{iT}}\right)^{-1} \left(L_t^{2T} + \frac{\theta_i}{e^{kT} + e^{iT}} \left(e^{iT} \tilde{L}_t^T - \Omega_d^i L_t^T\right)\right) \\
&= \frac{e^{kT} + e^{iT}}{e^{kT} + e^{\mu T} + \theta_i(e^{iT} - \Omega_d^i)} \left(L_t^{2T} + \frac{\theta_i}{e^{kT} + e^{iT}} \left(e^{iT} \tilde{L}_t^T - \Omega_d^i L_t^T\right)\right) \\
&= \frac{\theta_i e^{iT} \tilde{L}_t^T}{e^{kT} + e^{\mu T} + \theta_i(e^{iT} - \Omega_d^i)} + \frac{e^{kT} + e^{iT}}{e^{kT} + e^{\mu T} + \theta_i(e^{iT} - \Omega_d^i)} \left(L_t^{2T} - \frac{\theta_i \Omega_d^i L_t^T}{e^{kT} + e^{iT}}\right)
\end{aligned}
$$

which gives

$$
\begin{aligned}
\tilde{L}_t^T &= \frac{e^{kT} + e^{iT}}{e^{kT} + e^{\mu T} - \theta_i \Omega_d^i} \left(L_t^{2T} - \frac{\theta_i \Omega_d^i L_t^T}{e^{kT} + e^{iT}}\right) \\
&= \frac{e^{kT} + e^{iT}}{e^{kT} + (1-\theta_i)\Omega_u^i} \left(L_t^{2T} - \frac{\theta_i \Omega_d^i L_t^T}{e^{kT} + e^{iT}}\right) \\
&= \frac{e^{kT} + e^{iT}}{e^{kT} + (1-\theta_i)\Omega_u^i}\ L_t^{2T} - \frac{\theta_i \Omega_d^i}{e^{kT} + (1-\theta_i)\Omega_u^i}\ L_t^T \\
&= \frac{e^{kT} + e^{iT}}{e^{kT} + (1-\theta_i)\Omega_u^i}\ L_t^{2T} - \frac{\theta_i \Omega_d^i}{e^{kT} + (1-\theta_i)\Omega_u^i}\ \frac{1 + e^{(i-k)T}}{1 + e^{(\mu-k)T}} L_t^{2T} \\
&= \frac{e^{kT}(1 + e^{(i-k)T})}{e^{kT} + (1-\theta_i)\Omega_u^i}\ L_t^{2T} - \frac{\theta_i \Omega_d^i}{e^{kT} + (1-\theta_i)\Omega_u^i}\ \frac{1 + e^{(i-k)T}}{1 + e^{(\mu-k)T}} L_t^{2T} \\
&= \frac{1 + e^{(i-k)T}}{e^{kT} + (1-\theta_i)\Omega_u^i} \left(e^{kT} - \frac{\theta_i \Omega_d^i}{1 + e^{(\mu-k)T}}\right) L_t^{2T} \\
&= \frac{1 + e^{(i-k)T}}{e^{kT} + (1-\theta_i)\Omega_u^i} \left(\frac{e^{kT} + e^{\mu T} - \theta_i \Omega_d^i}{1 + e^{(\mu-k)T}}\right) L_t^{2T} \\
&= \frac{1 + e^{(i-k)T}}{e^{kT} + (1-\theta_i)\Omega_u^i} \left(\frac{e^{kT} + (1-\theta_i)\Omega_u^i}{1 + e^{(\mu-k)T}}\right) L_t^{2T} \\
&= \frac{1 + e^{(i-k)T}}{1 + e^{(\mu-k)T}}\ L_t^{2T},
\end{aligned}
$$

and thus $\tilde{L}_t^T < L_t^{2T}$ when $i < \mu$, which we assumed at the beginning. □

C Descriptive Statistics

Table 5: Selected Detailed Descriptive Statistics for Chicago, 2007-2015

	Mean			
	Unit Gross Rent	**Surface (Sq. Ft)**	**Lease Term (Months)**	**Freemonths**
Lease Type				
New lease	$17.23	5,661.11	52.46	2.58
Renewal	$18.10	15,326.45	55.7	2.58
Total	**$17.32**	**6,640.45**	**52.79**	**2.58**
Spatial Cluster				
East/West Corridor	$15.22	5,920.28	50.13	2.64
Indiana	$12.90	3,323.68	46.24	1.11
Kenosha County	$10.21	2,103.83	36	1.33
Metro Chicago	$20.61	10,415.39	66.58	3.7
Near West	$16.53	3,735.94	41.06	1.35
North	$17.37	3,366.03	44.09	1.91
North Chicago	$17.34	3,383.11	34.87	1.38
Northwest	$15.59	6,065.52	49.25	2.2
O'Hare	$16.18	7,290.89	63.25	3.33
South Chicago	$19.19	5,293.66	50.55	0.86
South Suburban	$14.33	4,585.97	40.47	1.61
Total	**$17.32**	**6,640.45**	**52.79**	**2.58**
Bulding Class				
Class A	$19.69	15,991.89	74.2	4.71
Class B	$17.01	4,720.56	49.95	2.18
Class C	$16.06	3,019.57	41.42	1.7
Total	**$17.32**	**6,640.45**	**52.79**	**2.58**
Occupancy Status Status				
Direct	$17.18	6,488.65	53.39	2.59
Sublease	$19.66	9,156.64	42.74	2.42
Total	**$17.32**	**6,640.45**	**52.79**	**2.58**
Quarter				
Q1	$17.59	6,306.25	54.19	2.65
Q2	$16.64	5,296.04	50.85	2.17
Q3	$17.26	6,871.29	52.12	2.61
Q4	$17.78	7,926.65	53.97	2.86
Total	**$17.32**	**6,640.45**	**52.79**	**2.58**

Table 6: Selected Detailed Descriptive Statistics for New York, 2007-2015

	Mean			
	Unit Gross Rent	**Surface (Sq. Ft)**	**Lease Term (Months)**	**Freemonths**
Lease Type				
new lease	$42.57	7,717.77	242.2	2.4
renewal	$47.23	31,085.19	199.23	2.56
Total	**$43.01**	**9,908.81**	**238.17**	**2.41**
Spatial Cluster				
Downtown	$37.14	26,902.07	206.26	2.53
Midtown	$46.94	8,036.31	239.42	2.64
Midtown South	$38.51	6,803.05	248.1	1.97
Uptown	$38.70	5,506.36	244.75	2.11
Total	**$43.01**	**9,908.81**	**238.17**	**2.41**
Bulding Class				
Class A	$58.84	22,885.89	197.1	3.86
Class B	$39.87	6,176.35	239.06	2.09
Class C	$33.58	3,588.99	271.38	1.6
Total	**$43.01**	**9,908.81**	**238.17**	**2.41**
Occupancy Status				
Direct	$43.12	9,909.71	237	2.44
Sublease	$42.04	9,900.93	248.44	2.18
Total	**$43.01**	**9,908.81**	**238.17**	**2.41**
Quarter				
Q1	$42.62	9,175.19	237.89	2.67
Q2	$42.89	9,788.92	232.57	2.32
Q3	$43.46	7,870.96	241.51	2.28
Q4	$43.01	12,746.90	241.06	2.43
Total	**$43.01**	**9,908.81**	**238.17**	**2.41**